

An overview on the SMIR Project: Structural Music Information Research

Moreno Andreatta
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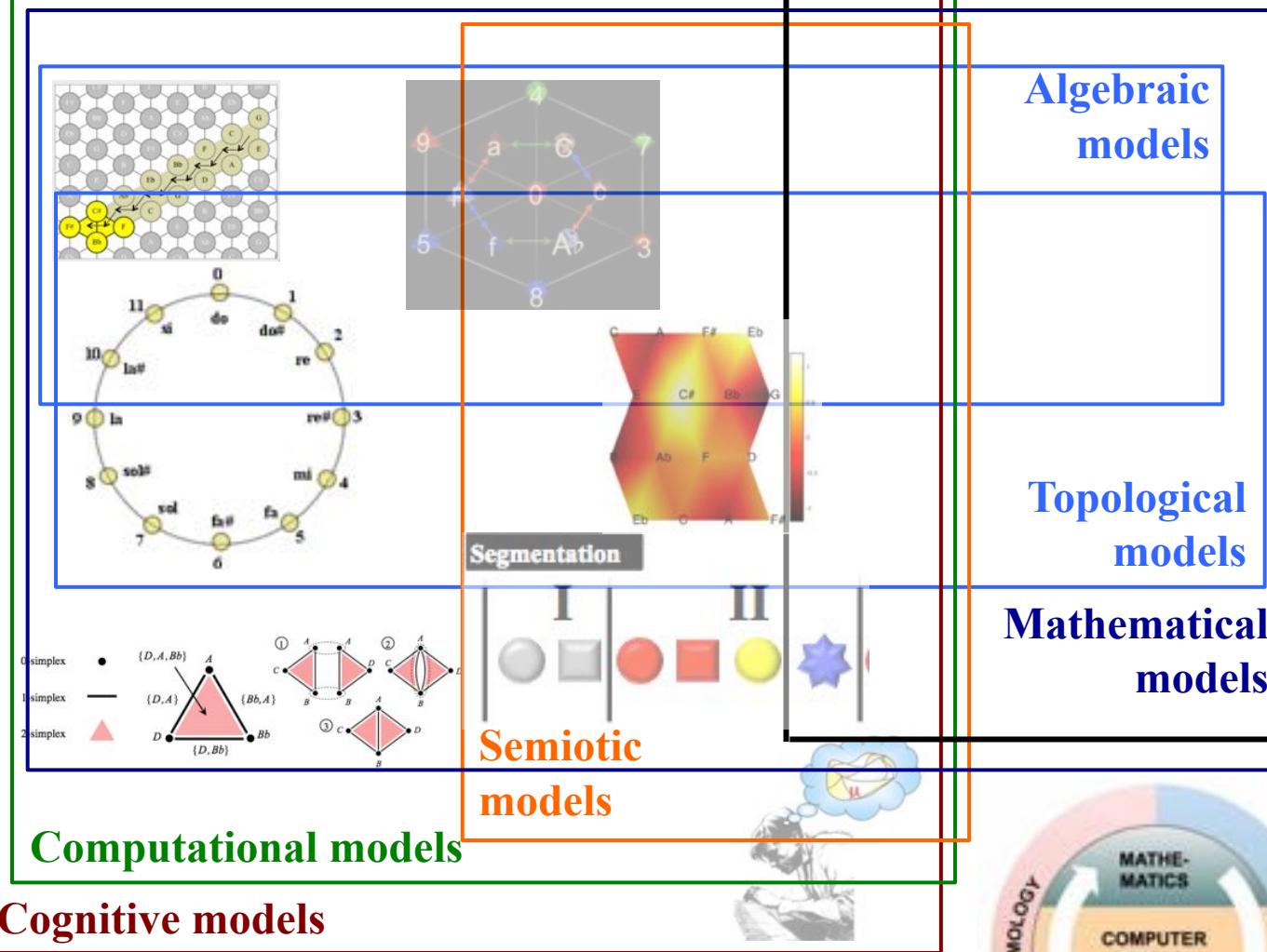
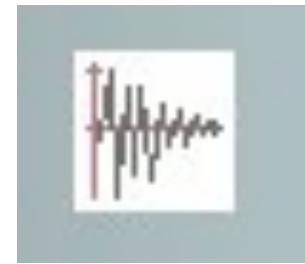
<http://repmus.ircam.fr/moreno/smir>

Programme

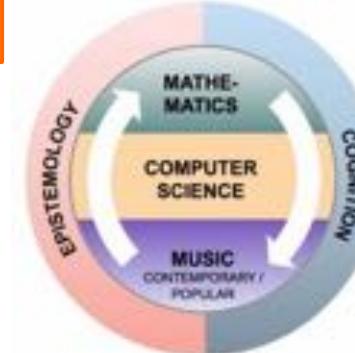
- **Mardi 10 octobre**
 - 9h30-10h30 accueil avec présentation du projet SMIR (Moreno)
 - 10h30-11h45 techniques d'homologie persistante pour l'analyse musicale (Davide Stefani et Pierre Guillot)
- Discussion
- Après-midi : rencontre avec Sonia Cannas (Tonnetz 3D et cycles hamiltoniens) et Pierre Relano (treillis et morphologie mathématique en analyse musicale)
- **Mercredi 11 octobre matin**
 - 9h30-11h00 Rencontre avec Jose-Louis Besada (post-doc GREAM/IRMA) et Nathalie Herold (post-doc GREAM) - conceptual blending et processus compositionnels (aspects épistémologiques et cognitives), systèmes complexes et musique, ...
 - 11h00-12h00 Théorie des catégories en analyse musicale (K-nets et théories transformationnelles) - Moreno
- Petite Discussion

The SMIR Project: Structural Music Information Research

Signal-based
Music Information
Retrieval



Structural Symbolic Music
Information Research



The 3+1 main research axes

- **Generalized Tonnetze, Persistent Homology and automatic classification of musical styles**
 - Sonia Cannas (PhD candidate)
 - Davide Stefani (PhD candidate)
 - Pierre Guillot (Researcher, IRMA)
- **Mathematical Morphology, Formal Concept Analysis and computational musicology**
 - Pierre Relaño (former Master student & composer)
 - Isabelle Bloch (Télécom ParisTech)
 - Jamal Atif (University of Dauphine)
- **Category theory and transformational (computer-aided) music analysis**
 - Andrée Ehresmann (mathematician)
 - Alexandre Popoff (mathematical music theorist)
 - Tom Fiore (University of Michigan, editor or JMM)
 - Thomas Noll (ESMuC, Barcelona)
- **Epistemology and cognitive musicology**
 - José-Luis Besada (post-doc GREAM/IRMA)
 - Nathalie Herold (post-doc GREAM)

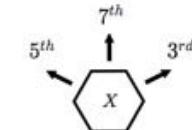
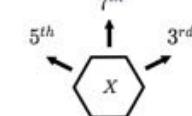
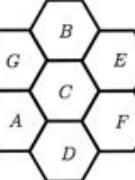
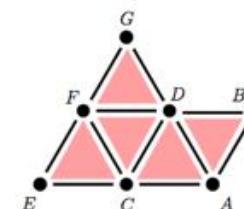
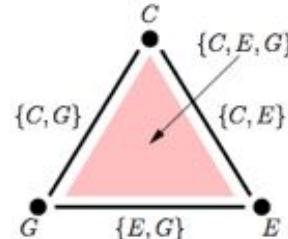
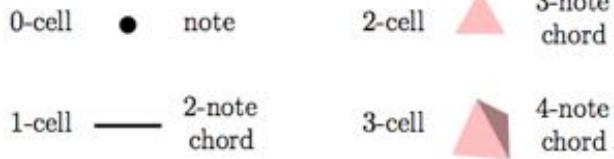
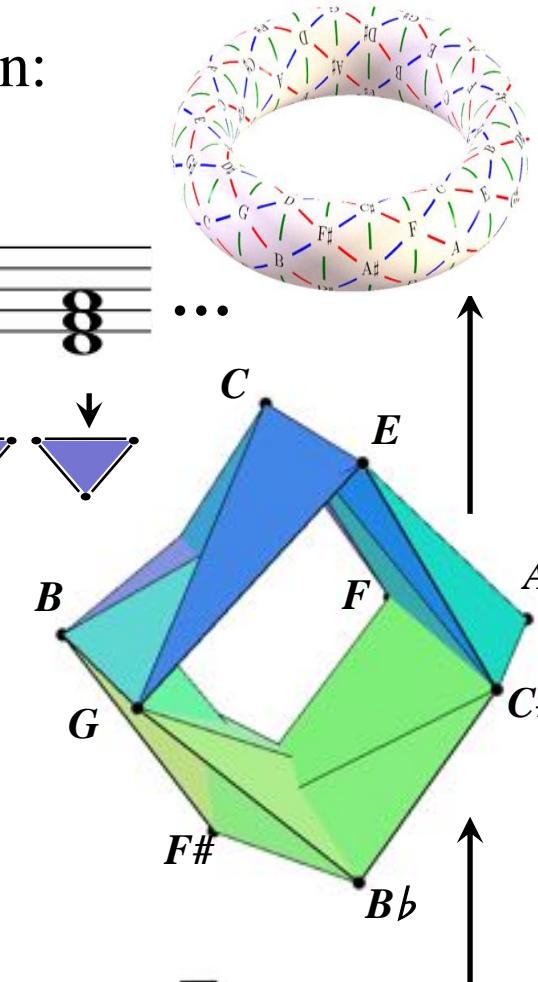
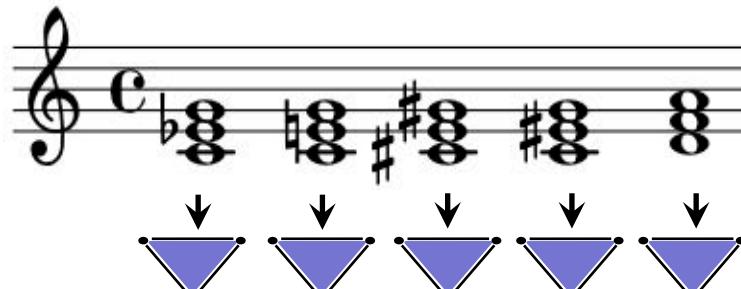
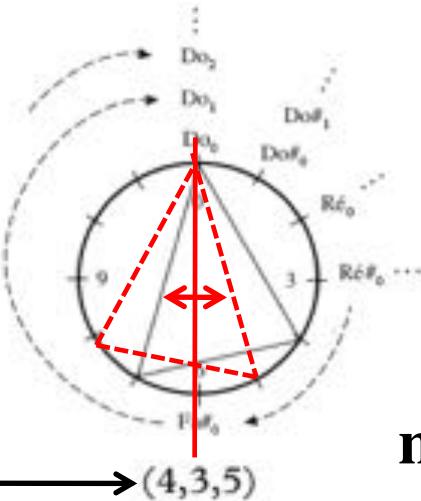


The Tonnetz as a simplicial complex

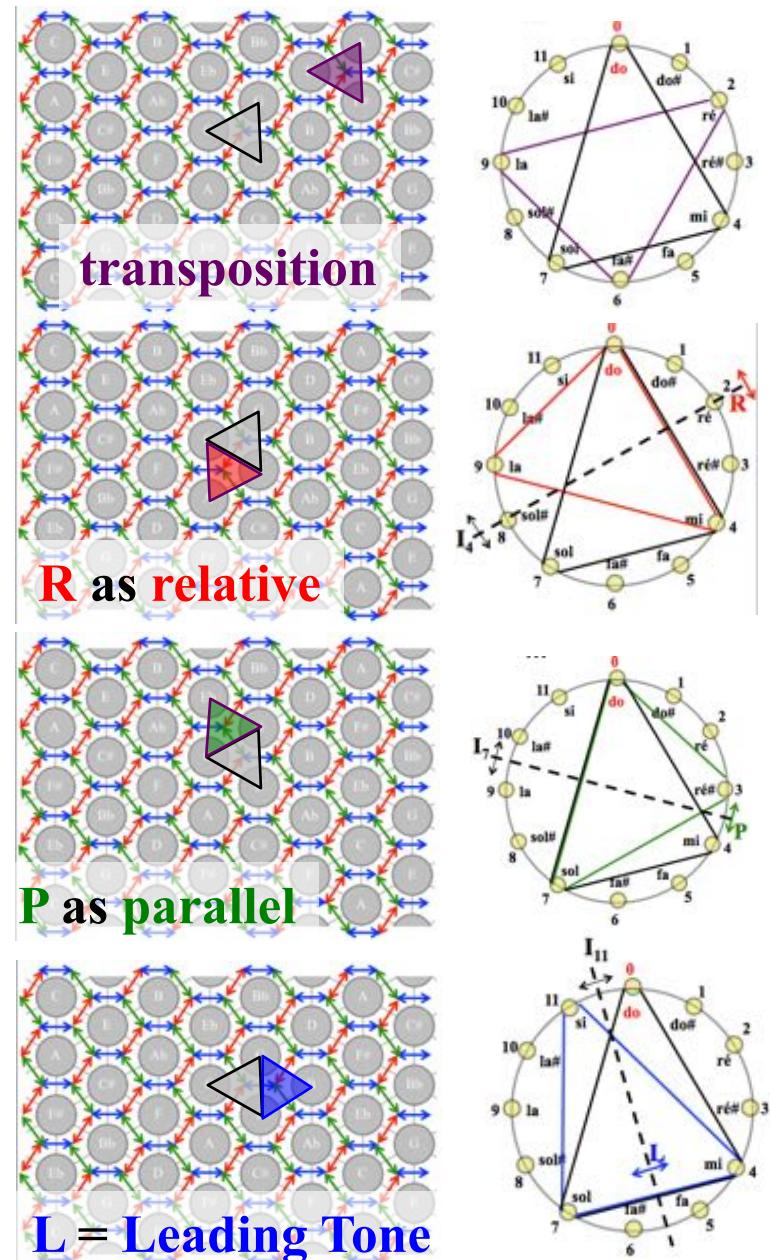
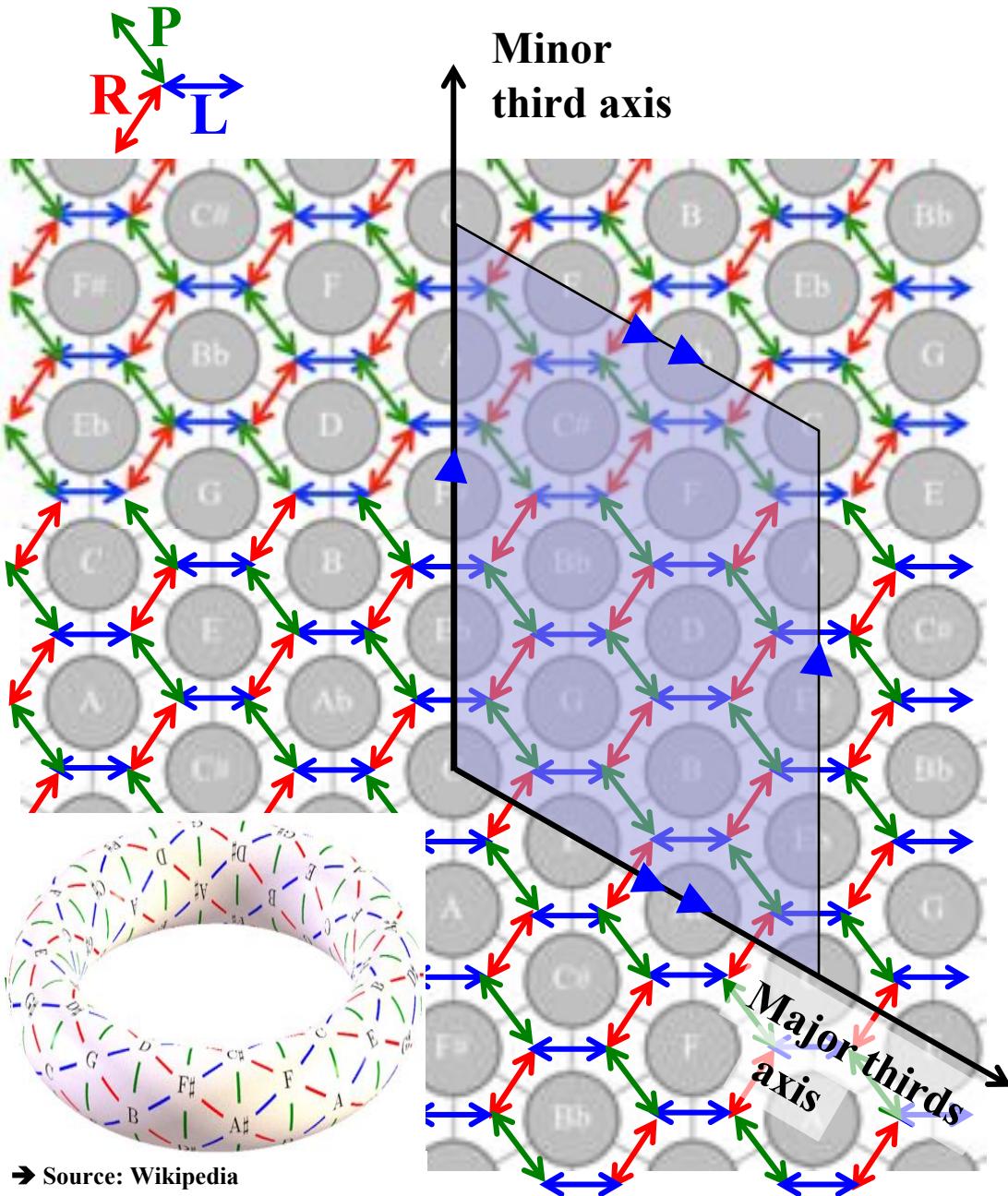
L. Bigo, *Représentation symboliques musicales et calcul spatial*, PhD, Ircam / LACL, 2013

Louis Bigo

- Assembling chords related by some equivalence relation
 - Equivalence up to transposition/inversion:



The Tonnetz, its symmetries and its topological structure



The collection of 28 « redundant » Hamiltonian Cycles

1. C-Cm-Ab-Abm-E-C#m-A-Am-F-Fm-C#-Bbm-F#-F#m-D-Dm-Bb-Gm-Eb-Ebm-B-Bm-G-Em--PLPLRL
2. C-Cm-Ab-Fm-C#-C#m-A-Am-F-Dm-Bb-Bbm-F#-F#m-D-Bm-G-Gm-Eb-Ebm-B-Abm-E-Em--PLRLPL
3. C-Cm-Eb-Ebm-F#-F#m-A-C#m-E-Em-G-Gm-Bb-Bbm-C#-Fm-Ab-Abm-B-Bm-D-Dm-F-Am--PRPRPRLR
4. C-Cm-Eb-Ebm-F#-Bbm-C#-C#m-E-Em-G-Gm-Bb-Dm-F-Fm-Ab-Abm-B-Bm-D-F#m-A-Am--PRPRLRPR
5. C-Cm-Eb-Ebm-F#-Bbm-C#-Fm-Ab-Abm-B-Bm-D-F#m-A-C#m-E-Em-G-Gm-Bb-Dm-F-Am--PRPRLRLR
6. C-Cm-Eb-Gm-Bb-Bbm-C#-C#m-E-Em-G-Bm-D-Dm-F-Fm-Ab-Abm-B-Ebm-F#-F#m-A-Am--PRLRPRPR
7. C-Cm-Eb-Gm-Bb-Bbm-C#-Fm-Ab-Abm-B-Ebm-F#-F#m-A-C#m-E-Em-G-Bm-D-Dm-F-Am--PRLRLR
8. C-Cm-Eb-Gm-Bb-Dm-F-Fm-Ab-Abm-B-Ebm-F#-Bbm-C#-C#m-E-Em-G-Bm-D-F#m-A-Am--PRLRLRPR
9. C-Em-E-Abm-Ab-Cm-Eb-Gm-G-Bm-B-Ebm-F#-Bbm-Bb-Dm-D-F#m-A-C#m-C#-Fm-F-Am--LPLPLR
10. C-Em-E-Abm-B-Ebm-Eb-Gm-G-Bm-D-F#m-F#-Bbm-Bb-Dm-F-Am-A-C#m-C#-Fm-Ab-Cm--LPLRLP
11. C-Em-G-Gm-Bb-Bbm-C#-C#m-E-Abm-B-Bm-D-Dm-F-Fm-Ab-Cm-Eb-Ebm-F#-F#m-A-Am--LRPRPRPR
12. C-Em-G-Gm-Bb-Bbm-C#-Fm-Ab-Cm-Eb-Ebm-F#-F#m-A-C#m-E-Abm-B-Bm-D-Dm-F-Am--LRPRPRLR
13. C-Em-G-Gm-Bb-Dm-F-Fm-Ab-Cm-Eb-Ebm-F#-Bbm-C#-C#m-E-Abm-B-Bm-D-F#m-A-Am--LRPR
14. C-Em-G-Bm-B-Ebm-Eb-Gm-Bb-Dm-D-F#m-F#-Bbm-C#-Fm-F-Am-A-C#m-E-Abm-Ab-Cm--LRLPLP
15. C-Em-G-Bm-D-Dm-F-Fm-Ab-Cm-Eb-Gm-Bb-Bbm-C#-C#m-E-Abm-B-Ebm-F#-F#m-A-Am--LRLRPRPR
16. C-Em-G-Bm-D-F#m-A-C#m-E-Abm-B-Ebm-F#-Bbm-C#-Fm-Ab-Cm-Eb-Gm-Bb-Dm-F-Am--LR
17. C-Am-A-F#m-F#-Ebm-Eb-Cm-Ab-Fm-F-Dm-D-Bm-B-Abm-E-C#m-C#-Bbm-Bb-Gm-G-Em--RPRPRPRL
18. C-Am-A-F#m-F#-Ebm-B-Abm-Ab-Fm-F-Dm-D-Bm-G-Em-E-C#m-C#-Bbm-Bb-Gm-Eb-Cm--RPRPRLRP
19. C-Am-A-F#m-F#-Ebm-B-Abm-E-C#m-C#-Bbm-Bb-Gm-Eb-Cm-Ab-Fm-F-Dm-D-Bm-G-Em--RPRPRLRL
20. C-Am-A-F#m-D-Bm-B-Abm-Ab-Fm-F-Dm-Bb-Gm-G-Em-E-C#m-C#-Bbm-F#-Ebm-Eb-Cm--RPRLRPRP
21. C-Am-A-F#m-D-Bm-B-Abm-E-C#m-C#-Bbm-F#-Ebm-Eb-Cm-Ab-Fm-F-Dm-Bb-Gm-G-Em--RPRPL
22. C-Am-A-F#m-D-Bm-G-Em-E-C#m-C#-Bbm-F#-Ebm-B-Abm-Ab-Fm-F-Dm-Bb-Gm-Eb-Cm--RPRLRLRP
23. C-Am-F-Fm-C#-C#m-A-F#m-D-Dm-Bb-Bbm-F#-Ebm-B-Bm-G-Gm-Eb-Cm-Ab-Abm-E-Em--RLPLPL
24. C-Am-F-Dm-D-Bm-B-Abm-Ab-Fm-C#-Bbm-Bb-Gm-G-Em-E-C#m-A-F#m-F#-Ebm-Eb-Cm--RLRPRPRP
25. C-Am-F-Dm-D-Bm-B-Abm-E-C#m-A-F#m-F#-Ebm-Eb-Cm-Ab-Fm-C#-Bbm-Bb-Gm-G-Em--RLRPRPRL
26. C-Am-F-Dm-D-Bm-G-Em-E-C#m-A-F#m-F#-Ebm-B-Abm-Ab-Fm-C#-Bbm-Bb-Gm-Eb-Cm--RLRP
27. C-Am-F-Dm-Bb-Gm-G-Em-E-C#m-A-F#m-D-Bm-B-Abm-Ab-Fm-C#-Bbm-F#-Ebm-Eb-Cm--RLRLRPRP
28. C-Am-F-Dm-Bb-Gm-Eb-Cm-Ab-Fm-C#-Bbm-F#-Ebm-B-Abm-E-C#m-A-F#m-D-Bm-G-Em--RL

Hamiltonian Cycles and the « thirds circle »

1. C-Cm-Ab-Abm-E-C#m-A-Am-F-Fm-C#-Bbm-F#-F#m-D-Dm-Bb-Gm-Eb-Ebm-B-Bm-G-Em--PLPLRL
2. C-Cm-Ab-Fm-C#-C#m-A-Am-F-Dm-Bb-Bbm-F#-F#m-D-Bm-G-Gm-Eb-Ebm-B-Abm-E-Em--PLRLPL
3. C-Cm-Eb-Ebm-F#-F#m-A-C#m-E-Em-G-Gm-Bb-Bbm-C#-Fm-Ab-Abm-B-Bm-D-Dm-F-Am--PRPRPRLR
4. C-Cm-Eb-Ebm-F#-Bbm-C#-C#m-E-Em-G-Gm-Bb-Dm-F-Fm-Ab-Abm-B-Bm-D-F#m-A-Am--PRPRLRPR
5. C-Cm-Eb-Ebm-F#-Bbm-C#-Fm-Ab-Abm-B-Bm-D-F#m-A-C#m-E-Em-G-Gm-Bb-Dm-F-Am--PRPRLRLR
6. C-Cm-Eb-Gm-Bb-Bbm-C#-C#m-E-Em-G-Bm-D-Dm-F-Fm-Ab-Abm-B-Ebm-F#-F#m-A-Am--PRLRPLR
7. C-Cm-Eb-Gm-Bb-Bbm-C#-Fm-Ab-Abm-B-Ebm-F#-F#m-A-C#m-E-Em-G-Bm-D-Dm-F-Am--PRLRPLR
8. C-Cm-Eb-Gm-Bb-Dm-F-Fm-Ab-Abm-B-Ebm-F#-Bbm-C#-C#m-E-Em-G-Bm-D-F#m-A-Am--PRLRPLR
9. C-Em-E-Abm-Ab-Cm-Eb-Gm-G-Bm-B-Ebm-F#-Bbm-Bb-Dm-D-F#m-A-C#m-C#-Fm-F-Am--LPLPLR
10. C-Em-E-Abm-B-Ebm-Eb-Gm-G-Bm-D-F#m-F#-Bbm-Bb-Dm-F-Am-A-C#m-C#-Fm-Ab-Cm--LPLRLP
11. C-Em-G-Gm-Bb-Bbm-C#-C#m-E-Abm-B-Bm-D-Dm-F-Fm-Ab-Cm-Eb-Ebm-F#-F#m-A-Am--LRPRPRLR
12. C-Em-G-Gm-Bb-Bbm-C#-Fm-Ab-Cm-Eb-Ebm-F#-F#m-A-C#m-E-Abm-B-Bm-D-Dm-F-Am--LRPRPRLR
13. C-Em-G-Gm-Bb-Dm-F-Fm-Ab-Cm-Eb-Ebm-F#-Bbm-C#-C#m-E-Abm-B-Bm-D-F#m-A-Am--LRPR
14. C-Em-G-Bm-B-Ebm-Eb-Gm-Bb-Dm-D-F#m-F#-Bbm-C#-Fm-F-Am-A-C#m-E-Abm-Ab-Cm--LRLPLP
15. C Em C Bm D Dm F Fm Ab Cm Eb Cm Bb Bbm C# C#m E Abm B Ebm F# F#m A Am LRLPRPPR
16. C-Em-G-Bm-D-F#m-A-C#m-E-Abm-B-Ebm-F#-Bbm-C#-Fm-Ab-Cm-Eb-Gm-Bb-Dm-F-Am--LR
17. C-Am-A-F#m-F#-Ebm-Eb-Cm-Ab-Fm-F-Dm-D-Bm-B-Abm-E-C#m-C#-Bbm-Bb-Gm-G-Em--RPRPRLR
18. C-Am-A-F#m-F#-Ebm-B-Abm-Ab-Fm-F-Dm-D-Bm-G-Em-E-C#m-C#-Bbm-Bb-Gm-Eb-Cm--RPRPLRP
19. C-Am-A-F#m-F#-Ebm-B-Abm-E-C#m-C#-Bbm-Bb-Gm-Eb-Cm-Ab-Fm-F-Dm-D-Bm-G-Em--RPRPLRL
20. C-Am-A-F#m-D-Bm-B-Abm-Ab-Fm-F-Dm-Bb-Gm-G-Em-E-C#m-C#-Bbm-F#-Ebm-Eb-Cm--RPRPLR
21. C-Am-A-F#m-D-Bm-B-Abm-E-C#m-C#-Bbm-F#-Ebm-Eb-Cm-Ab-Fm-F-Dm-Bb-Gm-G-Em--RPR
22. C-Am-A-F#m-D-Bm-G-Em-E-C#m-C#-Bbm-F#-Ebm-B-Abm-Ab-Fm-F-Dm-Bb-Gm-Eb-Cm--RPRPLRP
23. C-Am-F-Fm-C#-C#m-A-F#m-D-Dm-Bb-Bbm-F#-Ebm-B-Bm-G-Gm-Eb-Cm-Ab-Abm-E-Em--RLPLPL
24. C-Am-F-Dm-D-Bm-B-Abm-Ab-Fm-C#-Bbm-Bb-Gm-G-Em-E-C#m-A-F#-Ebm-Eb-Cm--RLRPRP
25. C-Am-F-Dm-D-Bm-B-Abm-E-C#m-A-F#m-F#-Ebm-Eb-Cm-Ab-Fm-C#-Bbm-Bb-Gm-G-Em--RLRPRPL
26. C-Am-F-Dm-D-Bm-G-Em-E-C#m-A-F#m-F#-Ebm-B-Abm-Ab-Fm-C#-Bbm-Bb-Gm-Eb-Cm--RLR
27. C-Am-F-Dm-Bb-Gm-G-Em-E-C#m-A-F#m-D-Bm-B-Abm-Ab-Fm-C#-Bbm-F#-Ebm-Eb-Cm--RLRPLR
28. C-Am-F-Dm-Bb-Gm-Eb-Cm-Ab-Fm-C#-Bbm-F#-Ebm-B-Abm-E-C#m-A-F#m-D-Bm-G-Em--RL



THÈSE / UNIVERSITÉ DE RENNES 1
sous le sens de l'Université Bretagne Loire

pour le grade de

DOCTEUR DE L'UNIVERSITÉ DE RENNES 1

Mention : Informatique

École doctorale MATISSE

présentée par

Corentin Guichaoua

préparée à l'unité de recherche 6074 (IRISA)

Institut de Recherche en Informatique et Systèmes Aléatoires
(Composante universitaire)

Modèles de compression
et critères de complexité
pour la description et
l'inférence de structure
musicale

Soutenance prévue à Rennes
le 28 septembre 2017

devant le jury composé de :

Moreno ANDREATTI

DR CNRS, IRISA / rapporteur

Myriam DESAINTE-CATHERINE

PR CNRS, Bordeaux INP / rapporteur

Matthew DAVIES

Senior Researcher, INESC TEC / examinateur

Mathieu GERAUD

CR CNRS, CRISAL / examinateur

François PACHET

Délégué de laboratoire de recherche, Sony CSL Paris / examinateur

Frédéric BIMBOT

DR CNRS, IRISA / directeur de thèse

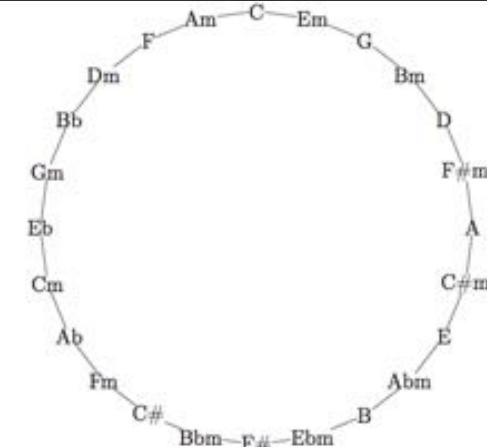
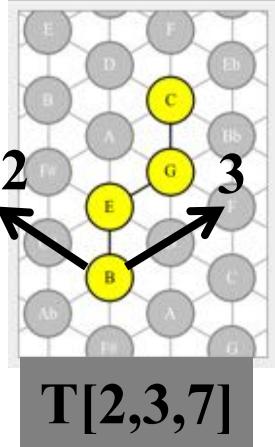


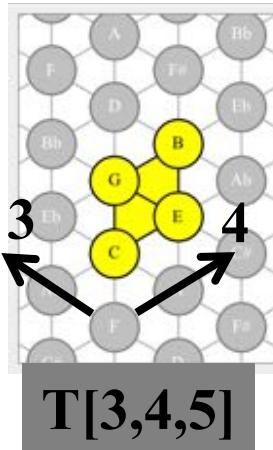
FIGURE 6.15 – Le cercle utilisé pour représenter l'espace des accords

The spatial character of the « musical style »

1

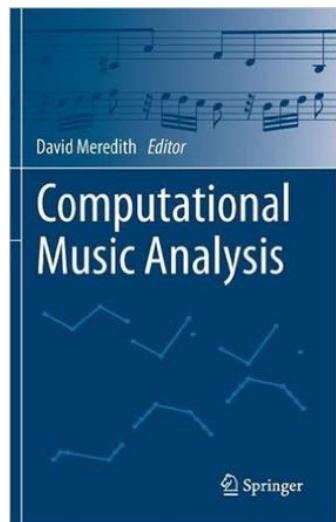
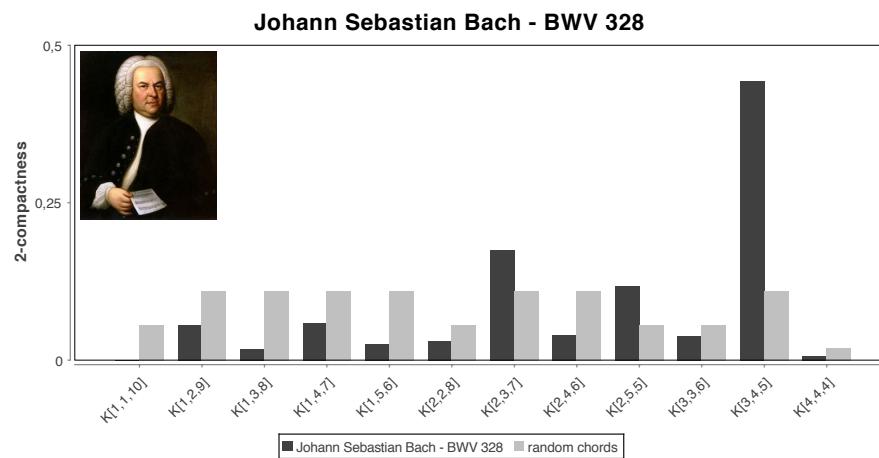


2

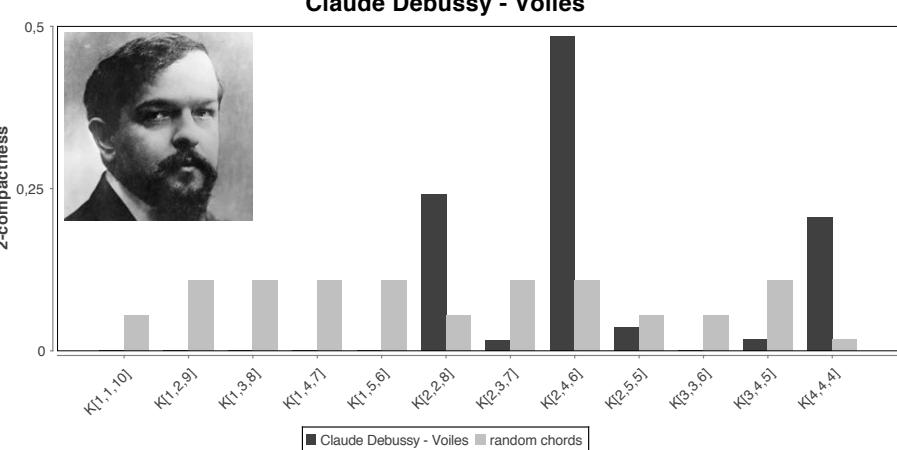


3

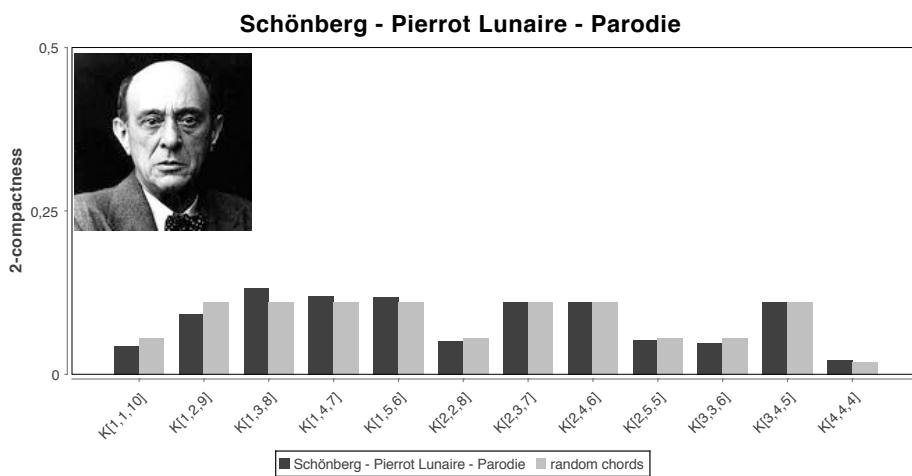
4



5



6



Spatial music analysis via *Hexachord*

The image shows a composite screenshot of the Hexachord software interface and a copy of the 'Computer Music Journal'.

Hexachord Software Components:

- Hex Viewer:** A 3D visualization of a geometric polyhedron, likely representing a complex or set class.
- Tessellation:** A hexagonal grid representation of musical data, with specific hexagons highlighted in yellow and labeled with letters (A through H).
- InfoBox:** A control panel for a MIDI file named "bwv0281.mid". It includes:
 - Tempo slider (set to 10).
 - Play and Stop buttons.
 - Select midi file input field.
 - Chromatic complexes and Heptatonic complexes dropdown menus (both set to CM).
 - Trace off and Harmonization ON buttons.
 - Display graph button.
 - Vertical compactness section with compactness dimension (2), 2-compactness, compute compactness, and absolute compactness buttons.
 - Path Transformation section with Origin complex (K[3,4,5]) and Destination complex (K[3,4,5]), Rotation (0), North translation (0), and North-east translation (0) buttons, along with a Path Transformation button.
 - Chart section titled "2-compactness : bwv0281" showing a bar chart of 2-compactness values over time.

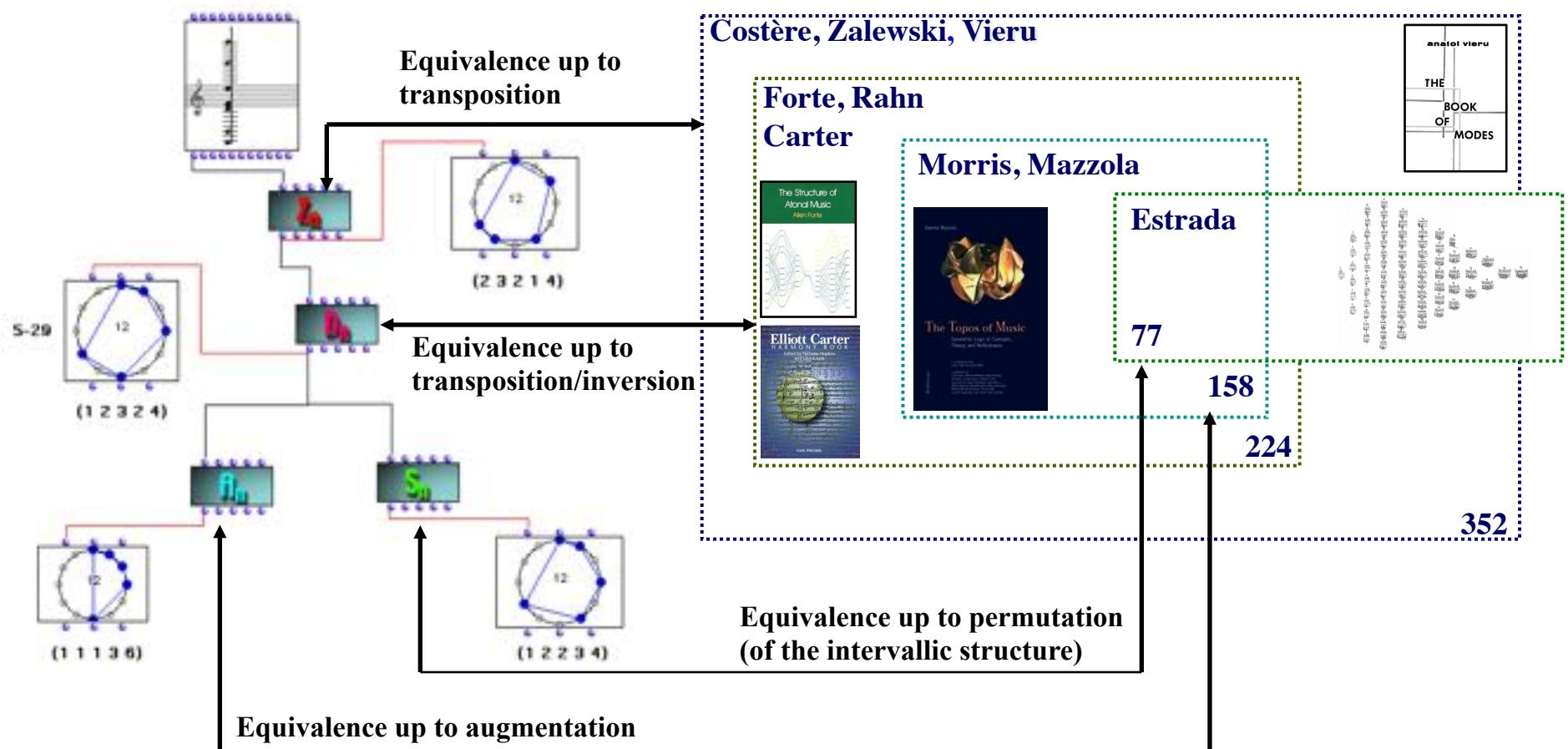
Computer Music Journal Cover:

Volume 30 Number 1, Spring 2006, pp. 1-20
Digital Harmonies and Discreet Sound Sources

The journal cover features a green background with a white header and footer. It includes a small graphic of a geometric shape and a hexagonal grid pattern.

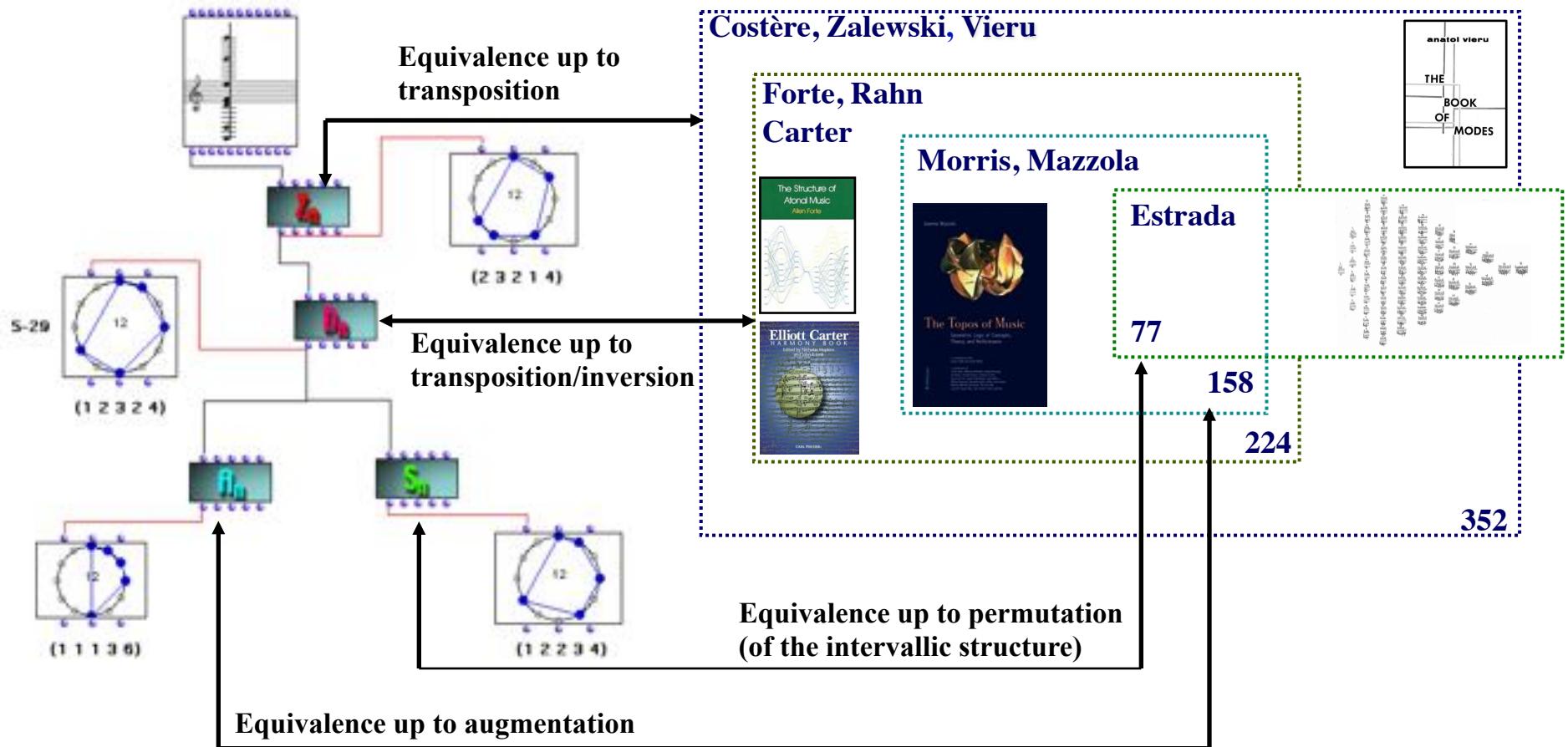
→ <http://www.lacl.fr/~lbigo/hexachord>

The catalogues of musical structures (from Costère to Estrada)



	1	2	3	4	5	6	7	8	9	10	11	12
	→	1	6	19	43	66	80	66	43	19	6	1
	→	1	6	12	29	38	50	38	29	12	6	1
	→	1	5	9	21	25	34	25	21	9	5	1
	→	1	6	12	15	12	11	7	5	3	2	1

Epistemological aspects of a group-theoretical approach



« [C'est la **notion de groupe** qui] donne un sens précis à l'idée de **structure** d'un ensemble [et] permet de déterminer les éléments efficaces des transformations en réduisant en quelque sorte à son **schéma opératoire** le domaine envisagé. [...] L'objet véritable de la science est le **système des relations** et non pas les termes supposés qu'il relie. [...] Intégrer les résultats - symbolisés - d'une expérience nouvelle revient [...] à créer un canevas nouveau, un **groupe de transformations** plus complexe et plus compréhensif » (G.-G. Granger : « Pygmalion. Réflexions sur la pensée formelle », 1947)



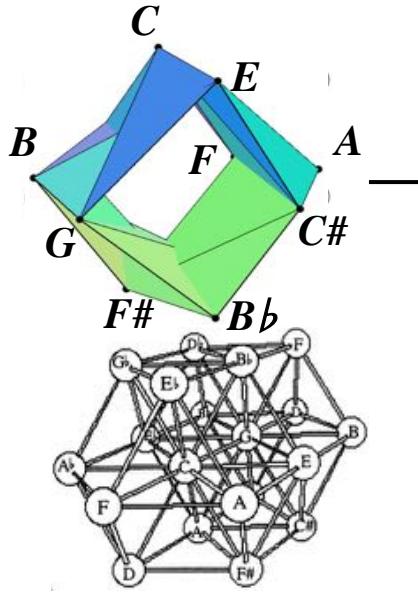
G.-G. Granger

Classifying Chord Complexes

L. Bigo, *Représentation symboliques musicales et calcul spatial*, PhD, Ircam / LACL, 2013

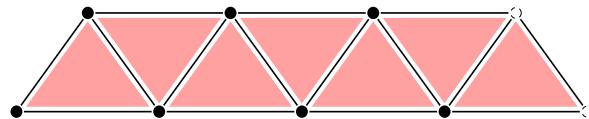
- Complexes enumeration in the chromatic system

$K_{TI}[3,4,5]$
[Cohn – 1997]



$K_{TI}[2,3,3,4]$
[Gollin - 1998]

$K_T[2,2,3]$
[Mazzola – 2002]



...

d	complexe	taille	b_n	p-v	χ
-	K_\emptyset	0	[0]		0
0	$K_{TI}[0]$	0	[0]		0
1	$K_{TI}[1, 11]$	12	[1, 1]	x	0
	$K_{TI}[2, 10]$	12	[2, 2]		0
	$K_{TI}[3, 9]$	12	[3, 3]		0
	$K_{TI}[4, 8]$	12	[4, 4]		0
	$K_{TI}[5, 7]$	12	[1, 1]	x	0
	$K_{TI}[6, 6]$	6	[6, 0]		6
2	$K_{TI}[1, 1, 10]$	12	[1, 1, 0]	x	0
	$K_{TI}[1, 2, 9]$	24	[1, 2, 1]	x	0
	$K_{TI}[1, 3, 8]$	24	[1, 2, 1]	x	0
	$K_{TI}[1, 4, 7]$	24	[1, 2, 1]	x	0
	$K_{TI}[1, 5, 6]$	24	[1, 1, 6]		6
	$K_{TI}[2, 2, 8]$	12	[2, 2, 0]		0
	$K_{TI}[2, 3, 7]$	24	[1, 2, 1]	x	0
	$K_{TI}[2, 4, 6]$	24	[2, 2, 6]		6
	$K_{TI}[2, 5, 5]$	12	[1, 1, 0]	x	0
	$K_{TI}[3, 3, 6]$	12	[3, 0, 3]		6
	$K_{TI}[3, 4, 5]$	24	[1, 2, 1]	x	0
	$K_{TI}[4, 4, 4]$	4	[4, 0, 0]		4
3	$K_{TI}[1, 1, 1, 9]$	12	[1, 1, 0, 0]	x	0
	$K_{TI}[1, 1, 2, 8]$	24	[1, 1, 12, 0]		12
	$K_{TI}[1, 1, 3, 7]$	24	[1, 2, 13, 0]		12
	$K_{TI}[1, 1, 4, 6]$	24	[1, 1, 18, 0]		18
	$K_{TI}[1, 1, 5, 5]$	12	[1, 1, 6, 0]		6

On the group of transformations of classical types of seventh chords

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² IRMA/Université de Strasbourg

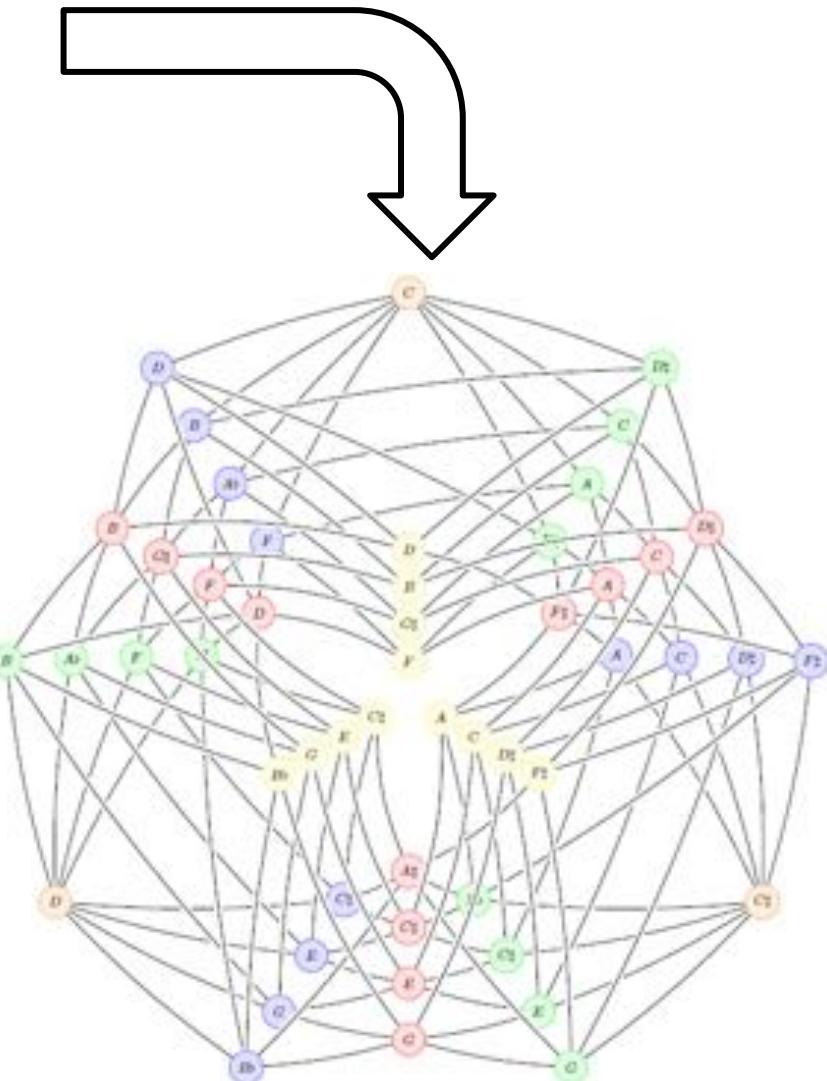
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Abstract. This paper presents a generalization of the well-known neo-Riemannian group PLR to the classical five types of seventh chord (dominant, minor, half-diminished, major, diminished) considered as tetra-chords with a marked root and proving that it is isomorphic to the abstract group $S_5 \times \mathbb{Z}_{12}^4$. This group includes as subgroups the PLR group and several other groups already appeared in the literature.

Keywords: Transformational theory; neo-Riemannian group; semi-direct product; seventh chord



→ Après-midi
Hamiltonian paths/cycles?

Chordal limitations of an isotropic Tonnetz model

Diagram illustrating the chordal limitations of an isotropic Tonnetz model, showing the relationship between musical chords, their geometric representations in the Tonnetz space, and their corresponding modes.

Top Row: Musical chords on a staff. The first chord is $C\flat, G\flat, E\flat$ (purple box). The second chord is $B\sharp, G\sharp, E\flat$ (green box). The third chord is $B\flat, G\flat, E\flat$ (red box).

Middle Row: Isotropic Tonnetz space graph. Nodes represent notes: $F\sharp, E\flat, C, A, C\sharp, G, B\flat, D, B, F\sharp, A, E, G\sharp, C, B\flat, G\sharp, E$. Edges connect adjacent notes. A purple diamond highlights the chord $C\sharp, G\sharp, F$ (top left). A green triangle highlights the chord $B, G\sharp, F$ (bottom left). A red triangle highlights the chord $B\flat, G\flat, E\flat$ (bottom right). A green arrow labeled "PV" points from the purple diamond to the green triangle. A red arrow labeled "MIII" points from the green triangle to the red triangle.

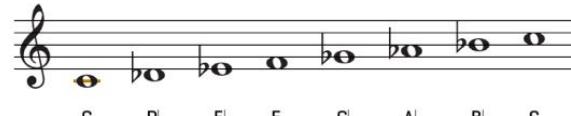
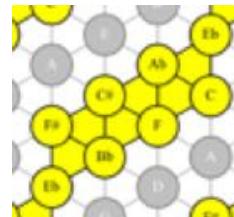
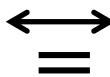
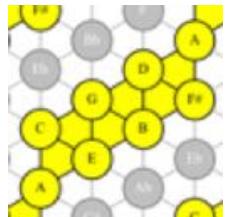
Bottom Row: Hexagonal lattice representation of modes.

- Left:** **Ionian mode** (green border). Shows a yellow hexagon with vertices A, B, C, D, E, F .
- Right:** **Locrian mode** (red border). Shows a yellow hexagon with vertices $E\flat, B\flat, G, C, F\flat, B$.
- Bottom:** A double-headed arrow indicates the equivalence between the Ionian and Locrian modes.

Bottom Staff: A staff with notes $C, D\flat, E\flat, F, G\flat, A\flat, B\flat, C$ corresponding to the notes in the Locrian mode hexagon.

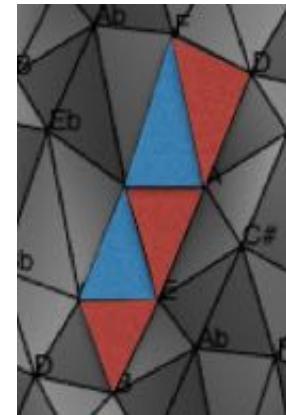
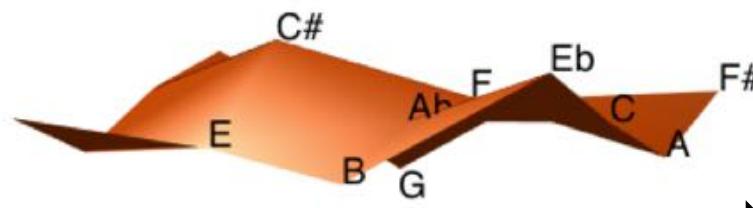
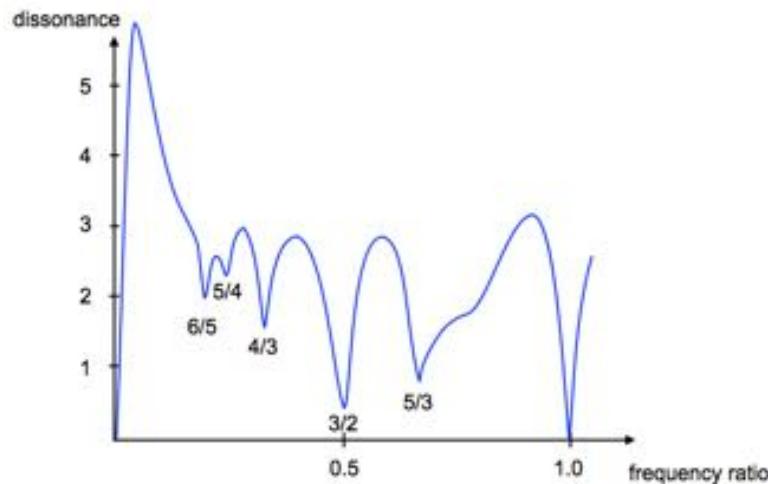
Towards a ‘morphological’ anisotropic *Tonnetz*

- M. Bergomi, *Dynamical and topological tools for (modern) music analysis*, UPMC-Ircam/LIM Milan, 2015.

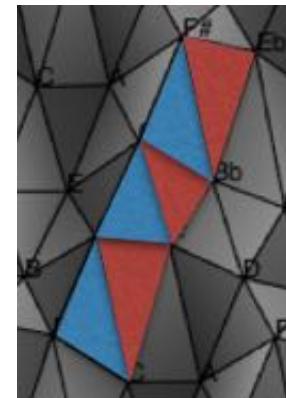
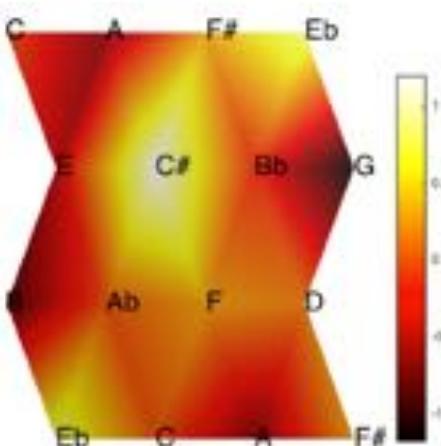


Ionian mode

Locrian mode



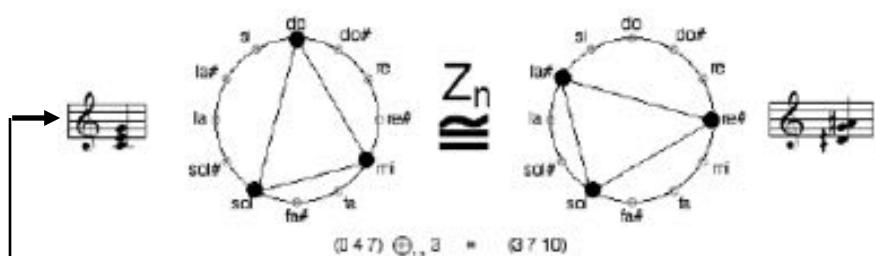
Ionian mode



Locrian mode

→ Persistent homology (Davide)

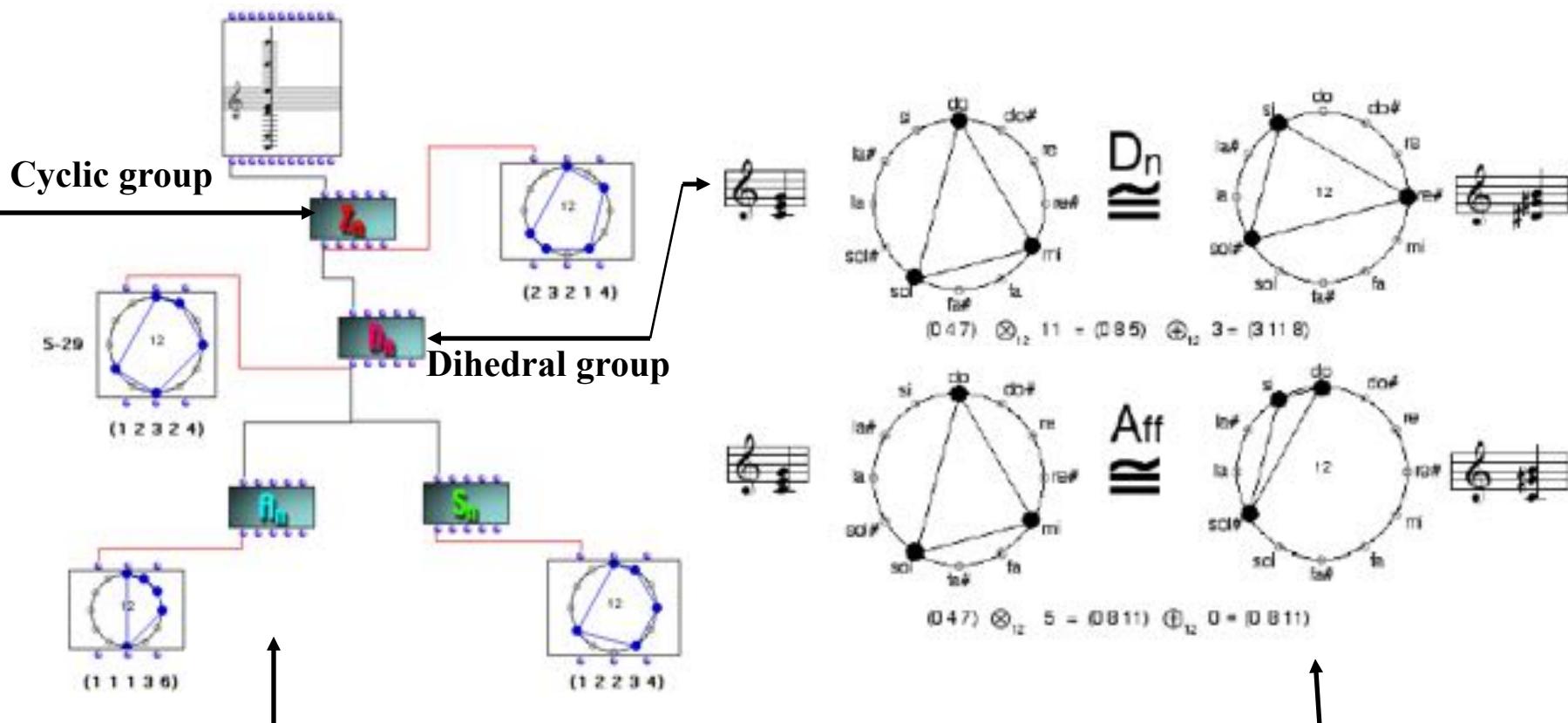
Equivalence classes of chords (up to a group action)



$$\mathbf{Z}_{12} = \langle T_k \mid (T_k)^{12} = T_0 \rangle$$

$$\mathbf{D}_{12} = \langle T_k, I \mid (T_k)^{12} = I^2 = T_0, ITI = I(IT)^{-1} \rangle$$

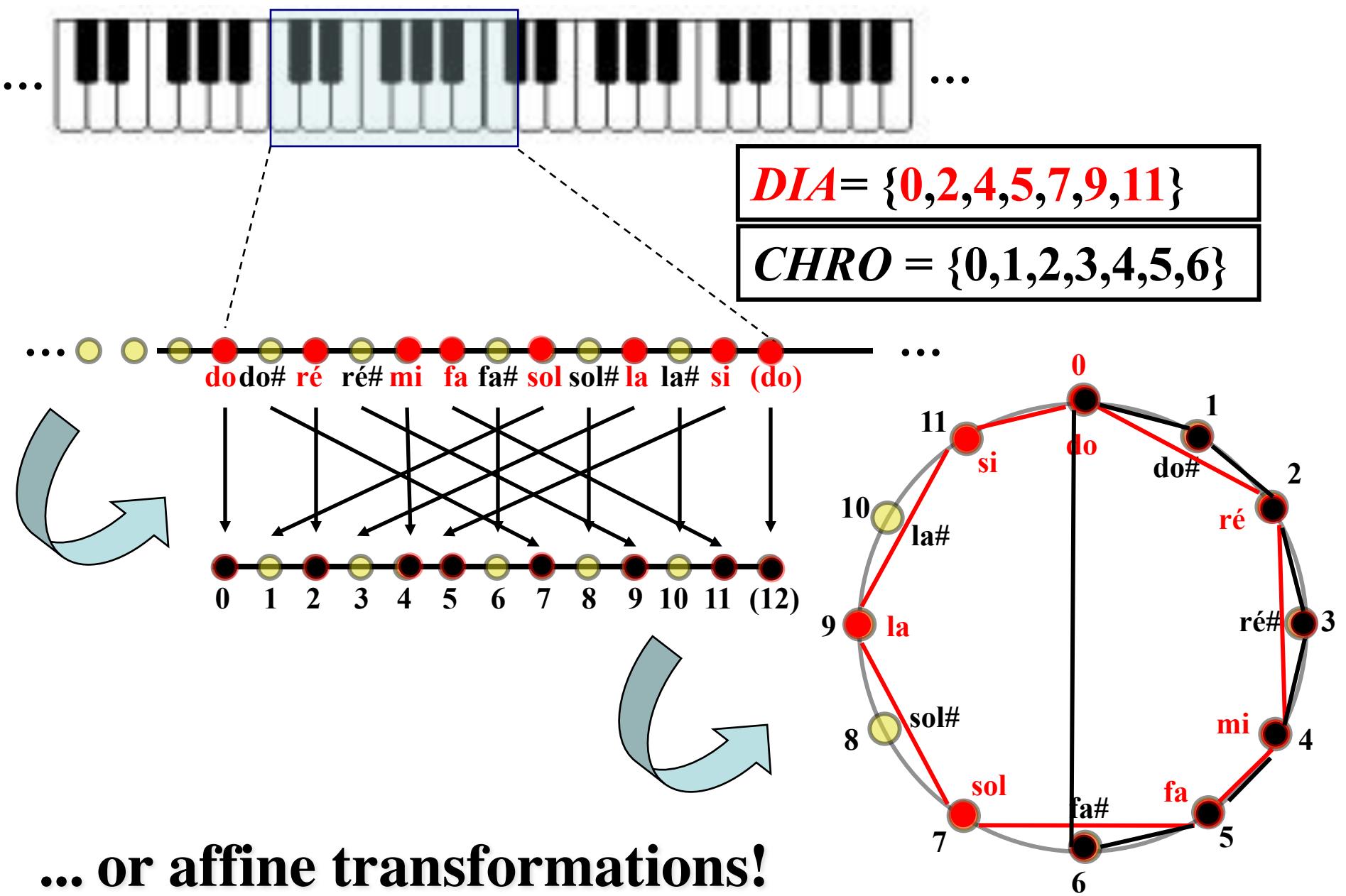
$$\mathbf{Aff} = \{f \mid f(x) = ax + b, a \in (\mathbf{Z}_{12})^*, b \in \mathbf{Z}_{12}\}$$



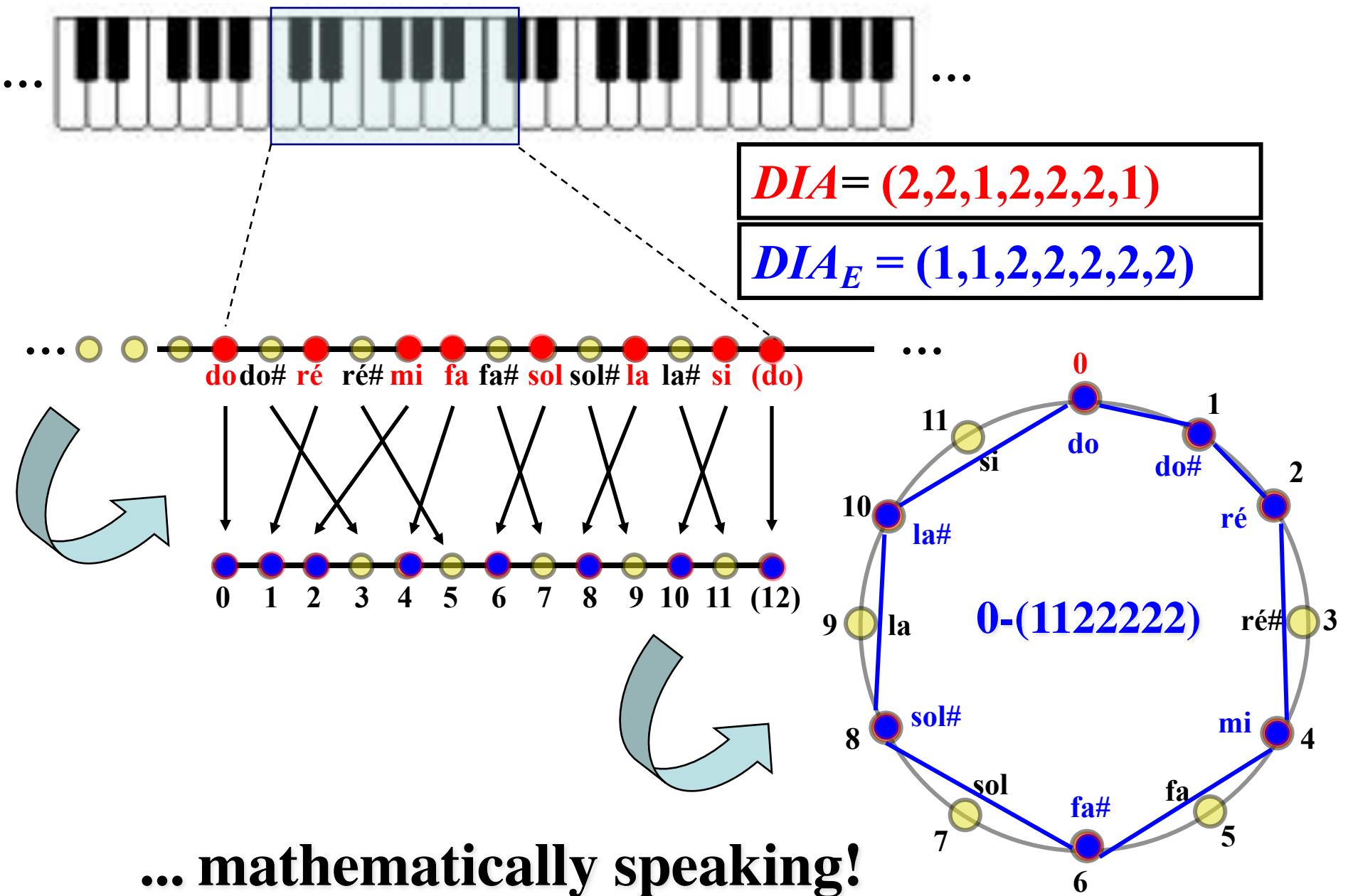
Paradigmatic architecture

Affine group

Augmentations are multiplications...

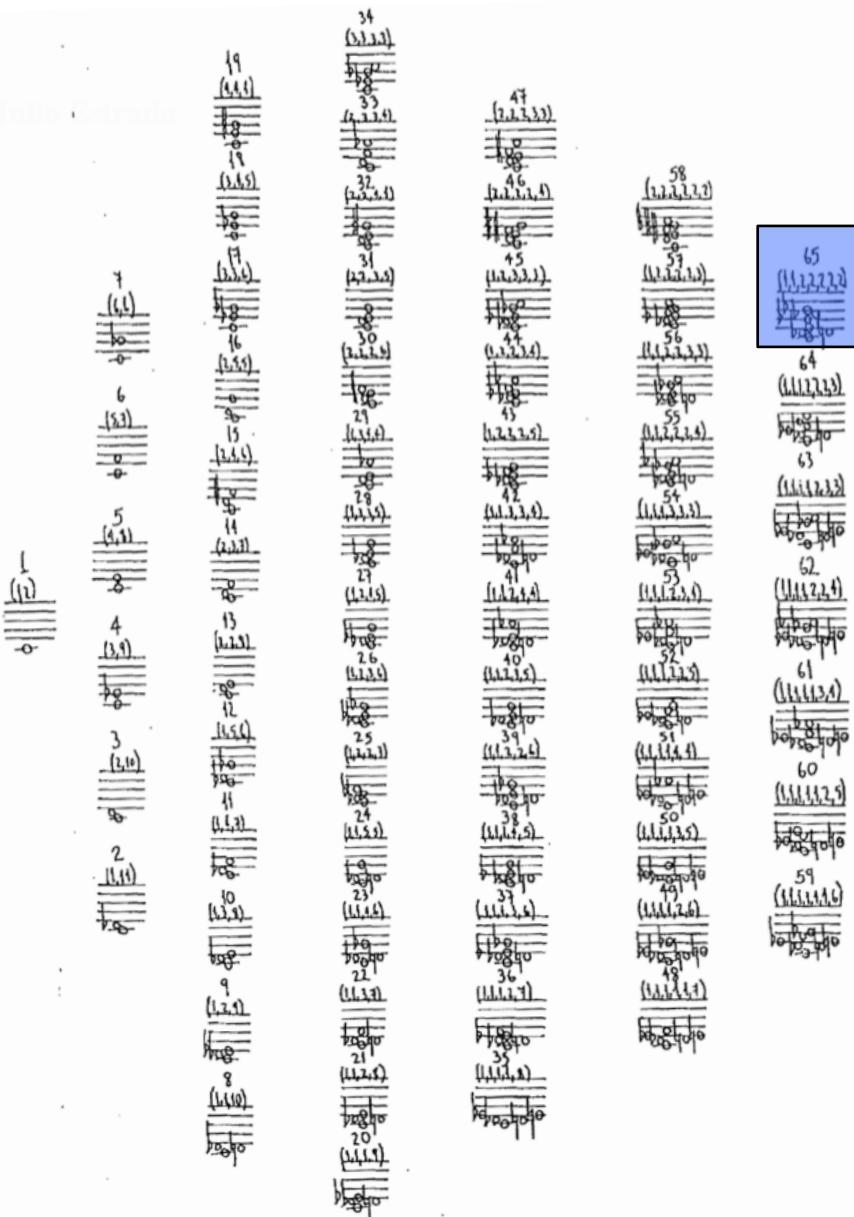


Permutations are ‘partitions’...



The permutohedron as a combinatorial space

Julio Estrada, *Théorie de la composition : discontinuum – continuum*, université de Strasbourg II, 1994



65

(1,1,1,2,2,2)

ILLUSTRATION III. REPRESENTATION EN NOTATION MUSICALE DE L'ENSEMBLE DE PARTITIONS DE L'ECHELLE DE HAUTEURS D12 :
12 NIVEAUX DE DENSITE, 77 IDENTITES.

64

(1,1,1,2,2,2)

70

(1,1,1,2,2,2)

63

(1,1,1,2,2,2)

73

(1,1,1,2,2,2)

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(1,1,1,2,2,2)

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211

(1,1,1,2,2,2)

212

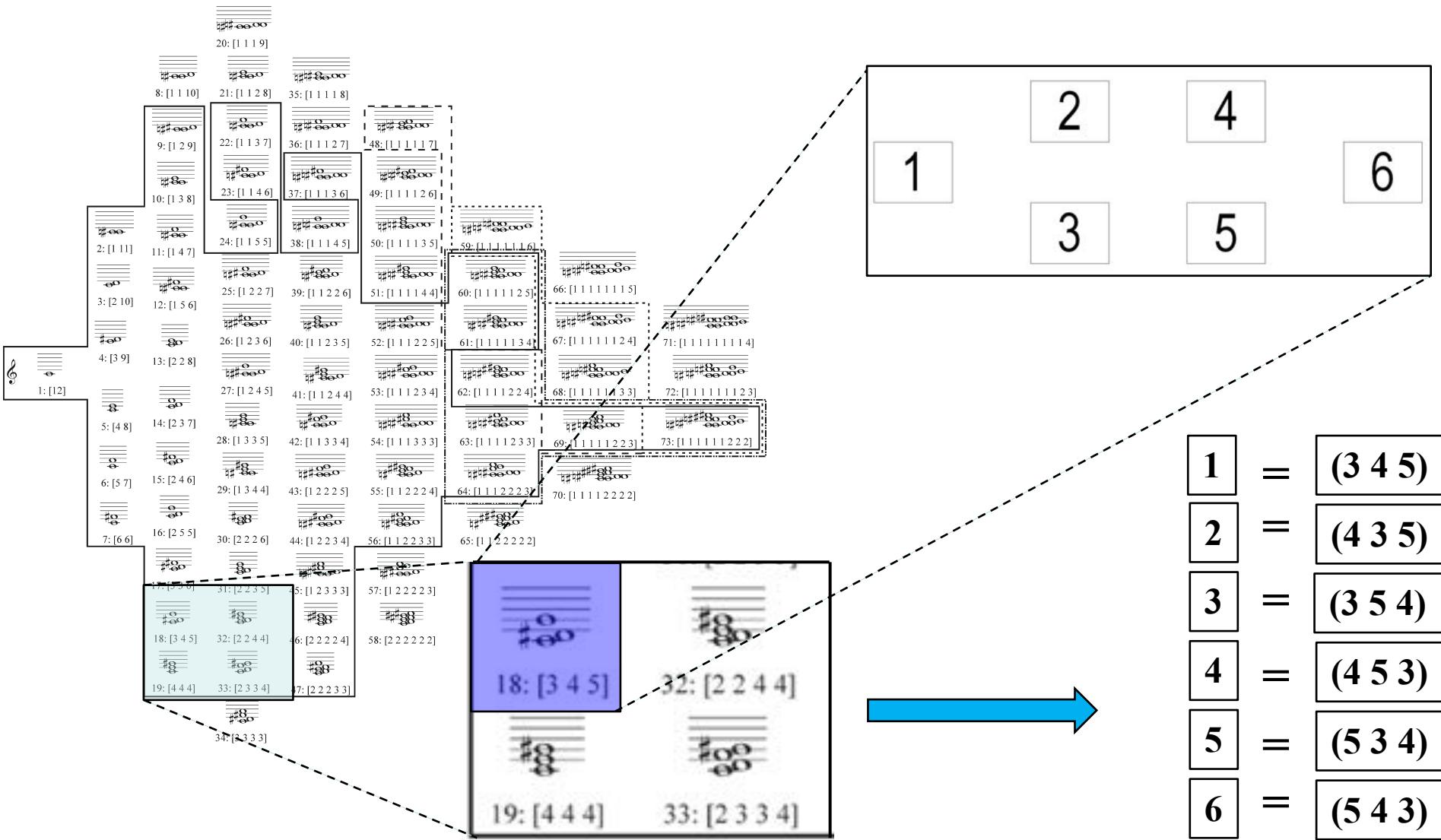
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213

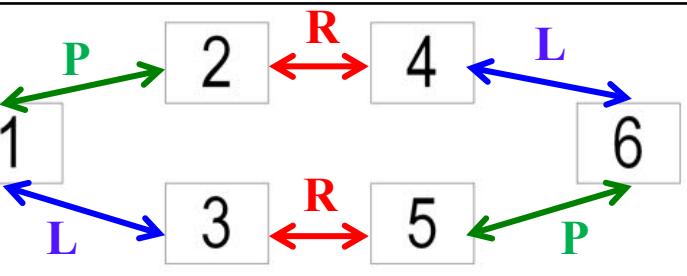
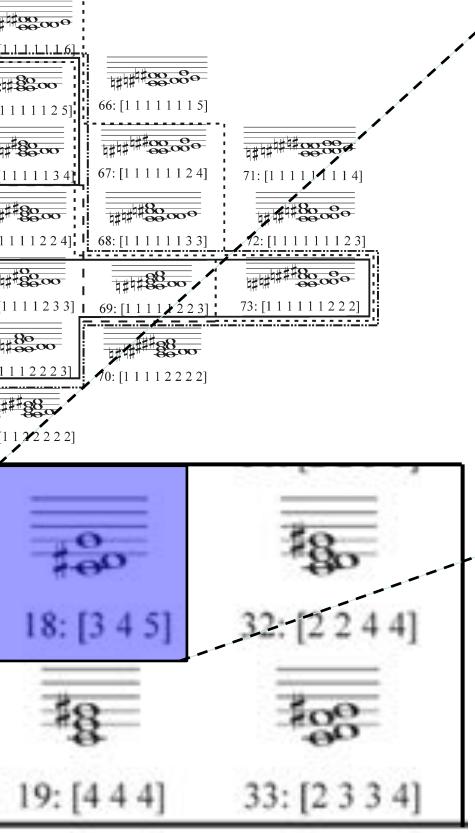
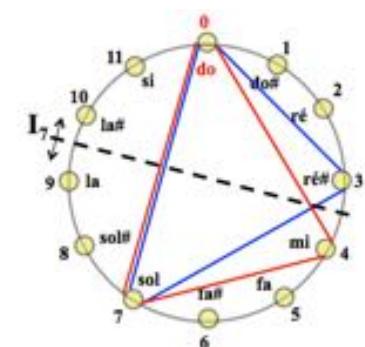
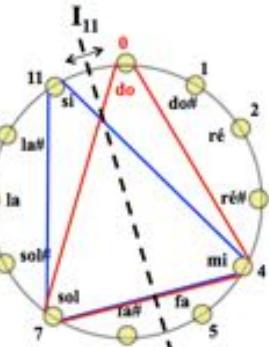
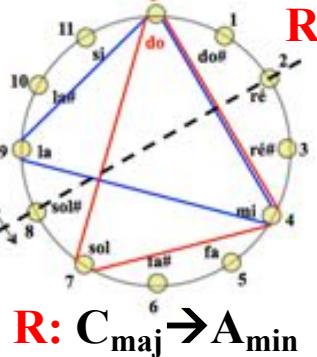
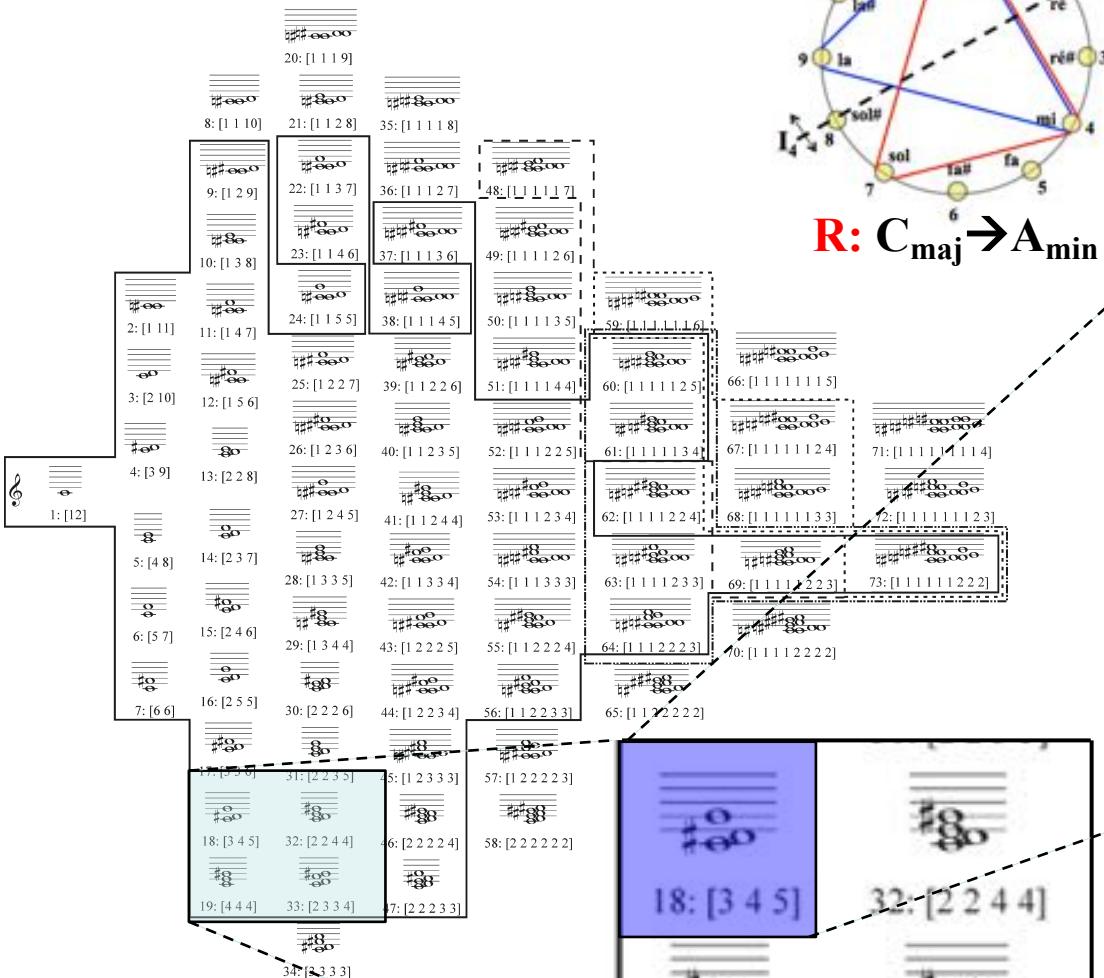
(1,1,1,2,2,2)

214

Permutohedron and *Tonnetz*: a structural inclusion

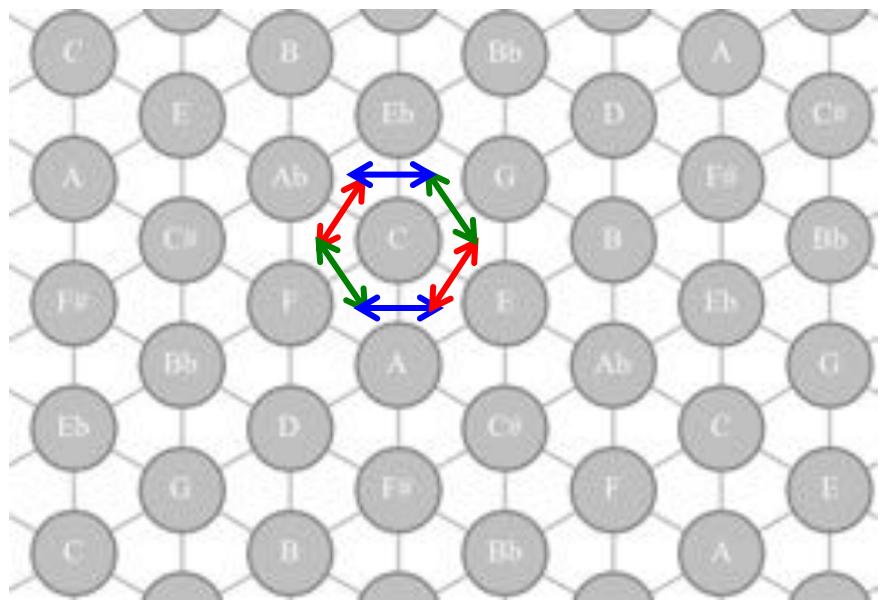
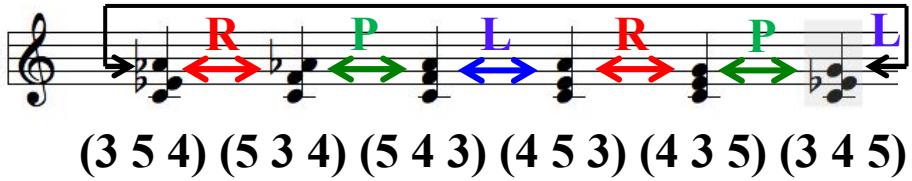
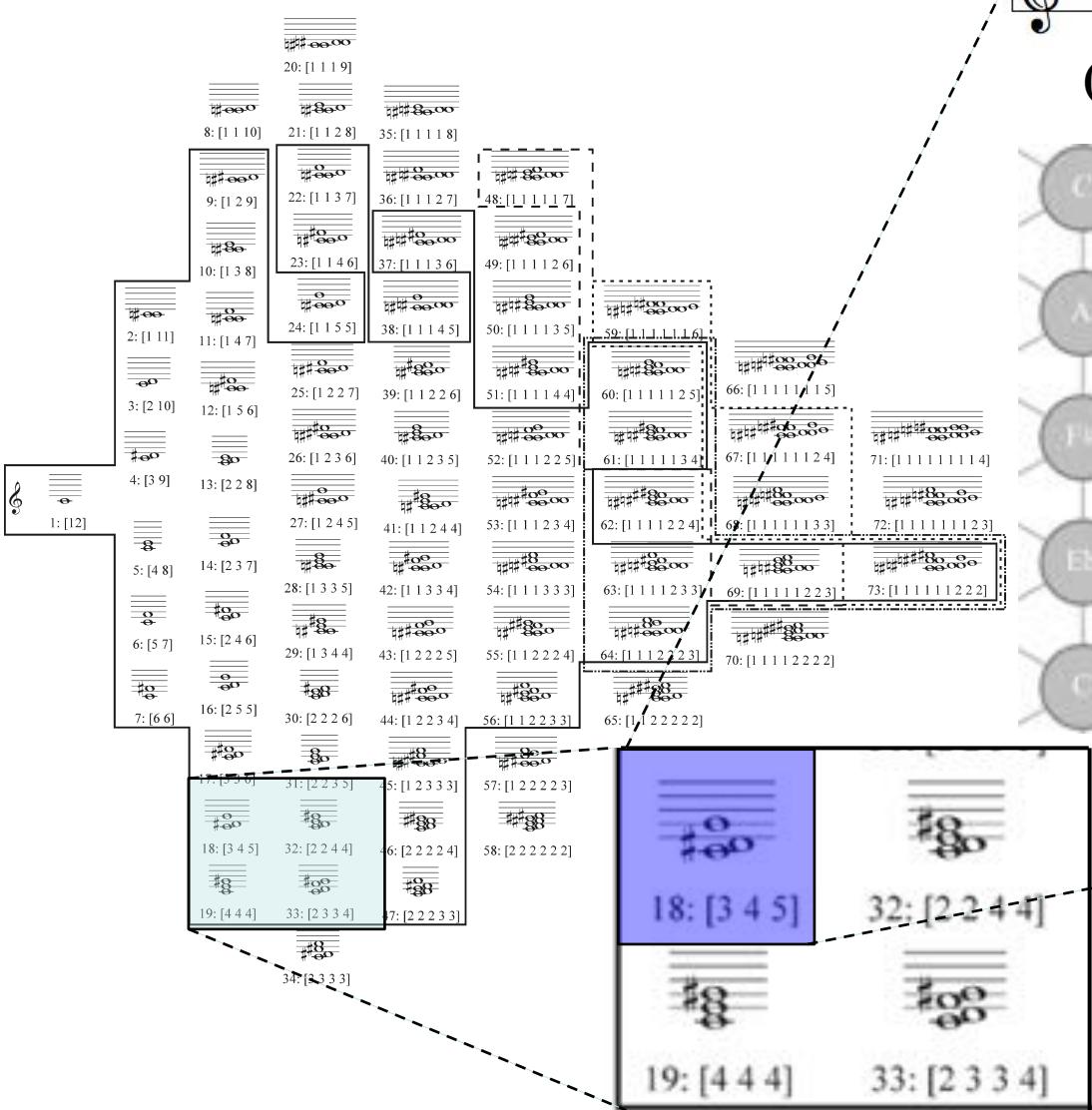


Permutohedron and Tonnetz: a structural inclusion



1	=	(3 4 5)
2	=	(4 3 5)
3	=	(3 5 4)
4	=	(4 5 3)
5	=	(5 3 4)
6	=	(5 4 3)

Permutohedron and *Tonnetz*: a structural inclusion

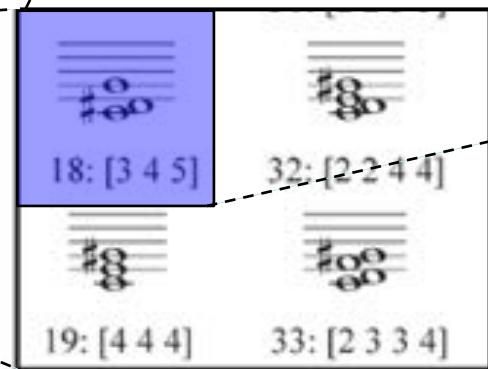
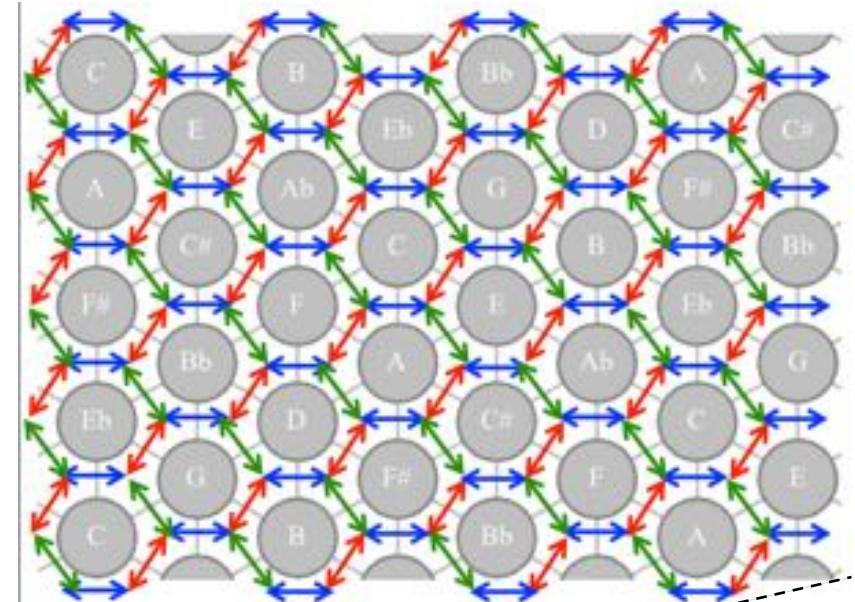
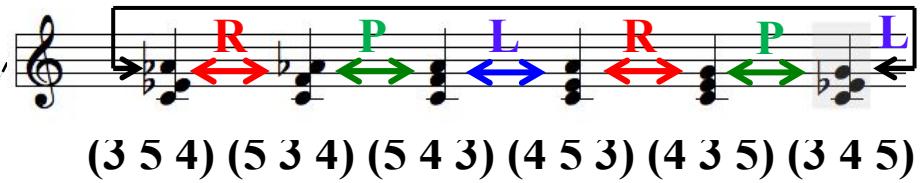
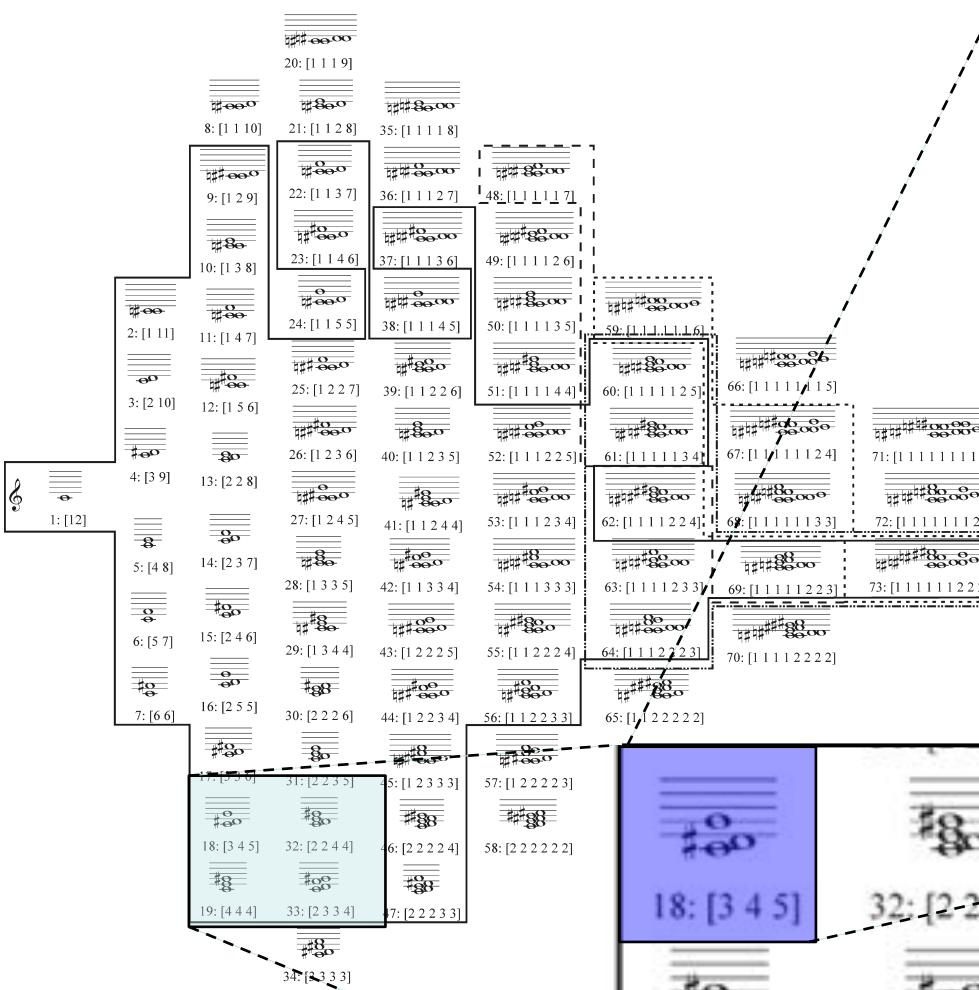


$$\mathbf{R}: \mathbf{C}_{\text{maj}} \rightarrow \mathbf{A}_{\text{min}}$$

$$L: C_{maj} \rightarrow E_{min}$$

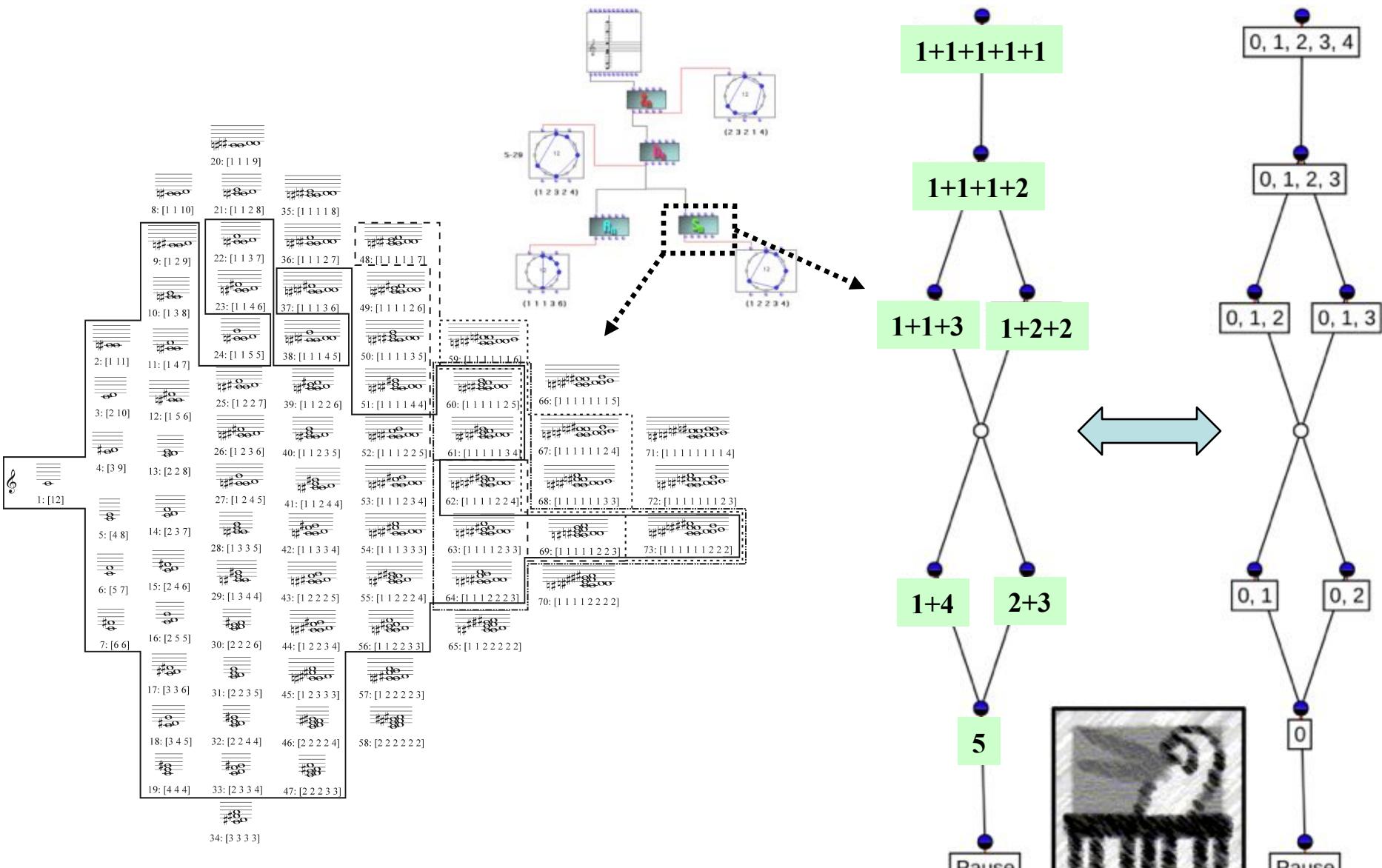
$$P: C_{\text{maj}} \rightarrow C_{\text{min}}$$

Permutohedron and Tonnetz: a structural inclusion



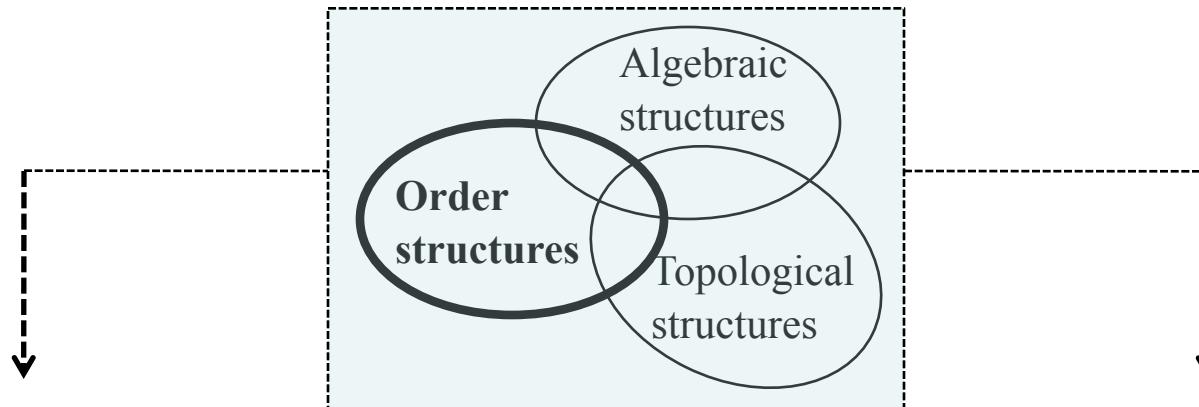
R: C_{maj} → A_{min}
L: C_{maj} → E_{min}
P: C_{maj} → C_{min}

The permutohedron as a lattice of formal concepts



• T. Schlemmer, M. Andreatta, « Using Formal Concept Analysis to represent Chroma Systems », MCM 2013, McGill Univ., Springer, LNCS.

Formal Concept Analysis: the double history



- M. Barbut, « Note sur l'algèbre des techniques d'analyse hiérarchique », in B. Matalon (éd.), *L'analyse hiérarchique*, Paris, Gauthier-Villars, 1965.
- M. Barbut, B. Monjardet, *Ordre et Classification. Algèbre et Combinatoire*, en deux tomes, 1970
- M. Barbut, L. Frey, « Techniques ordinaires en analyse des données », Tome I, *Algèbre et Combinatoire des Méthodes Mathématiques en Sciences de l'Homme*, Paris, Hachette, 1971.
- B. Leclerc, B. Monjardet, « Structures d'ordres et sciences sociales », *Mathématiques et sciences humaines*, 193, 2011, 77-97

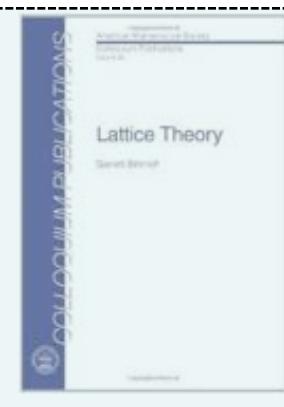
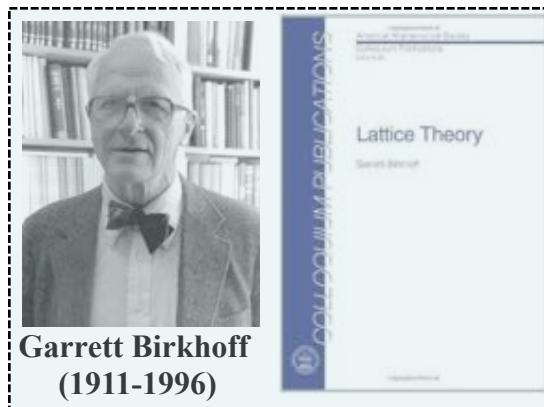
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- R. Wille, « Restructuring Lattice Theory: An approach based on Hierarchies of Concepts », I. Rival (ed.), *Ordered Sets*, 1982
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- B. Ganter & R. Wille, *Formal Concept Analysis: Mathematical Foundations*, Springer, Berlin, 1998



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Formal Concept Analysis: the common root



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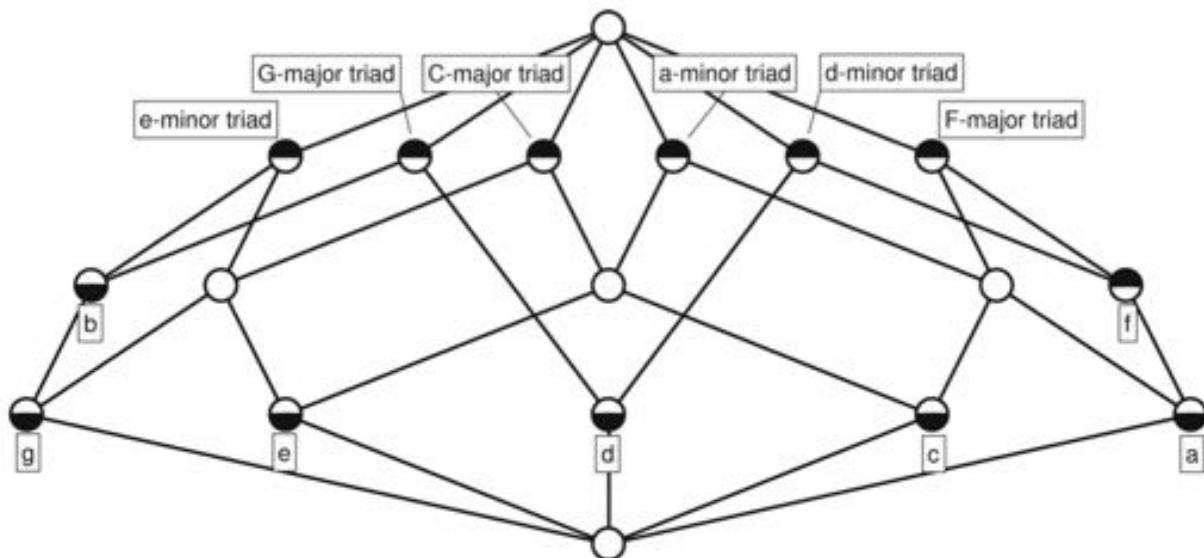
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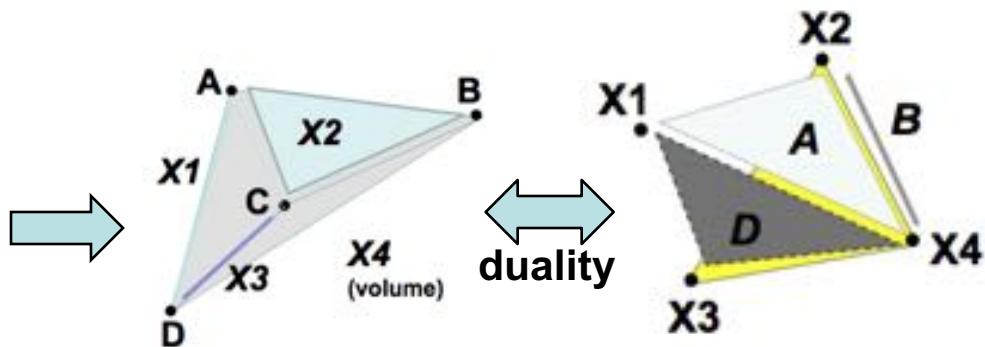
- R. Wille, « Mathematische Sprache in der Musiktheorie », in B. Fuchssteiner, U. Kulisch, D. Laugwitz, R. Liedl (Hrsg.): *Jahrbuch Überblicke Mathematik. B.I.-Wissenschaftsverlag*, Mannheim, 1980, p. 167-184.
- R. Wille, « Restructuring Lattice Theory: An approach based on Hierarchies of Concepts », I. Rival (ed.), *Ordered Sets*, 1982
- R. Wille, « Sur la fusion des contextes individuels », *Mathématiques et sciences humaines*, tome 85, 1984.
- B. Ganter & R. Wille, *Formal Concept Analysis: Mathematical Foundations*, Springer, Berlin, 1999

Formal Concept Analysis and topology: the Q-analysis

	C-major triad	d-minor triad	e-minor triad	F-major triad	G-major triad	a-minor triad
c	X			X	X	
d		X		X		
e	X	X			X	
f		X	X	X		
g	X	X	X			
a		X	X	X	X	X
b			X	X		



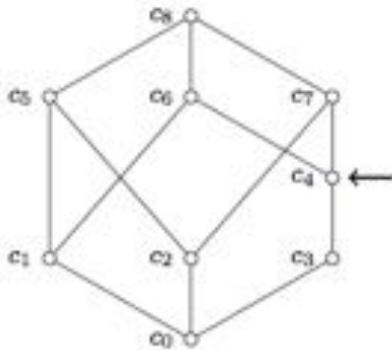
	A	B	C	D
X1	1	0	0	1
X2	1	1	1	0
X3	0	0	1	1
X4	1	1	1	1



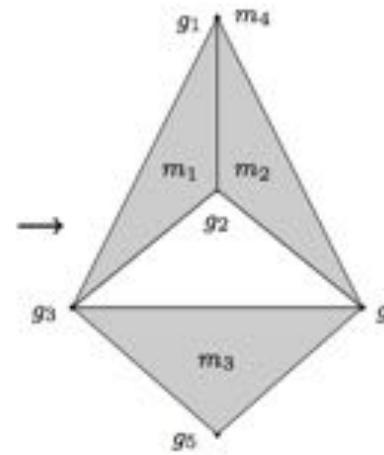
Concept lattice vs simplicial complex

Freund A., M. Andreatta, J.-L. Giavitto (2015), « Lattice-based and Topological Representations of Binary Relations with an Application to Music », *Annals of Mathematics and Artificial Intelligence*, 2015.

Lattice

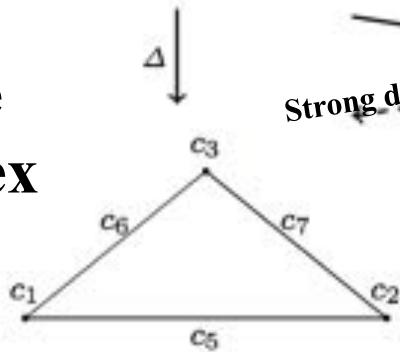


	m_1	m_2	m_3	m_4
g_1	x			
g_2	x	x		
g_3	x		x	
g_4		x	x	
g_5			x	



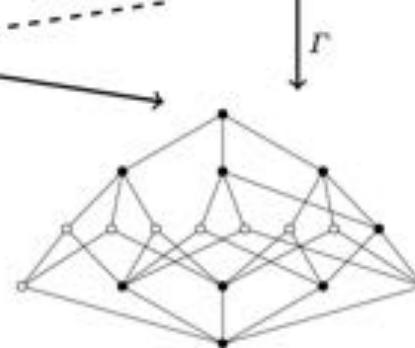
Simplicial complex

Lattice complex



$\Delta \downarrow$

ζ
Strong deformation retraction



Conclusions:

- The concept lattice alone cannot be fully reconstructed from the simplicial complex
- The simplicial complex cannot be fully determined from the concept lattice alone
- The concept lattice alone allows to determine the homotopy type of the simplicial complex

Concept lattice & mathematical morphology

RAPPORT DE STAGE RECHERCHE - M2 INFORMATIQUE FONDAMENTALE



Morphologie Mathématique, Analyse des Concepts Formels et Musicologie Computationale

Effectué au sein des structures :

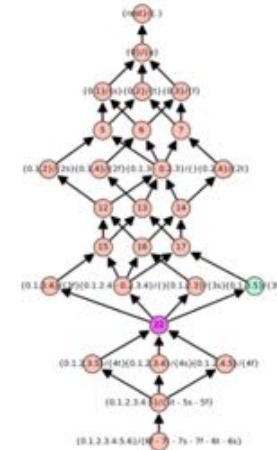
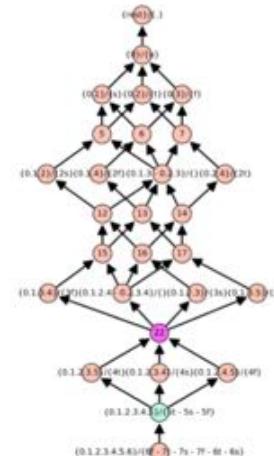
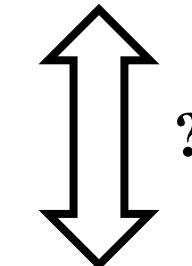
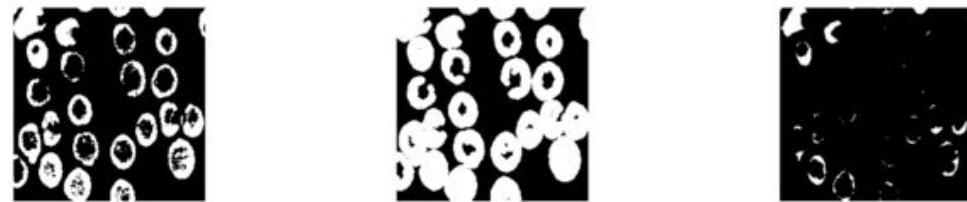


Présenté par :
Pierre Mascarade
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Télécom ParisTech
Morphologie Mathématique - FCA

Sous le tutorat de :
Márton Karsai,
ENS Lyon
Computer Science / Complex Systems

Effectué durant la période
Février 2017 - Juin 2017

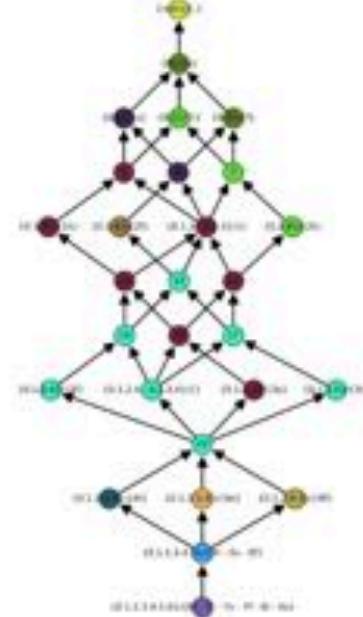
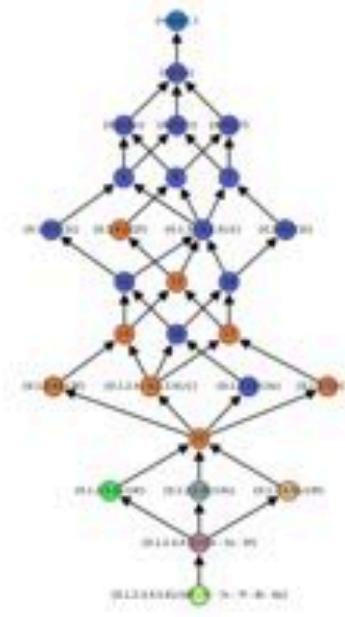
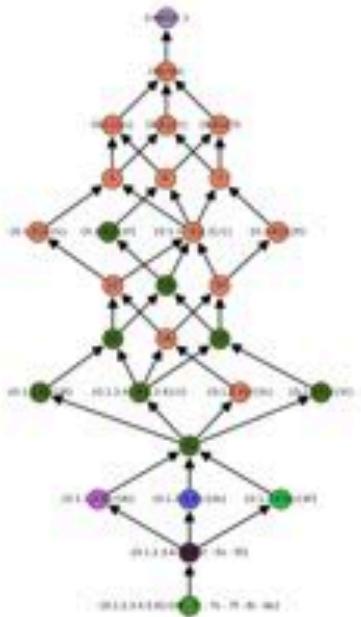


→ Après-midi

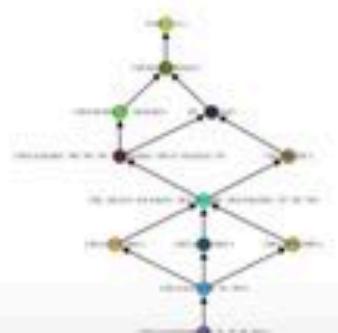
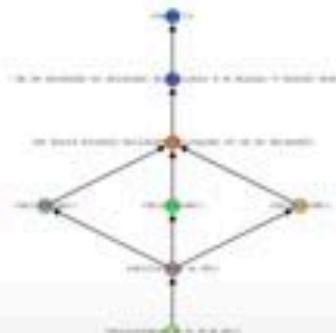
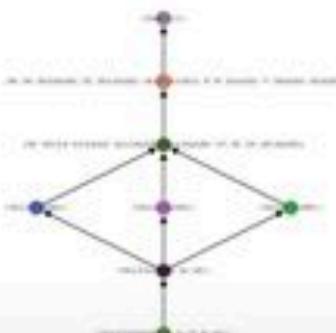
Concept lattice & mathematical morphology

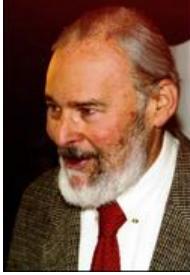
Exemple de descripteurs harmonico-morphologiques chez Ligeti

Congruence $\theta - \theta_\delta - \theta_\varepsilon$ sur $\mathbb{C}(7\text{-tet})$ engendrées par: $H_{\mathbb{C}}^{\mathfrak{M}}$ - $\delta(H_{\mathbb{C}}^{\mathfrak{M}})$ - $\varepsilon(H_{\mathbb{C}}^{\mathfrak{M}})$



Treillis-quotients: neutre $\mathbb{C}(\mathfrak{M})/\theta$ - dilatation $\mathbb{C}(\mathfrak{M})/\theta_\delta$ - érosion $\mathbb{C}(\mathfrak{M})/\theta_\varepsilon$.





D. Lewin

Action
simplément
transitive

Système d'Intervalles Généralisés - Système Généralisé d'Intervalles David Lewin's *Generalized Interval System* [GMIT, 1987]

$$\text{GIS} = (S, G, \text{int})$$

S = ensemble

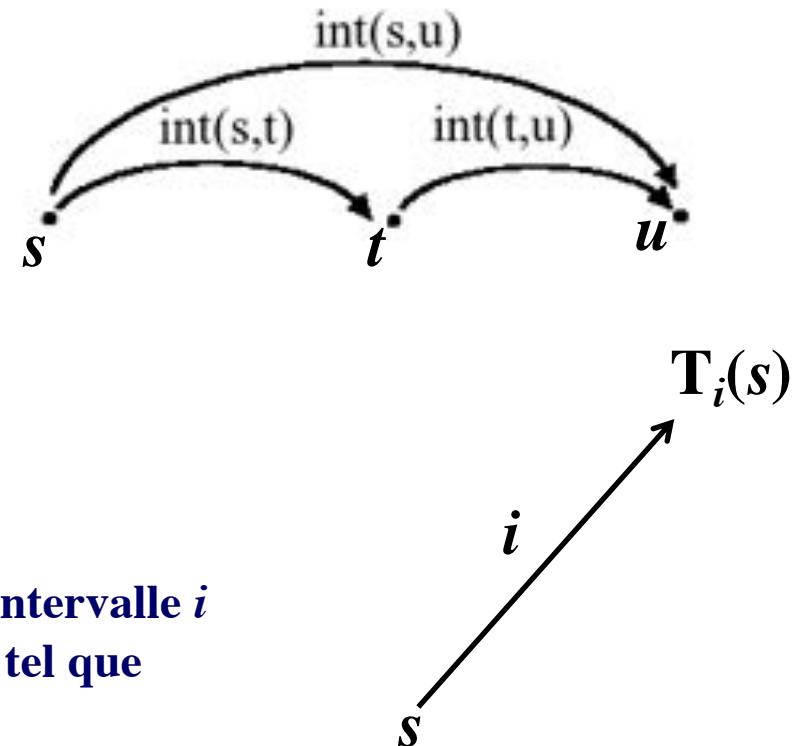
(G, \bullet) = groupe d'intervalles

int = fonction intervallique

$$S \times S \xrightarrow{\text{int}} G$$

1. Pour tous objets s, t, u dans S :
 $\text{int}(s, t) \bullet \text{int}(t, u) = \text{int}(s, u)$

2. Pour tout objet s dans S et tout intervalle i dans G il y a un seul objet t dans S tel que
 $\text{int}(s, t) = i$



Soit $\tau = \{T_i ; i \in G\}$ le groupe des transpositions

$\text{GIS} = (S, G, \text{int}) \Leftrightarrow \tau \times S \rightarrow S$ telle que $(T_i, s) \rightarrow T_i(s)$ où $\text{int}(s, T_i(s)) = i$

- Extension de la théorie transformationnelle aux groupoïdes et aux actions générales de groupoïdes (thèse J. Mandereau, 2011-2013)
- Liens avec les Systèmes Evolutifs à Mémoire (thèse G. Genuys, 2014-2017)



K-nets as a transformational construction

D. Lewin, "A Tutorial on K-nets using the Chorale in Schoenberg's Op.11, N°2 », JMT, 1994

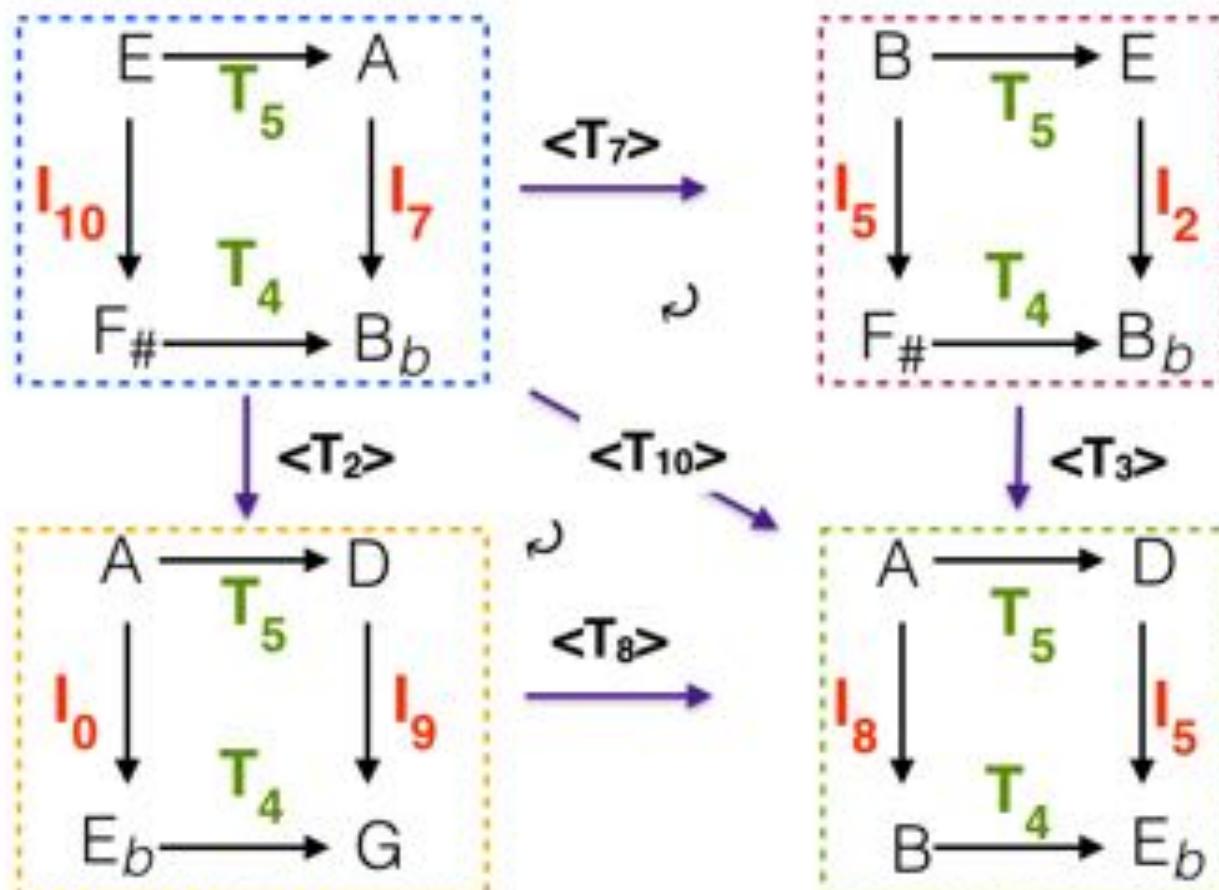


D. Lewin

H. Klumpenhouwer

A musical score excerpt consisting of two staves. The top staff is in treble clef and the bottom staff is in bass clef. The music includes various notes (eighth and sixteenth notes) and rests, with some notes having accidentals like sharps and flats. A dashed blue box highlights a segment of the melody.

$$\langle T_k \rangle : T_m \rightarrow T_m \\ I_m \rightarrow I_{k+m}$$



K-nets as a transformational construction

D. Lewin, "A Tutorial on K-nets using the Chorale in Schoenberg's Op.11, N°2 », JMT, 1994



D. Lewin

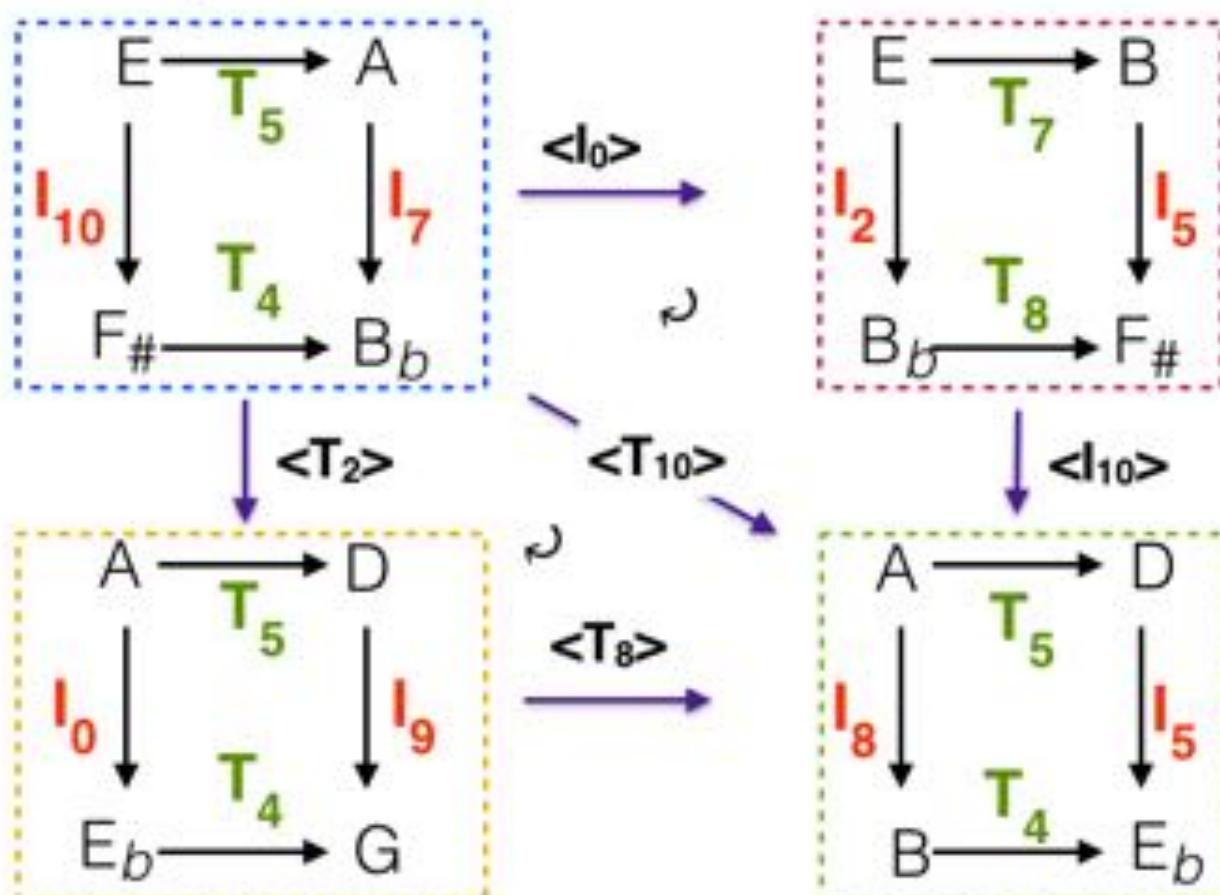
H. Klumpenhouwer



$$\langle T_k \rangle : T_m \rightarrow T_m \\ I_m \rightarrow I_{k+m}$$

$$\langle I_k \rangle : T_m \rightarrow T_{-m} \\ I_m \rightarrow I_{k-m}$$

$$\langle T_k \rangle \circ \langle T_m \rangle = \langle T_{k+m} \rangle \\ \langle I_k \rangle \circ \langle I_m \rangle = \langle I_{m-k} \rangle$$



K-nets as a transformational construction

D. Lewin, "A Tutorial on K-nets using the Chorale in Schoenberg's Op.11, N°2 », JMT, 1994



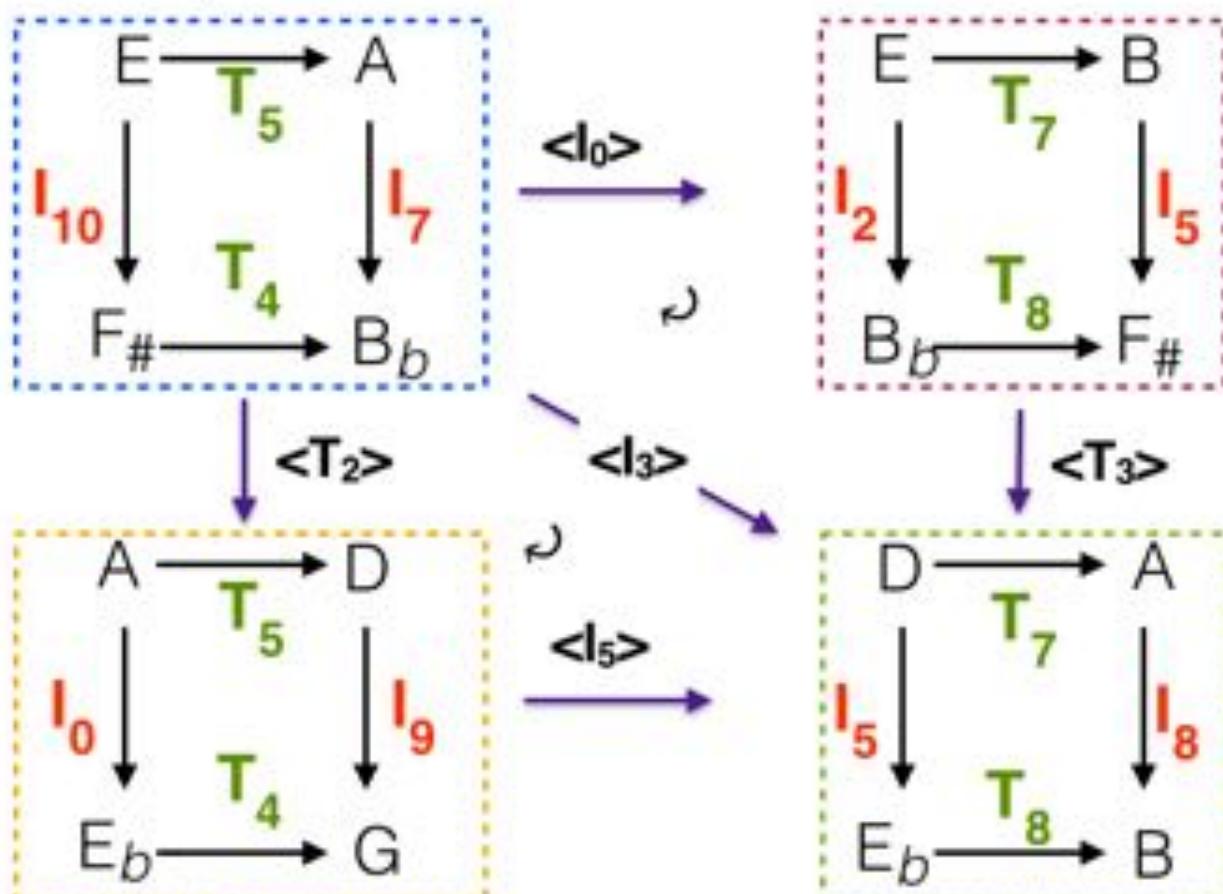
D. Lewin

H. Klumpenhouwer

$$\langle T_k \rangle : T_m \rightarrow T_m \\ I_m \rightarrow I_{k+m}$$

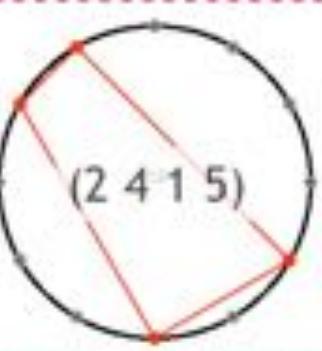
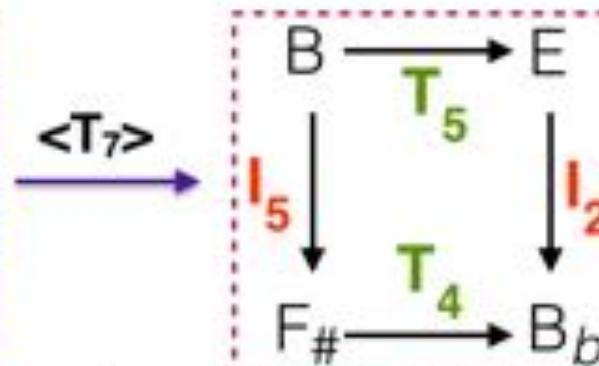
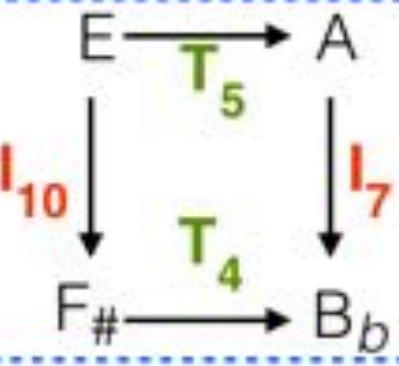
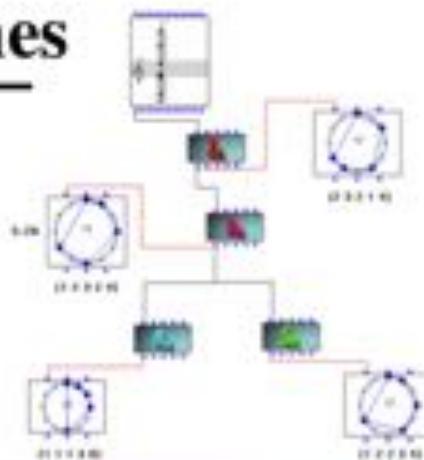
$$\langle I_k \rangle : T_m \rightarrow T_{-m} \\ I_m \rightarrow I_{k-m}$$

$$\begin{aligned} \langle T_k \rangle \circ \langle T_m \rangle &= \langle T_{k+m} \rangle \\ \langle T_k \rangle \circ \langle I_m \rangle &= \langle I_{m-k} \rangle \\ \langle I_m \rangle \circ \langle T_k \rangle &= \langle I_{k+m} \rangle \\ \langle I_k \rangle \circ \langle I_m \rangle &= \langle T_{m-k} \rangle \end{aligned}$$



Transformational vs set-theoretical approaches

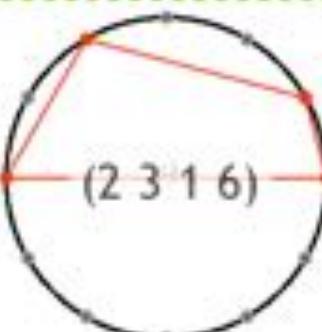
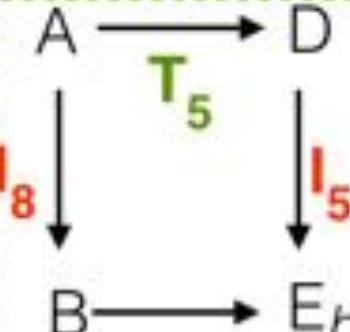
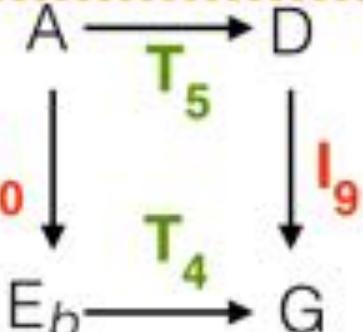
D. Lewin, "A Tutorial on K-nets using the Chorale in Schoenberg's Op.11, N°2 ", JMT, 1994



$\downarrow < T_2 >$

\curvearrowleft

$\downarrow < T_3 >$



$\downarrow < T_8 >$

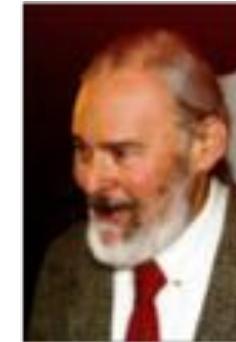
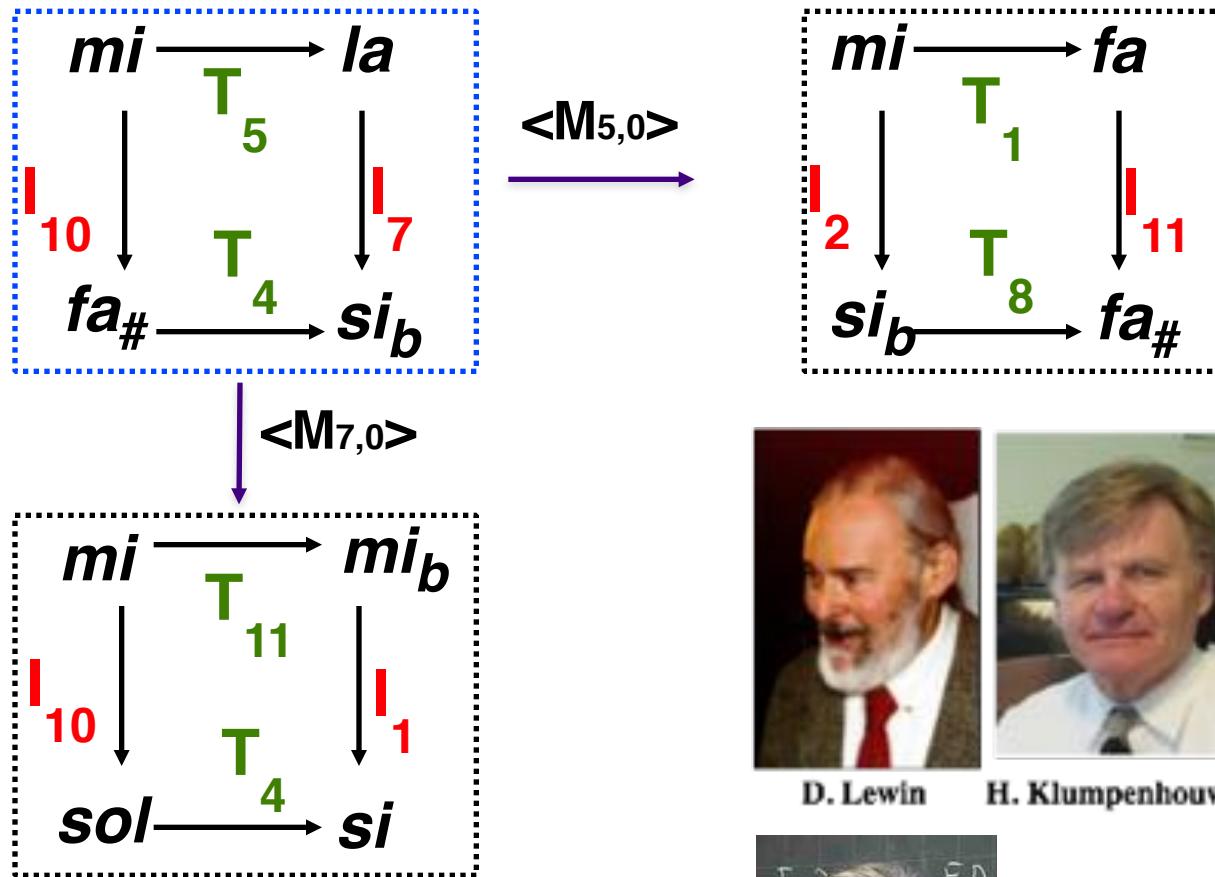
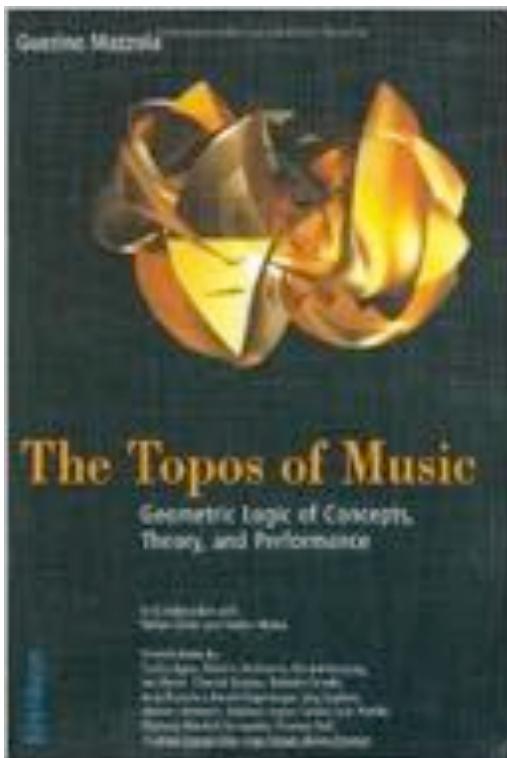
Affine isographies

$$\langle M_{5,k} \rangle : T_m \rightarrow T_{5m}$$

$$I_m \rightarrow I_{k+5m}$$

$$\langle M_{7,k} \rangle : T_m \rightarrow T_{7m}$$

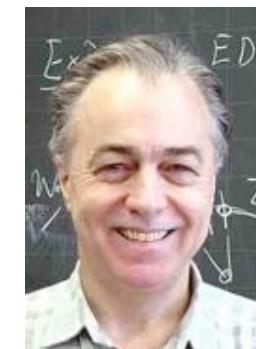
$$I_m \rightarrow I_{k+5m}$$



D. Lewin

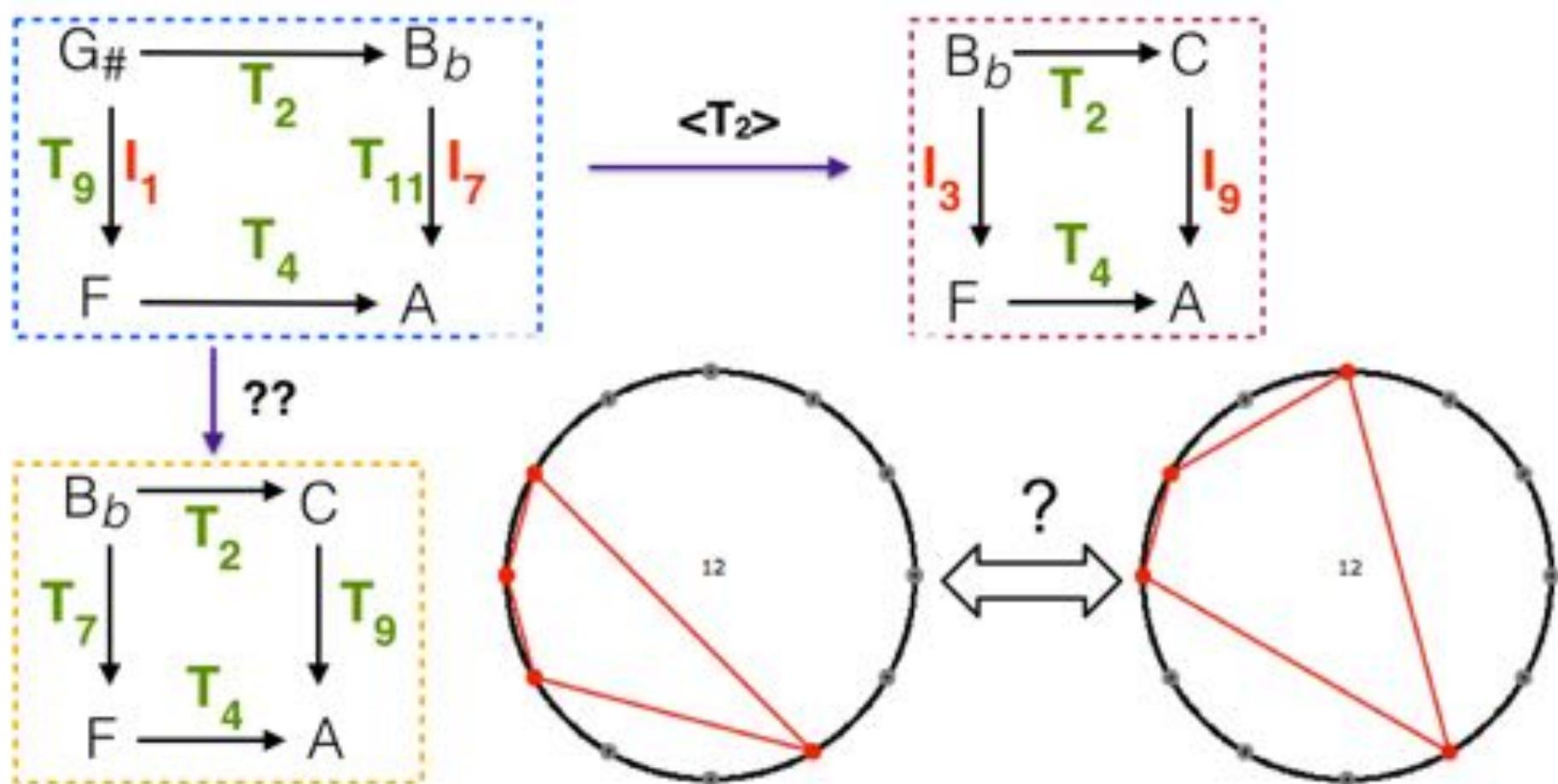


H. Klumpenhouwer



G. Mazzola

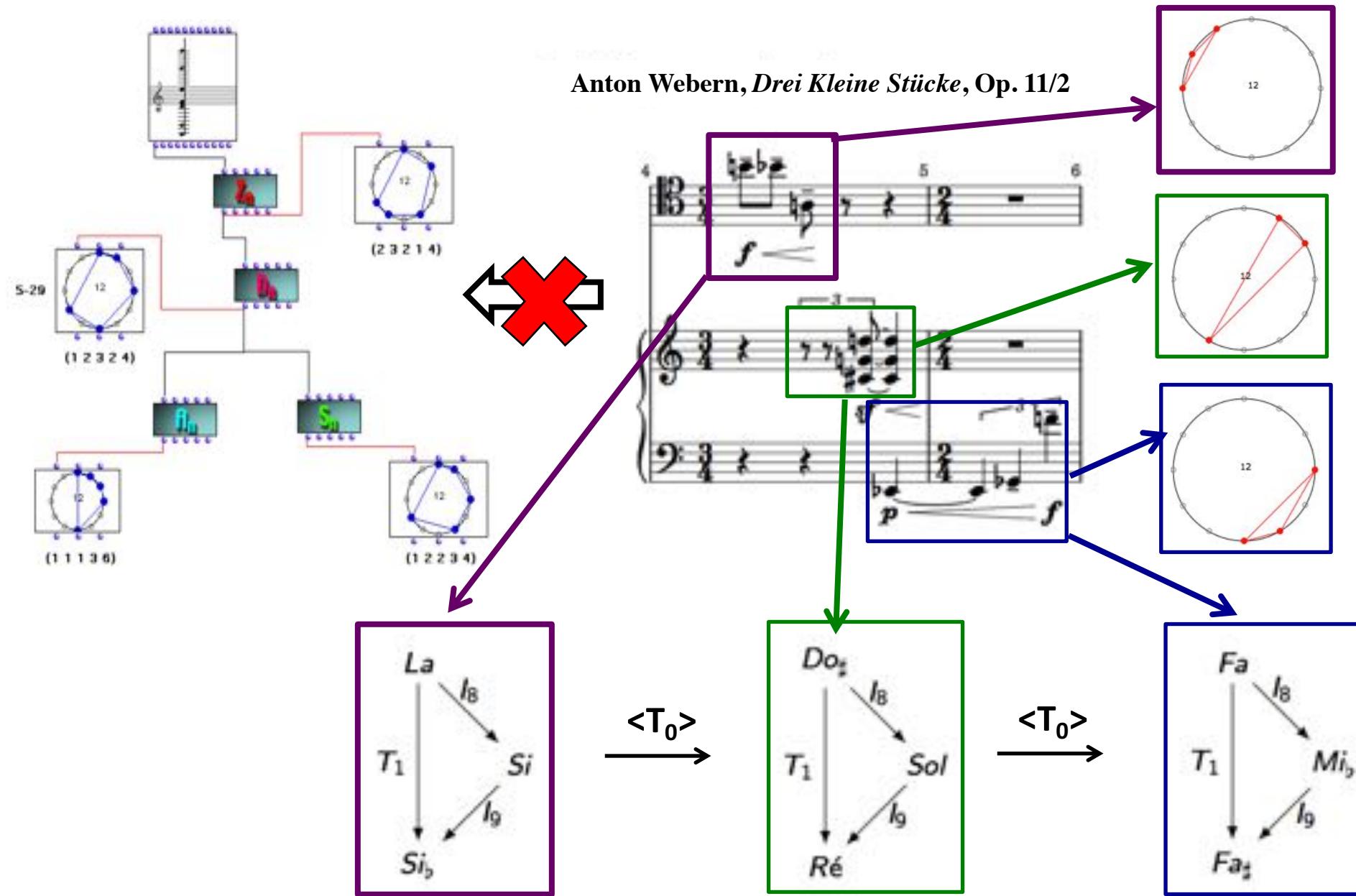
Some theoretical difficulties with the isographic relations



CONCLUSION

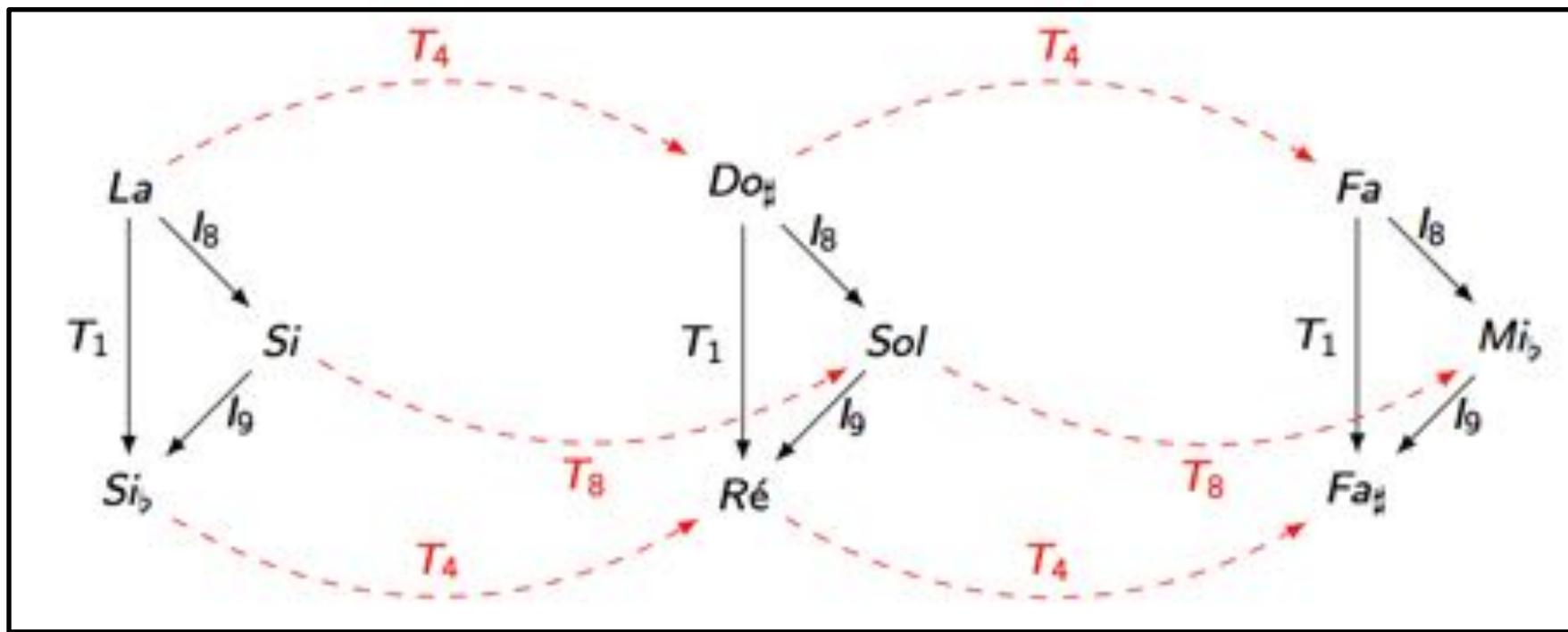
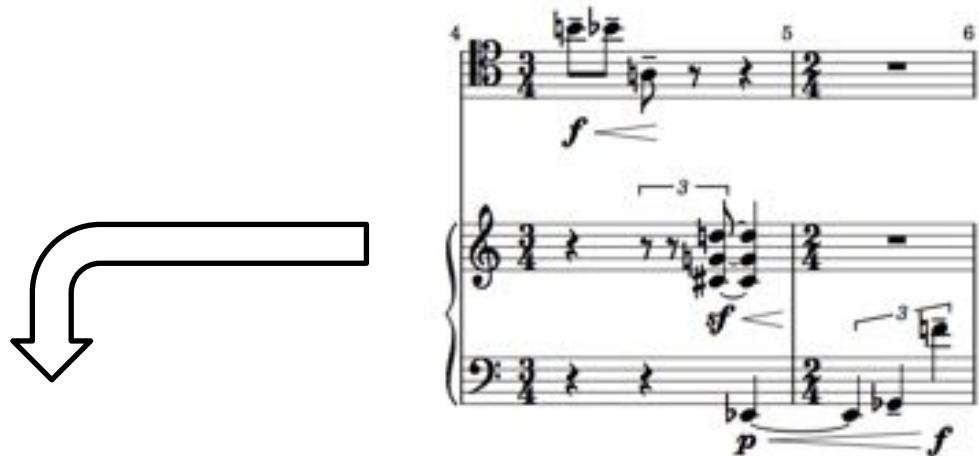
There are K-Nets which are not always isographic to a given one, i.e. the isographic relations are highly sensitive to the transformations used to label the arrows.
Is it possible to overstep this theoretical limitation? Which new definition of K-nets allows one to do that?

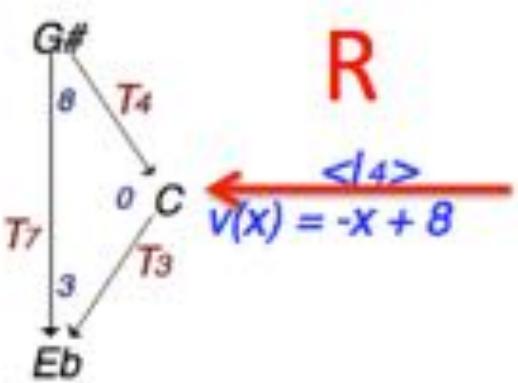
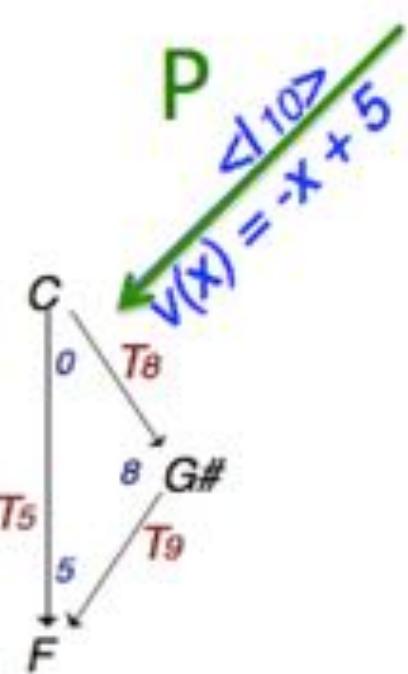
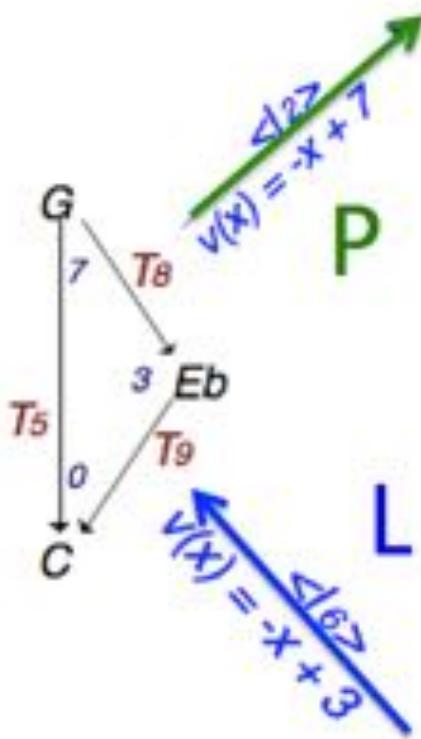
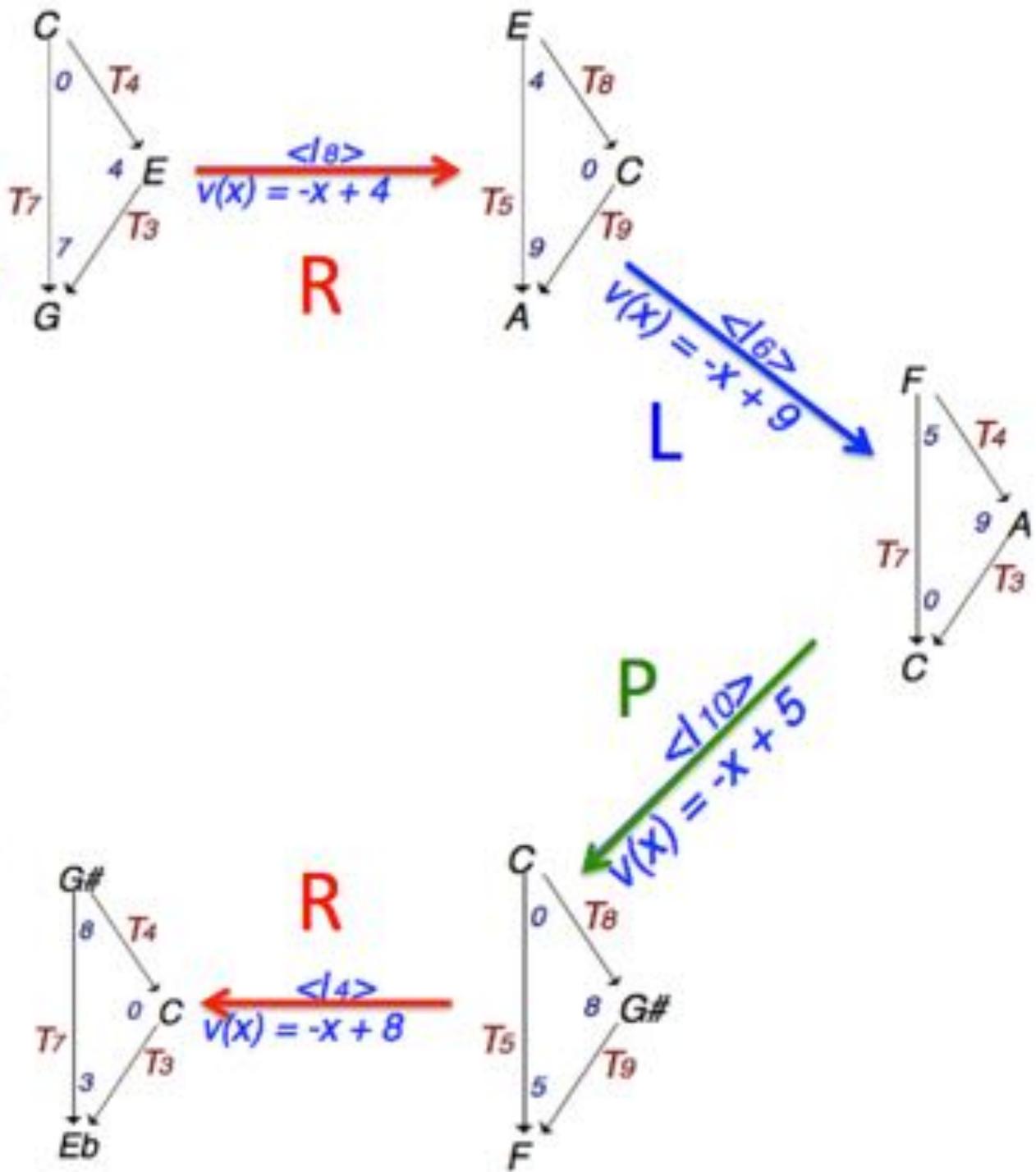
K-Nets and the paradigmatic approach

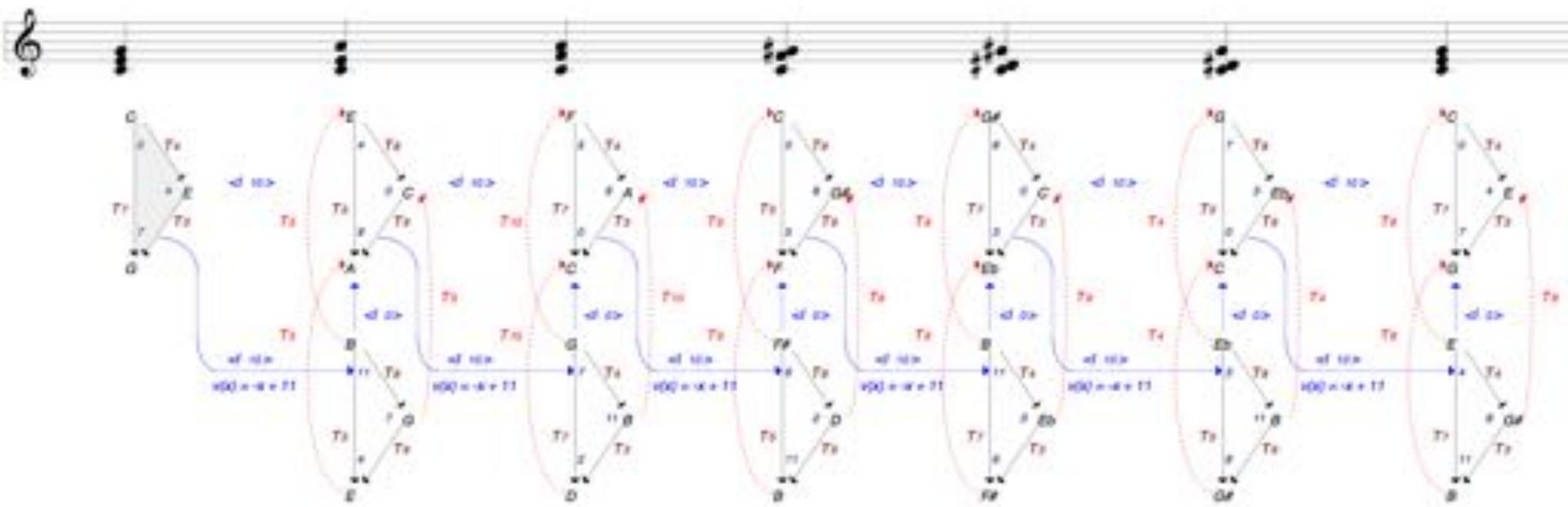
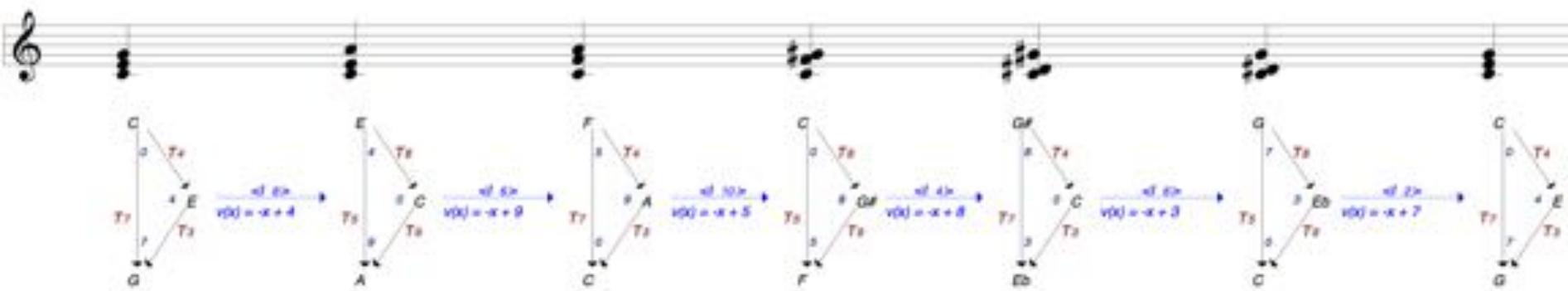


Local isographies: node-to-node applications

Webern, *Drei Kleine Stücke*, Op. 11/2

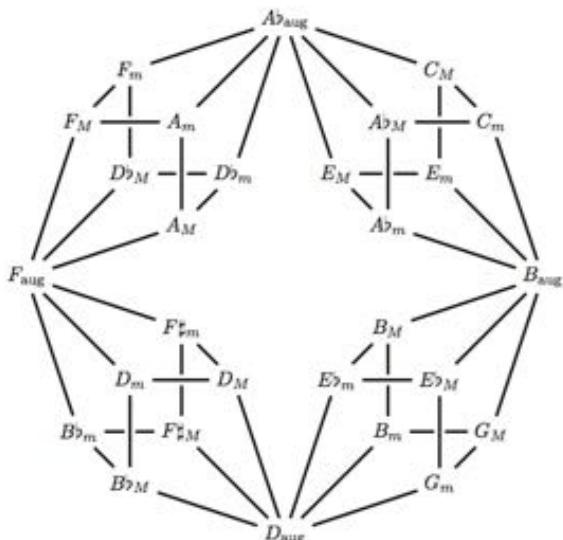








(a)



$$D_M \xrightarrow{\mathcal{U}} D_{\text{aug}} \xrightarrow{\mathcal{U}} G_m \xrightarrow{\mathcal{P}} G_M \xrightarrow{\mathcal{U}} B_{\text{aug}} \xrightarrow{\mathcal{U}} C_m$$

$$C_M \xleftarrow[\mathcal{U}]{\mathcal{P}} A_{\text{b}\text{aug}} \xrightarrow[\mathcal{U}]{\mathcal{U}} F_m \xrightarrow[\mathcal{P}]{\mathcal{U}} F_M \xrightarrow[\mathcal{U}]{\mathcal{U}} F_{\text{aug}} \xrightarrow[\mathcal{U}]{\mathcal{U}} B_{\text{b}m}$$

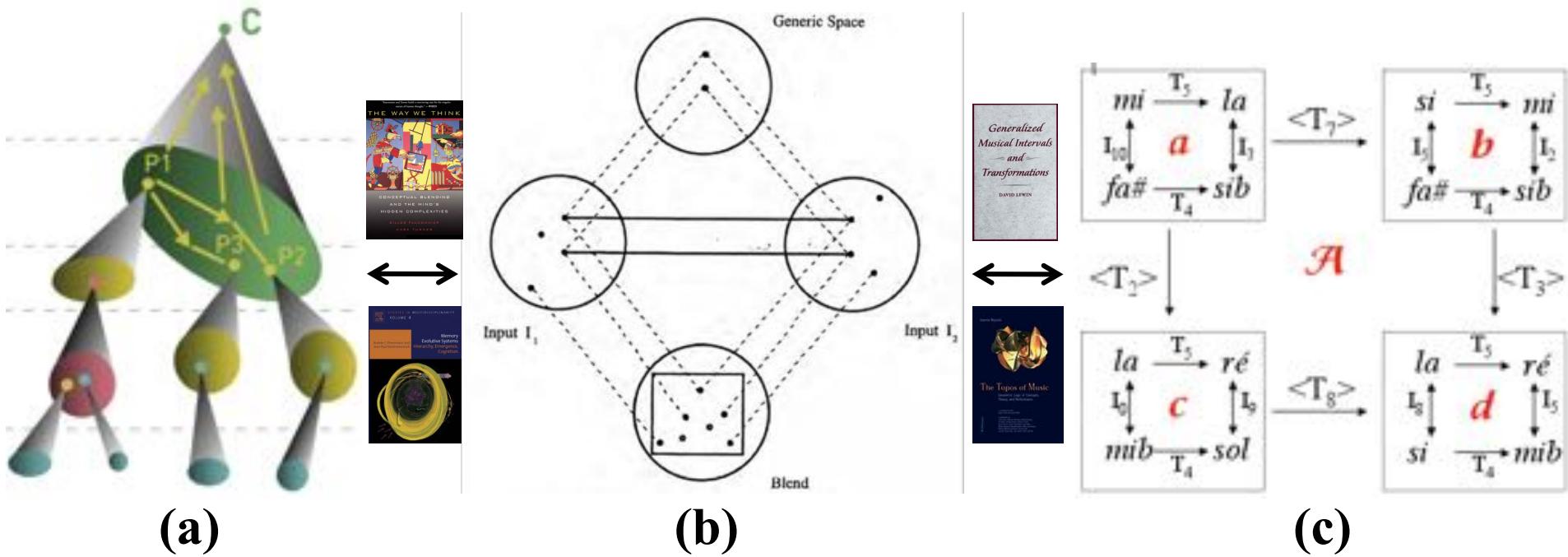
(b)

$$\begin{array}{ccccccc}
D_M & \xrightarrow{\mathcal{U}} & G_M & \xrightarrow{\mathcal{U}} & C_M & \xrightarrow{\mathcal{U}} & F_M \\
\downarrow \mathcal{U}^2 & \xrightarrow{(N=Id, F\nu)} & \downarrow \mathcal{U}^2 & \xrightarrow{(N=Id, F\nu)} & \downarrow \mathcal{U}^2 & \xrightarrow{(N=Id, F\nu)} & \downarrow \mathcal{U}^2 \\
D_{\text{aug}} & \xrightarrow{\mathcal{U}} & B_{\text{aug}} & \xrightarrow{\mathcal{U}} & A_{\text{b}\text{aug}} & \xrightarrow{\mathcal{U}} & F_{\text{aug}} \\
& \searrow \mathcal{U} & & \searrow \mathcal{U} & \searrow \mathcal{U} & \searrow \mathcal{U} & \\
& G_m & & C_m & & F_m & \\
& & \searrow \mathcal{U} & & \searrow \mathcal{U} & & \searrow \mathcal{U} \\
& & B_{\text{b}m} & & A_{\text{M}} & & F_{\text{b}m}
\end{array}$$

(c)

Figure 11. (a) Reduction of the opening progression of *Take A Bow* from Muse (the first twelve chords are represented here). (b) First transformational analysis in the $M_{\mathcal{UPL}}$ monoid showing the sequential regularity of the progression. (c) Second transformational analysis in the $M_{\mathcal{UPL}}$ monoid showing the successive transformations of the initial three-chord cell by the homography $(N = Id, F\nu)$ with $\nu(n_M) = (n + 5)_M$, $\nu(n_m) = (n + 5)_m$, and $\nu(n_{\text{aug}}) = (n + 1 \pmod 4)_{\text{aug}}$.

Towards a categorical explanation of music perception?



(a) Processus de « colimite » à la base des systèmes évolutifs à mémoire (Ehresmann et Vanbremersch, 2007) ; (b) réseau minimal pour le « blending conceptuel » (Fauconnier & Turner, 2002) et exemple de Klumpenhouwer Network (ou *K*-net).

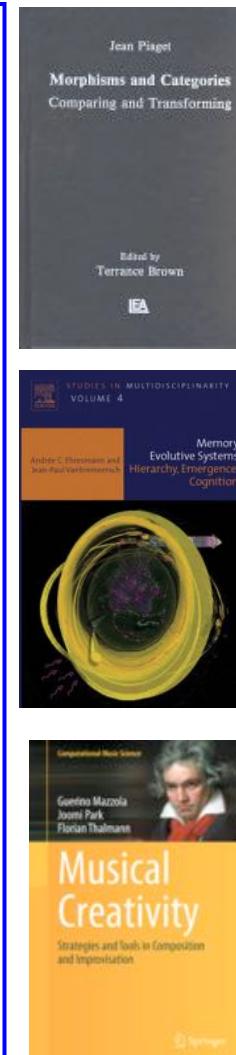
« La théorie des catégories est une théorie des constructions mathématiques, qui est macroscopique, et procède d'étage en étage. Elle est un bel exemple d'**abstraction réfléchissante**, cette dernière reprenant elle-même un principe constructeur présent dès le stade sensori-moteur. Le **style catégoriel** qui est ainsi à l'image d'un aspect important de la **genèse des facultés cognitives**, est un style adéquat à la description de cette genèse »



J. Piaget

Category Theory and Cognition

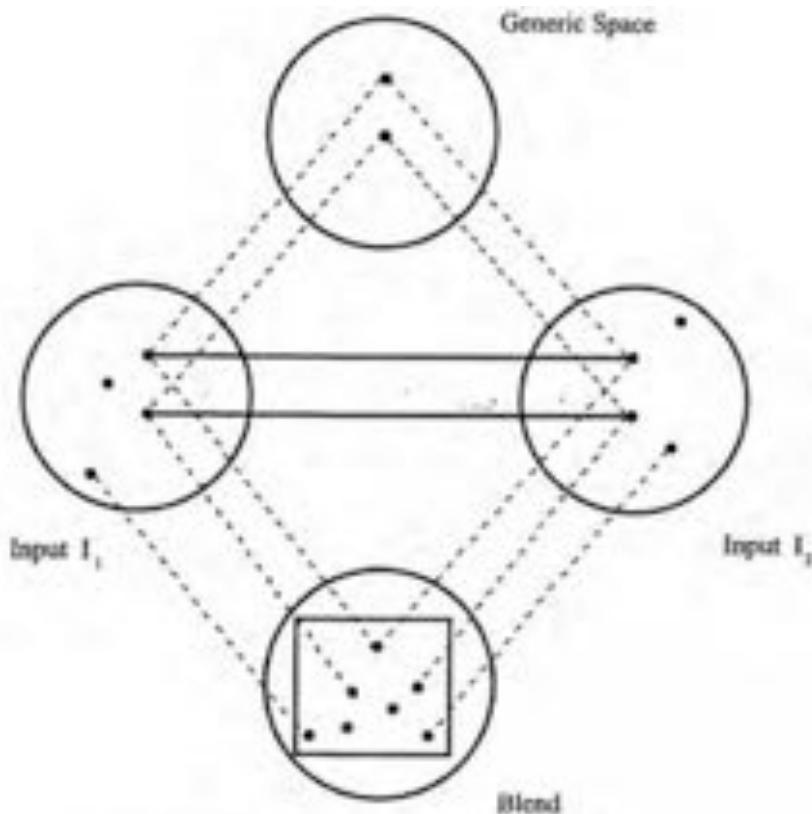
- G. S. Halford & W. H. Wilson, “A Category Theory Approach to Cognitive Development”, *Cognitive Psychology*, 12, 1980
- **J. Piaget, Gil Henriques et Edgar Ascher, *Morphisms and Categories: Comparing and Transforming* (orig. French, 1990)**
- J. Macnamara & G. E. Reyes, *The Logical Foundation of Cognition*, OUP, 1994
- A. Ehresmann, J.-P Vanbremecem, *Memory Evolutive Systems, Hierarchy, Emergence, Cognition*, 2007
- **A. Ehresmann, J.-P. Vanbremecem, “MENS, a mathematical model for cognitive systems”, *Journal of Mind Theory*, 2009**
- S. Phillips, W. H. Wilson, “Categorial Compositionality: A Category Theory Explanation for the Systematicity of Human Cognition”, PLoS Comp. Biology, 6(7), July 2010
- S. Phillips, W. H. Wilson, “Categorial Compositionality II: Universal Constructions and a General Theory of (Quasi-)Systematicity in Human Cognition, PLoS Comp. Biology, 7(8), August 2011
- A. Ehresmann, “MENS, an Info-Computational Model for (Neuro-)cognitive Systems Capable of Creativity”, *Entropy*, 2012
- **G. Mazzola, *Musical Creativity*, Springer, 2012**
- M. Andreatta, Andreatta M., A. Ehresmann, R. Guitart, G. Mazzola, “Towards a categorical theory of creativity”, Fourth International Conference, MCM 2013, McGill University, Montreal, June 12-14, 2013, Springer, 2013.



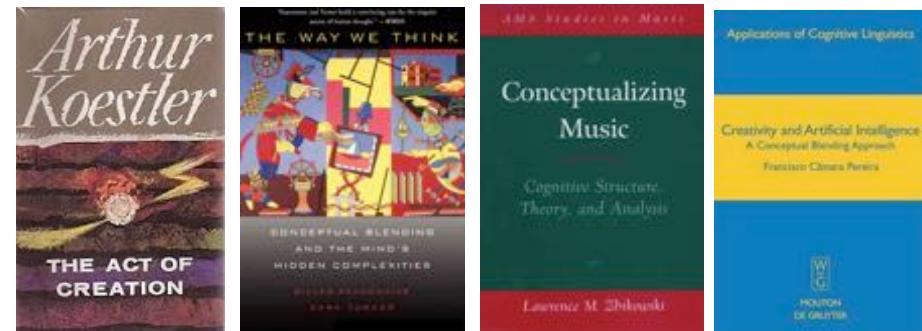
Category theory offers a re-conceptualization for cognitive science, analogous to the one that Copernicus provided for astronomy, where representational states are no longer the center of the cognitive universe —replaced by the relationships between the maps that transform them [S. Phillips, W. H. Wilson, 2010].

Creative processes and conceptual blending

- A. Koestler, *The act of creation*, 1964
- L. Zbikowski, « Seeger's Unitary Field Theory Reconsidered ». In: Yung, Bell & Helen Rees (eds). *Understanding Charles Seeger, Pioneer in American Musicology*. Illinois: University of Illinois Press. 1999: 130-149.
- G. Fauconnier & M. Turner, *The Way We Think*, 2002
- L. Zbikowski, *Conceptualizing Music: Cognitive Structure, Theory, and Analysis*, 2002
- F. C. Pereira, *Creativity and Artificial Intelligence - A Conceptual Blending Approach*, 2007



Minimal network for the *conceptual blending*
[Fauconnier & Turner, 2002]



[...] Conceptual Blending is as an elaboration of other works related to creativity, namely Bisociation, Metaphor and Conceptual Combination. As such, it attracts the attention of computational creativity modelers and, regardless of how Fauconnier and Turner describe its processes and principles, it is unquestionable that there is some kind of blending happening in the creative mind.

F. C. Pereira, *Creativity and Artificial Intelligence - A Conceptual Blending Approach*, 2007

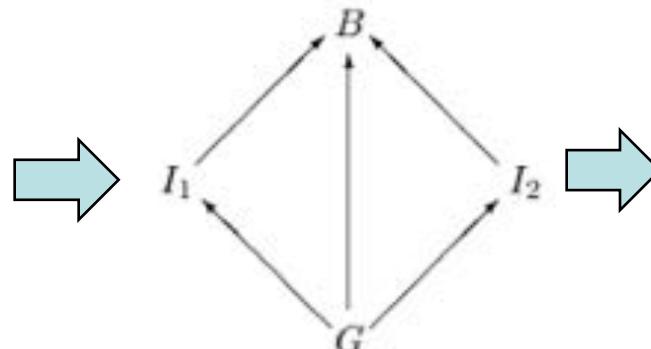
From conceptual to structural blending

- J. Goguen, « A Categorical Manifesto », *Math. Structures in Computer Science*, 1991.
- J. Goguen, « An Introduction to Algebraic Semiotics, with Applications to User Interface Design », 1999
- J. Goguen, « Musical Qualia, Context, Time, and Emotion », in *Journal of Consciousness Studies* 11, 3/4, 117-147, 2004
- J. Goguen, « What is a Concept? », *International Conference on Comp. Science*, 2005
- A. Ehresmann, J.-P Vanbremeerch, Memory Evolutive Systems, Hierarchy, Emergence, Cognition, 2007

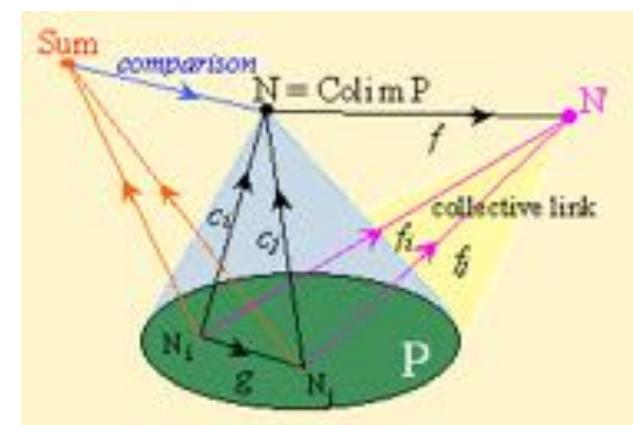


The **category of sign systems with semiotic morphisms** has some additional structure over that of a category: it is an *ordered category*, because of the orderings by quality of representation that can be put on its morphisms. This extra structure gives a richer framework for considering blends; I believe this approach captures what Fauconnier and Turner have called « emergent » structure, without needing any other machinery. [Goguen, 1999, p. 32]

Algebraic/
structural
semiotics

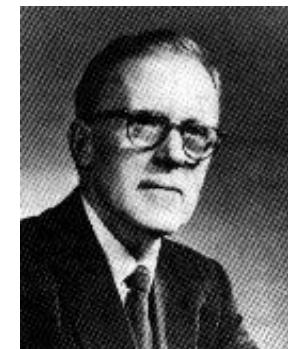
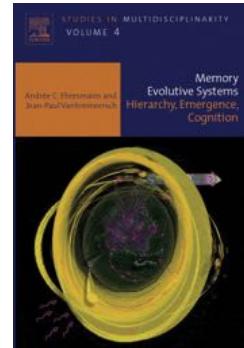
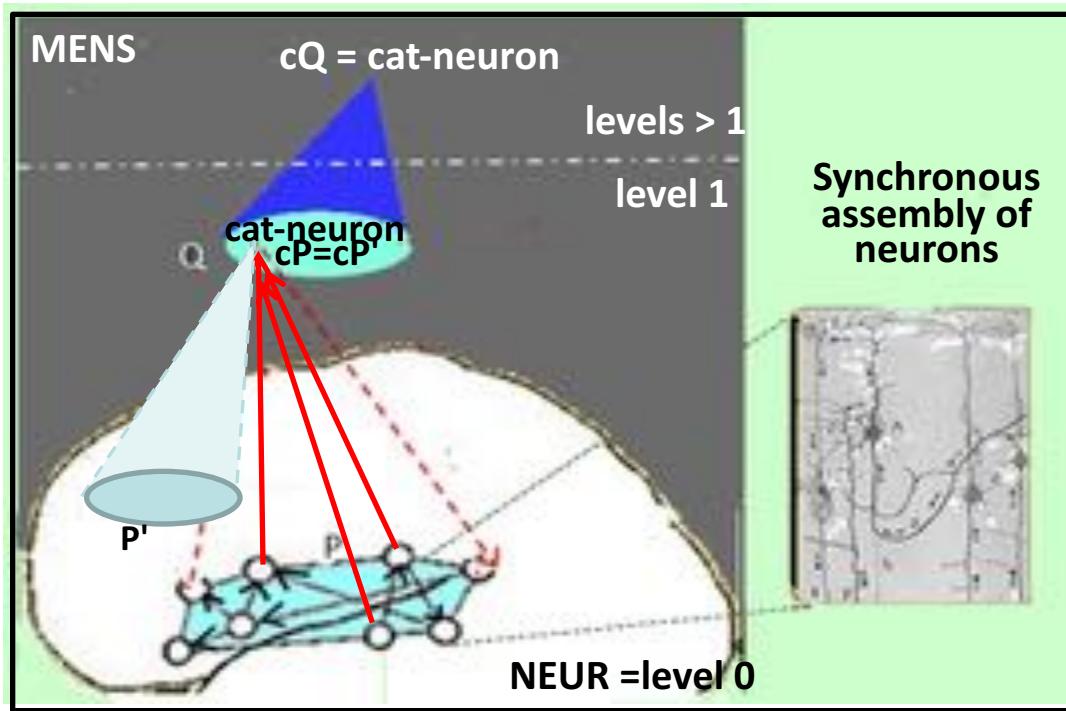


structural blending



Colimit of a diagram

The structure of Memory Evolutive Neural Systems

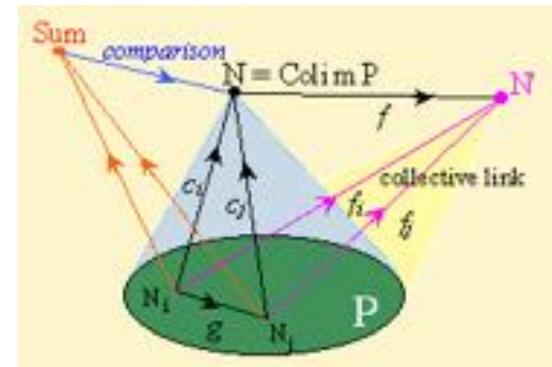


Donald Hebb

It is a MES whose level 0 is the Evolutive System of neurons NEUR. The components represent the neurons and their links are the synaptic paths.

At higher levels: there are more and more complex 'conceptual' objects, called *cat(egory)-neurons*, modeling a mental object as the colimit $cP = cP'$ of the synchronous assemblies of (cat-)neurons P, P' which activate them.

→ Why colimits for the study of creativity?



Hierarchical categories and Emergence



G. Edelman

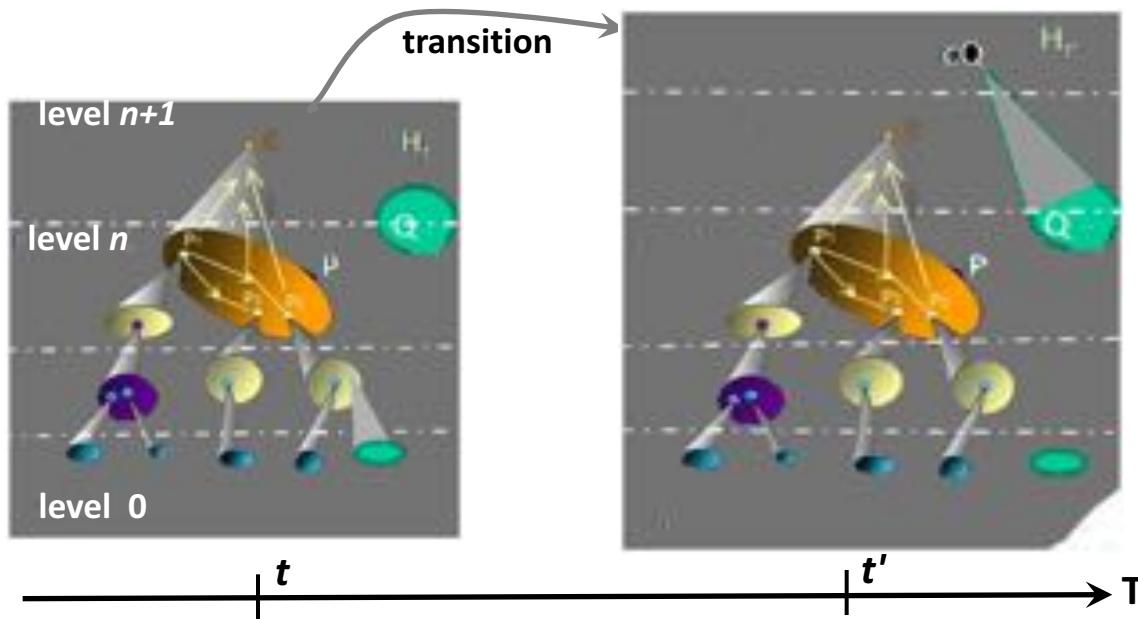
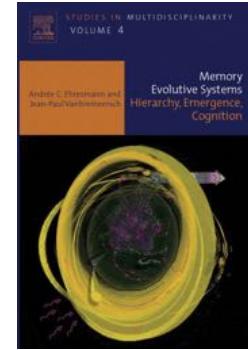
A category is *hierarchical* if its objects are partitioned into levels, so that M of level $n+1$ is the colimit of at least one pattern (= diagram) P of levels $\leq n$.

A morphism $M \rightarrow M'$ is a (P, P') -*simple link* (or n -*simple link*) if it binds a cluster of links between decompositions P and P' of M and M' (of levels $\leq n$).

Multiplicity Principle (or degeneracy principle by Edelman/Gally) = existence of *multiform objects* M which are the colimit of 2 patterns P and Q non isomorphic nor connected by a cluster; then M can switch between them. This 'flexible redundancy' gives flexibility to the system.

→ Existence of n -*complex links* composites of n -*simple links* representing properties 'emerging' at level $n+1$.

Memory Evolutive Systems (MES)

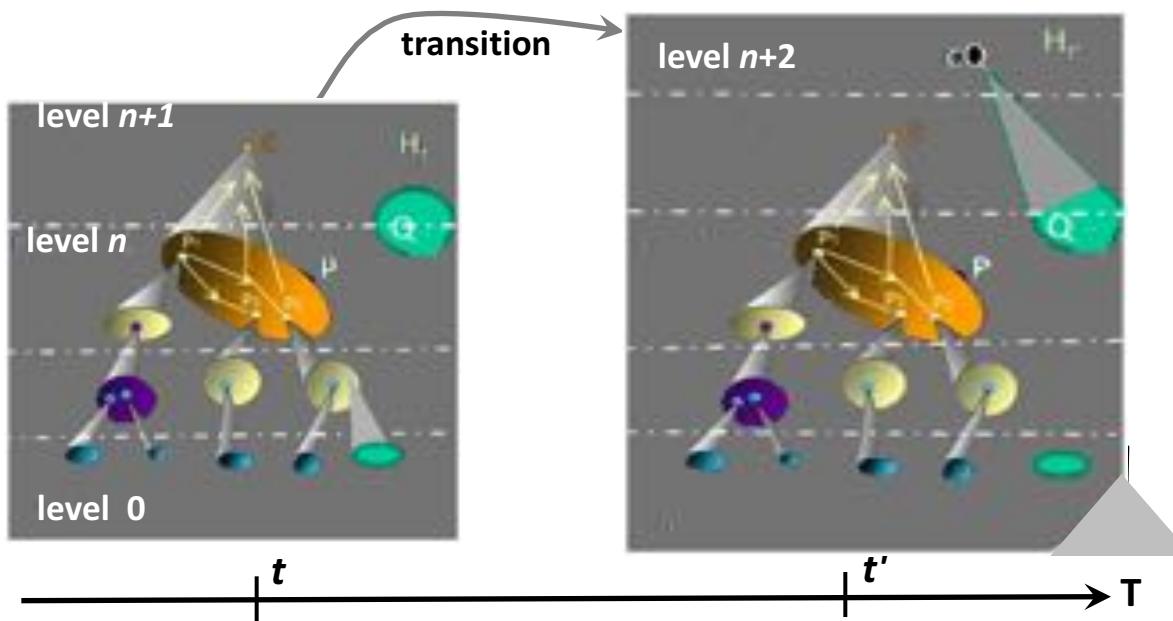
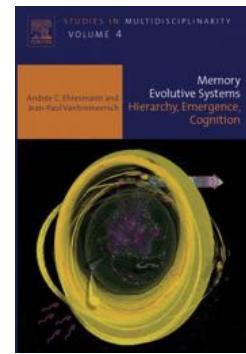


A MES H consists of:

- (i) a timescale T (included in R);
- (ii) a family $(H_t)_{t \in T}$ of hierarchical categories whose objects are the states C_t at t and its morphisms represent channels transmitting information or actions;
- (iii) for $t < t'$, a *transition* functor from a subcategory of H_t to $H_{t'}$. A component C is a maximal set of successive states C_t linked by transitions. Such a component C has at least one *ramification* down to level 0, and its *complexity order* is the shortest length of a ramification

The dynamic is modulated by the interactions between a net of specialized subsystems called *co-regulators* which develop a subsystem *Mem* acting as a long-term memory.

Complexification and Emergence Theorem



In a MES the transition from t to t' results from changes of the following types: ‘adding’ external elements, ‘suppressing’ or ‘decomposing’ some components, adding a colimit to some given patterns. Modeled by the *complexification process*: given a procedure Pr on H_t with objectives of the above kinds, the *complexification* of H_t for Pr is the category $H_{t'}$ in which these objectives are optimally satisfied. It is explicitly constructed in MES (2007).

EMERGENCE THEOREM. *The Multiplicity Principle is necessary for the existence of components of complexity order > 1 and it is preserved by complexification. Two successive complexifications do not reduce to a unique one. Iterated complexifications lead to the emergence of components of increasing orders.*

Towards a categorical theory of creativity (in music, cognition and discourse)

Abstract

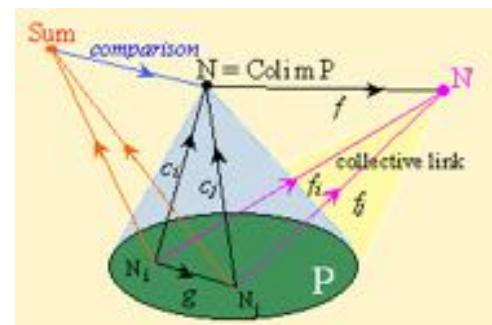
This article presents a first attempt at establishing a **category-theoretical model of creative processes**. The model, which is applied to musical creativity, discourse theory, and cognition, suggests the relevance of the notion of “colimit” as a unifying construction in the three domains as well as the central role played by the Yoneda Lemma in the categorical formalization of creative processes.



MAMUPHI

Séminaire MaMux

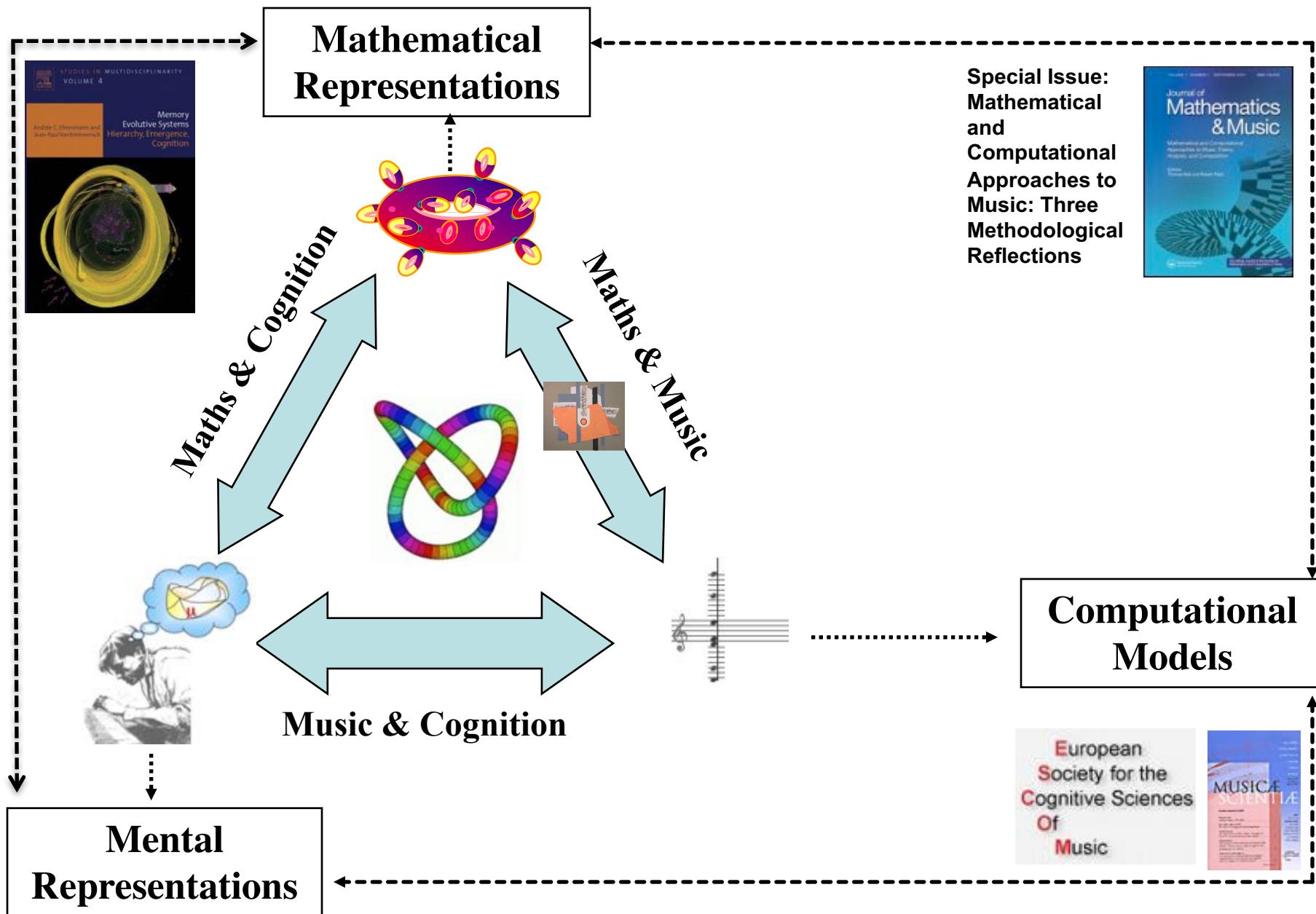
BNSC cherche soutien
STMS, recherche Musicales Bézout
théories théorie logique philosophie
sciences informatique autres logique philosophie
linguistiques mathématiques structures
sémantique cognitives représentation Systèmes musicaux rapport
traîns CNRS mathématique mathématique Représentations
Séminaire distinguées théorie hypothèse pampidou technologie biomimétique
perception UPMC analyse organisation
pertinence paragraphe
spécificité L'art National théâtre



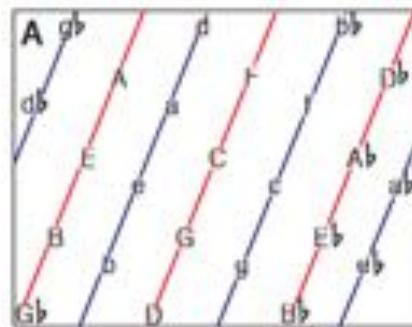
**Andreatta M., A. Ehresmann, R. Guitart, G. Mazzola,
« Towards a categorical theory of creativity », Fourth
International Conference, MCM 2013, McGill
University, Montreal, June 12-14, 2013, Springer, 2013.**

Bridging the gap: mathematical and cognitive approaches

<http://recherche.ircam.fr/equipes/repmus/mamux/Cognition.html>



The neuronal foundation of the Tonnetz



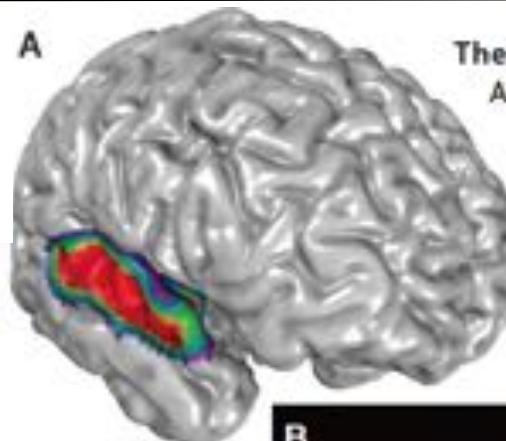
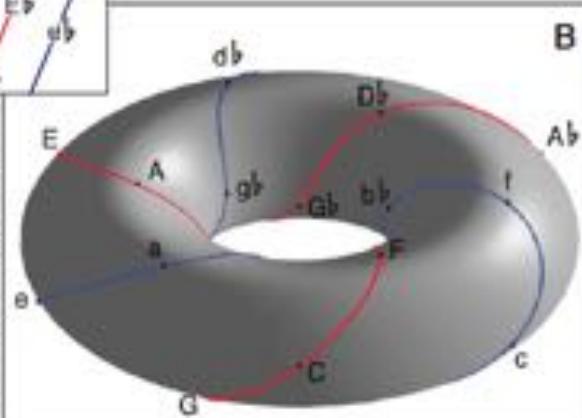
Mental key maps. (A) Unfolded version of the key map, with opposite edges to be considered matched. There is one circle of fifths for major keys (red) and one for minor keys (blue), each

wrapping the torus three times. In this way, every major key is flanked by its relative minor on one side (for example, C major and a minor) and its parallel minor on the other (for example, C major and c minor). (B) Musical keys as points on the surface of a torus.

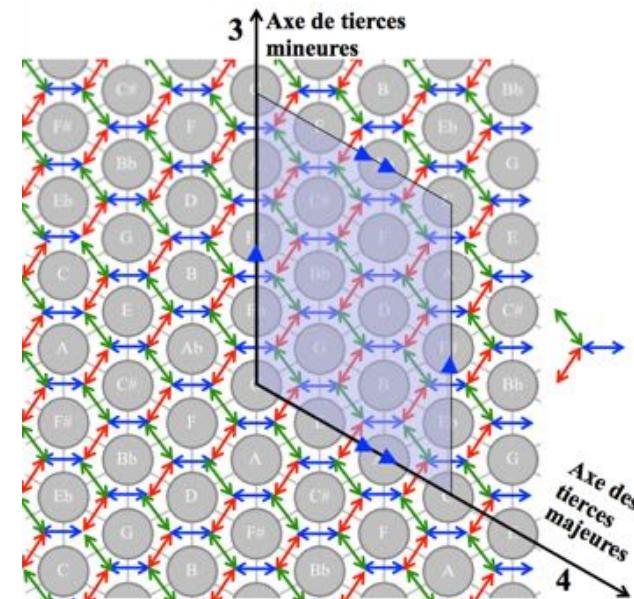
PERSPECTIVES: NEUROSCIENCE

Mental Models and Musical Minds

Robert J. Zatorre and Carol L. Krumhansl



The sensation of music. (A) Auditory cortical areas in the superior temporal gyrus that respond to musical stimuli. Regions that are most strongly activated are shown in red. (B) Metabolic activity in the ventromedial region of the frontal lobe increases as a tonal stimulus becomes more consonant.



Acotto E. et M. Andreatta (2012), « Between Mind and Mathematics. Different Kinds of Computational Representations of Music », *Mathematics and Social Sciences*, n° 199, 2012(3), p. 9-26.

