MATHEMATICAL MUSIC THEORY: AN OVERVIEW AND SOME PEDAGOGICAL EXPERIENCES

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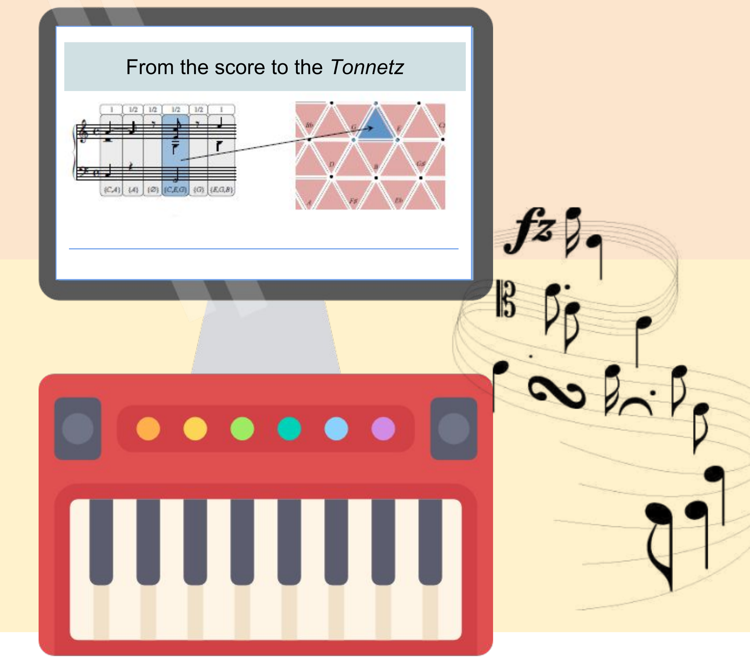


Geometric models in music

- Circular geometric-musical models:
  - musical clock
  - circle of fifths
- Examples of musical graphs:
  - **Tonnetz** (Fig. 1), note-based graph or 2-dimensional simplicial complex in which each 0-simplex represents a note and each 2-simplex represents a major or minor triad
  - **Chicken-wire Torus** (Fig. 2), chord-based graph whose vertices are major and minor triads
  - **Generalized Chicken-wire Torus** for seventh chords (Fig. 3), chord-based graph whose vertices are sevenths.

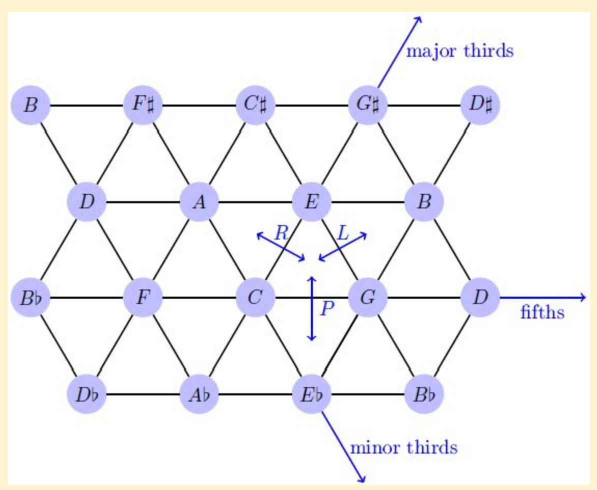


Fig. 1: *Tonnetz*

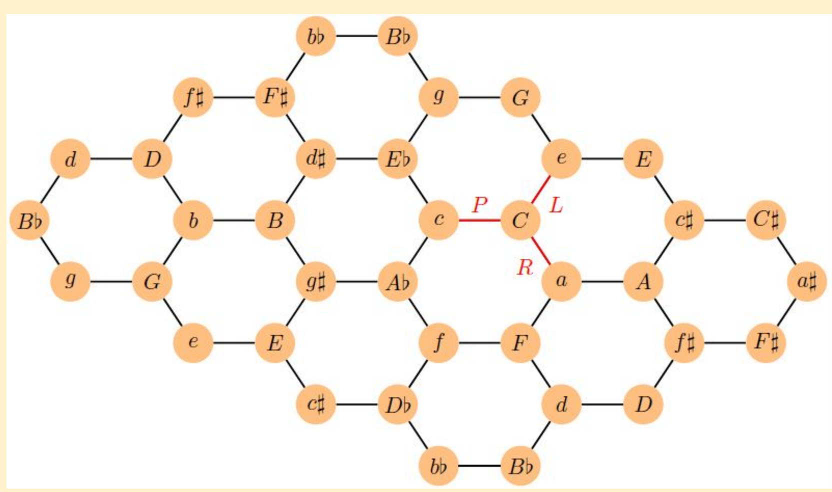


Fig. 2: *Chicken-wire Torus*

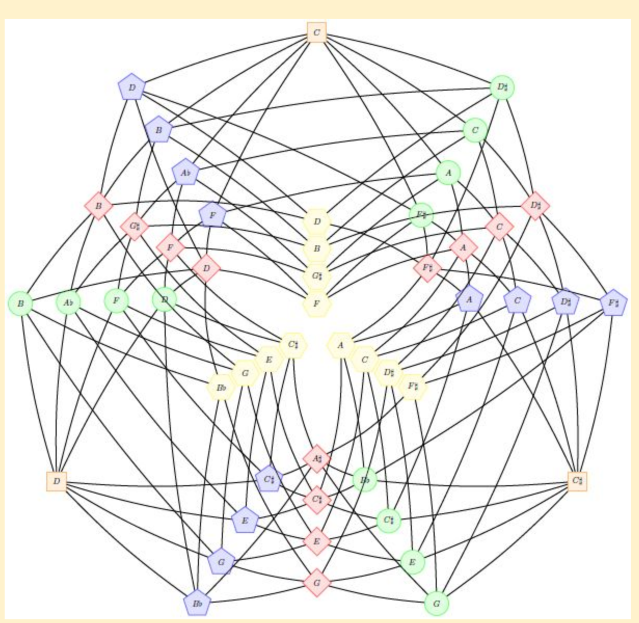


Fig. 3: *Generalized Chicken-wire Torus for seventh chords*

Why geometry in music?

- It is an intuitive tool to visualize and describe sequences of chords, in particular the property of stepwise motion in parsimonious voice leading (interaction among the different melodic lines creating the harmony).
- It provides music-analytical tools for compositional applications. Example: Hamiltonian cycles in chord-based graphs. Example: geometric transformations can modify musical sequences creating new musical ideas.

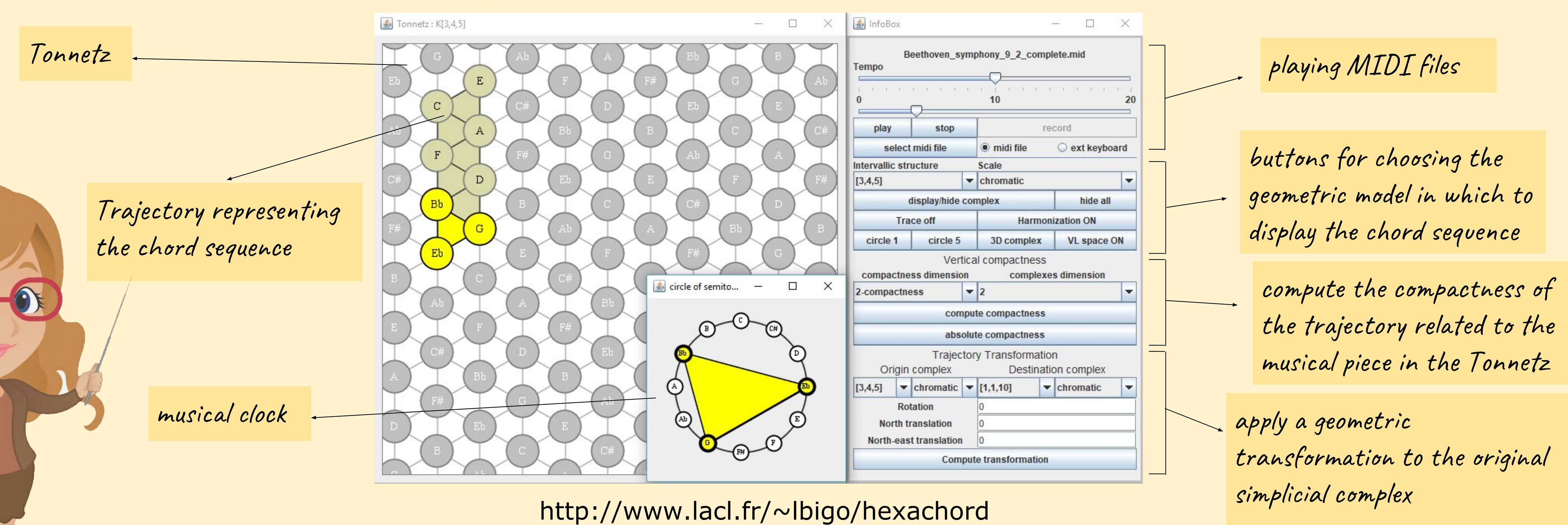
Introduction and aim

- Despite a long historical relationship between mathematics and music, the research in Mathematical Music Theory is very recent.
- Our aim is to give an overview on some main topics based on geometry and algebra, and some applications.

Pedagogical experiences and results

We have proposed some pedagogical mathemusical experiences via **HexaChord** in two Festival of Science, one in France (Strasbourg) and one in Italy (Cagliari).

HexaChord is a computer aided-music analysis environment developed by Louis Bigo, in which different geometric-musical models are integrated.



http://www.lacl.fr/~lbigo/hexachord

Festival of Science - Strasbourg

- **Participants**: children (different ages)
- **Materials**:
  - computer with HexaChord
  - piano keyboard connected to the pc
  - wooden musical-clock and Tonnetz.
- **Observations and results**:
  - children played more with the wooden models, more familiar to already known games
  - when a guide explained the features of HexaChord, the children showed interest and they understood very quickly its behavior
  - during the absence of a guide to the piano and the computer, the younger children did not understand how the musical sequences were displayed, then they moved on the geometric wooden models

Festival of Science - Cagliari

- **Participants**: two classes of students belonging to the last year of a high school with a musical program.
- **Materials**:
  - computer with HexaChord
  - video projector.
- **Observations and results:**
  - thanks to their musical background, they easily understood the features of HexaChord and its utility for music analysis
  - their reactions to computational approaches to music analysis based on geometric models were very positive
  - great surprise in discovering that mathematics can also be a useful tool for learning music

Algebraic tools

Musical graphs are related to musical transformations of the transformational theory, a branch of Mathematical Music Theory based on the use of algebraic structures to define musical transformations.

- The edges in the *Chicken-wire Torus* and the "edge-flips" in the Tonnetz correspond to the neo-Riemannian transformations *P*, *L* and *R*. These 3 transformations describe the parsimonious voice leading between major and minor triads.

> **Theorem**: the *PLR*-group generated by *P*, *L* and *R* acts on the set of the 24 major and minor triads generating a group isomorphic to the dihedral group D_{12} of order 24.

- We have extended the studies on triads to seventh chords. The edges of the generalized Chicken-wire Torus for sevenths correspond to the 17 parsimonious operations on dominant, minor, half-diminished, major and diminished sevenths. We have generalized the *PLR*-group for these types of sevenths.

> **Theorem**: Let *PLRQ* be the group generated by the 17 most parsimonious transformations acting on the set of dominant, minor, half-diminished, major and diminished sevenths. Then $PLRQ \simeq S_5 \ltimes \mathbb{Z}_{12}^3$

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