# Maths in song writing: a focus on some Oumupian experiences

W&K-FORUM-Symposium:
Music & Mathematics
Salzburg, 2 June 2023
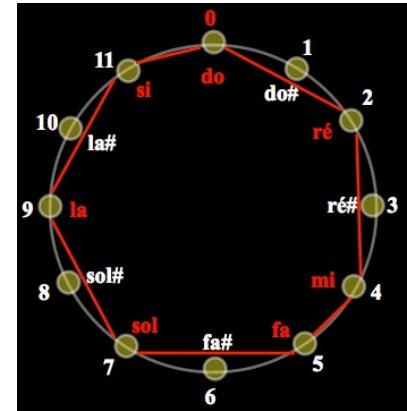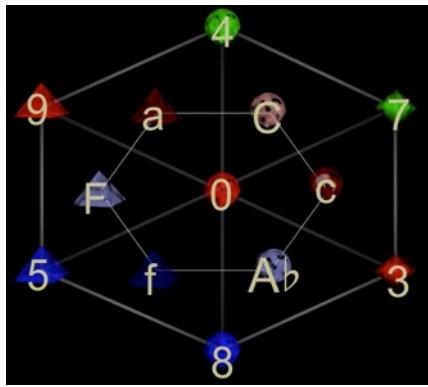
Moreno ANDREATTA

CNRS / IRMA / Université de Strasbourg
IRCAM / Sorbonne Université

www.morenoandreatta.com

A Song for π

1 2 3 4 5 6 7 8 9
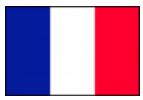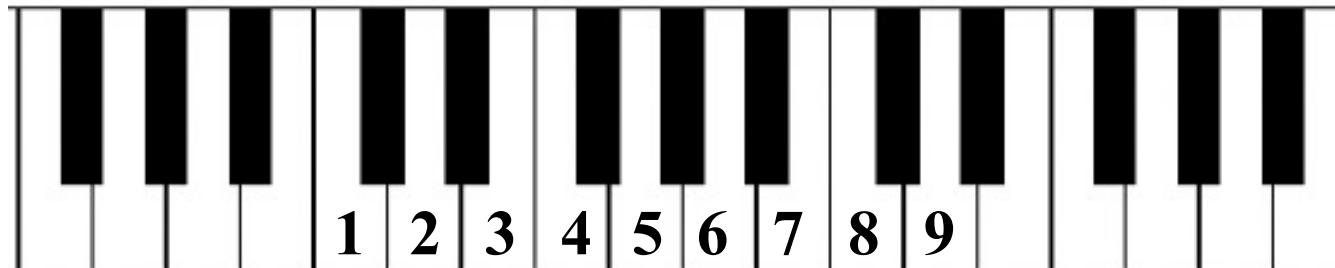
3,141592653589793238462643383279502884197169399375l...

(3,3,2)

El trecillo

INTERNATIONAL DAY OF MATHEMATICS
MARCH 14

(2022)

Le Tonnetz
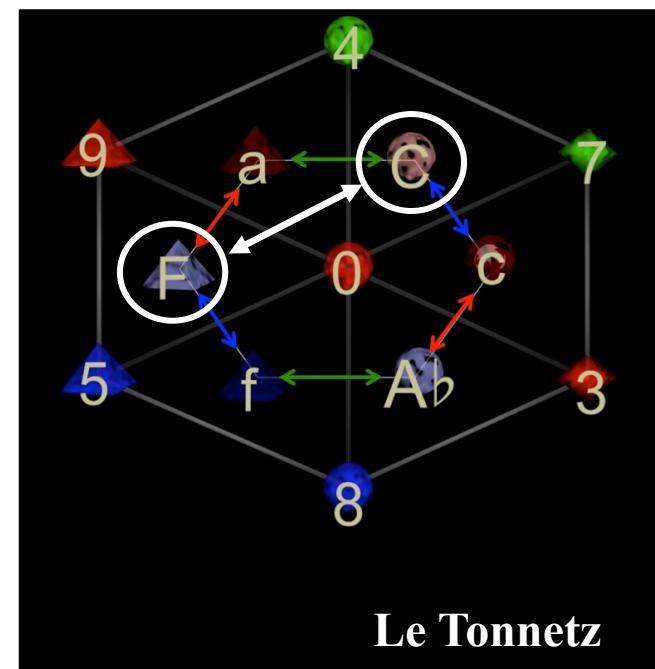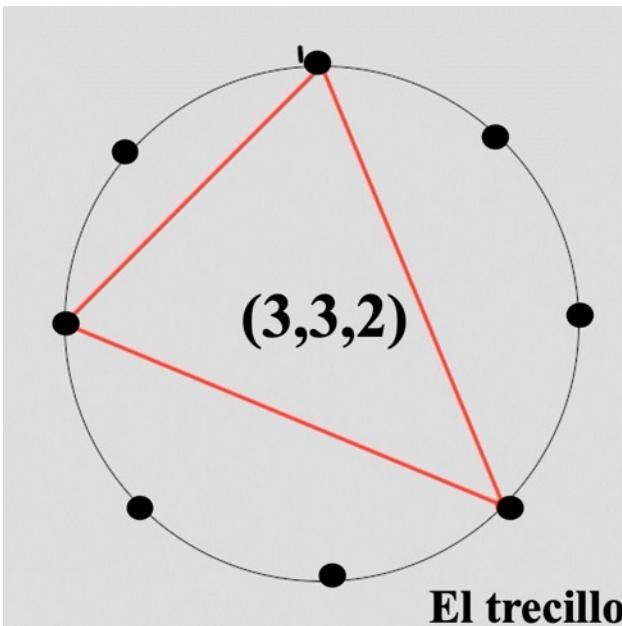
Marin Mersenne, the father of combinatorics
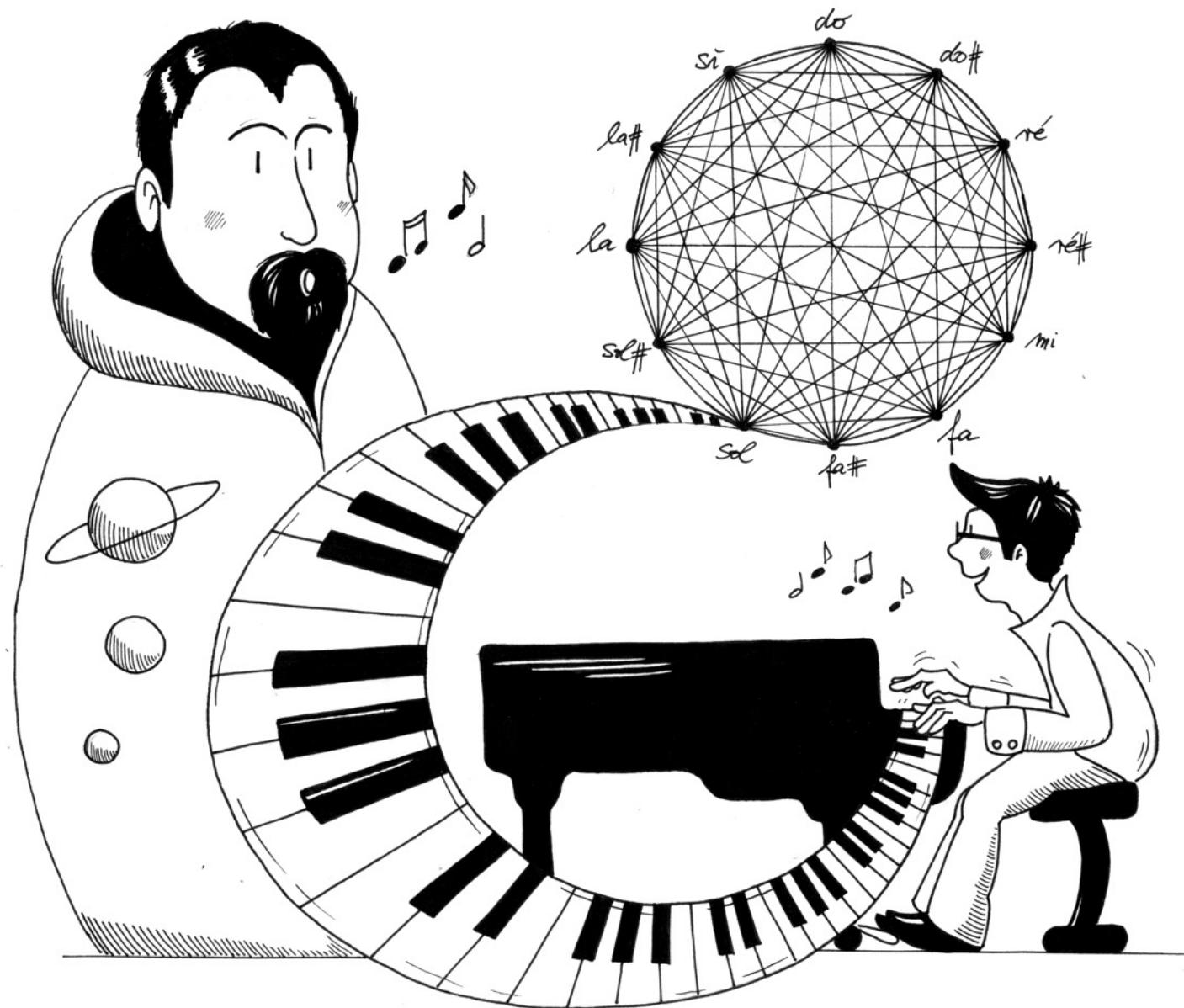
The circular representation of the pitch space

Harmonicorum Libri XII, 1648

Permutational melodies in contemporary (art) music

Marin Mersenne, *Harmonicorum Libri XII*, **1648**

II4
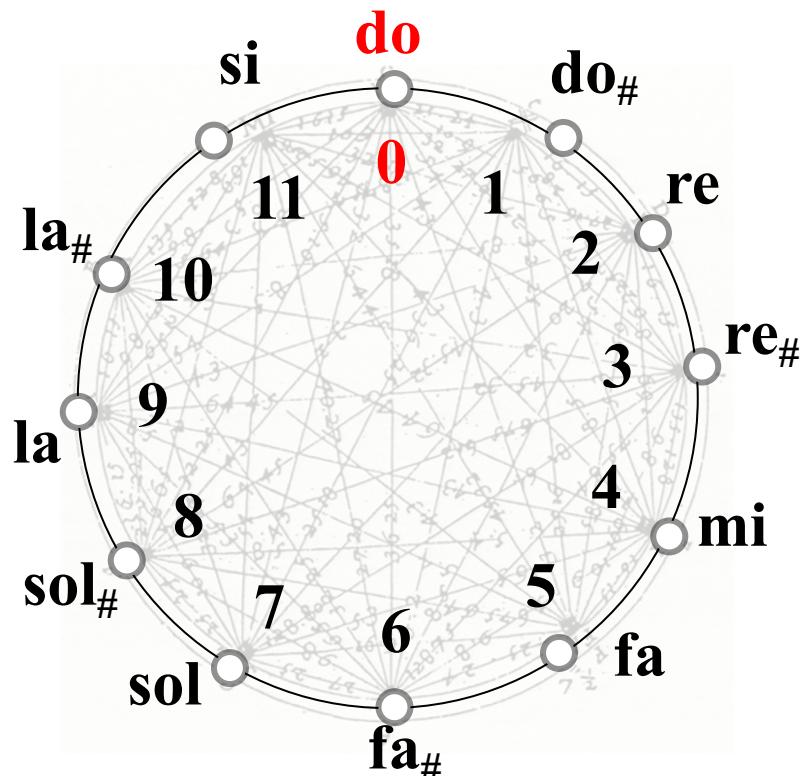
LIBER SEPTIMVS.

DE CANTIBVS, SEV CANTILENIS,
EARVMQ; NVMERO, PARTIBVS, ET SPECIEBVS.

Tabula Combinationis ab 1 ad 22.	
I	1
II	2
III	6
IV	24
V	120
VI	720
VII	5040
VIII	40320
IX	362880
X	3628800
XI	39916800
XII	479001600
XIII	6227020800
XIV	87178291200
XV	1307674368000
XVI	20922789888000
XVII	355687428096000
XVIII	6402373705728000
XIX	121645100408832000
XX	2432902008176640000
XXI	51090942171709440000
XXII.	1124000727777607680000

do

mi $_b$

mi

sol
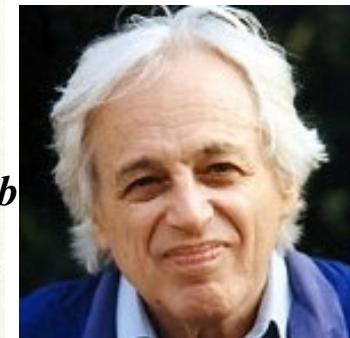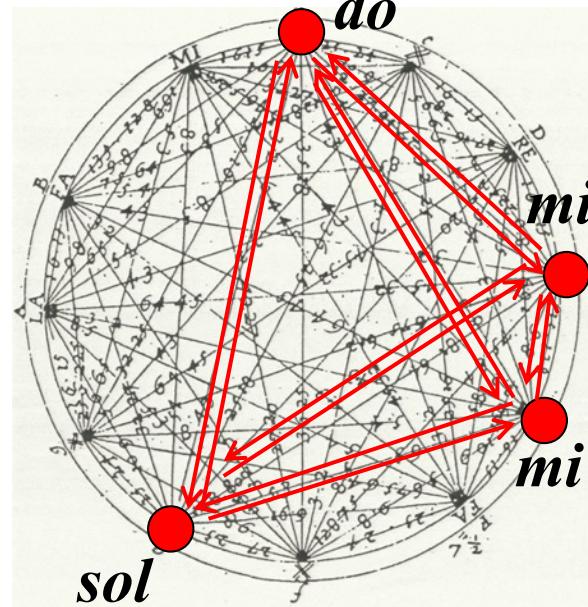
Six Bagatelles
(G. Ligeti, 1953)

Varietas, feu Combinatio quatuor notarum.

A permutational song: one sentence, one note (one note left!)

Una volta soltanto una storia d'amore finisce (M. Andreatta)

Una volta una storia d'amore
            soltanto una storia
Una storia d'amore
            soltanto una storia d'amore
Una storia
            soltanto una storia
Una storia d'amore
            soltanto
Una volta soltanto
            una storia d'amore soltanto

Un amore soltanto una volta
            soltanto una storia d'amore soltanto
Una storia d'amore
            soltanto una volta una storia
Un amore
            una volta soltanto
Un amore finisce
            soltanto
Una volta una storia d'amore finisce
            Un amore soltanto

Una volta soltanto una storia d'amore finisce
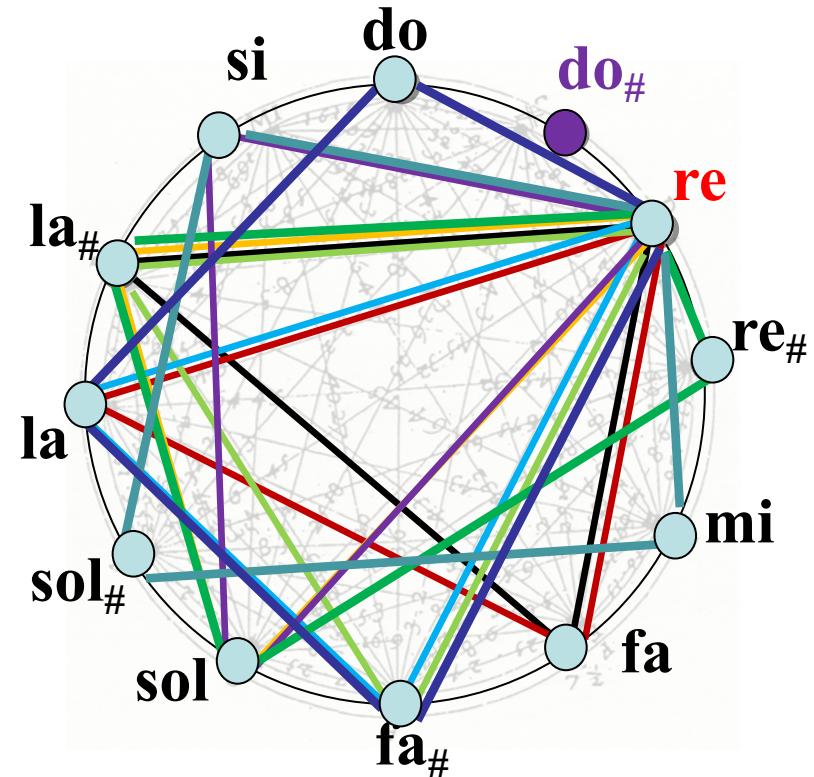
Rules and constraints in the artistic process

Cent mille milliards de poèmes, 1961

GEORGES PEREC
La Vie mode d'emploi
romans

Georges Perec

La vie mode d'emploi,

OuLiPo (Ouvroir de Littérature Potentielle)

Georges Perec
Roman
La disparition
Les Lettres Nouvelles
Denoël

Italo Calvino
Il castello dei destini incrociati, 1969

Raymond Queneau

From the OuLiPo to the OuMuPo (ouvroir de musique potentielle)

Cent mille milliards de poèmes, 1961
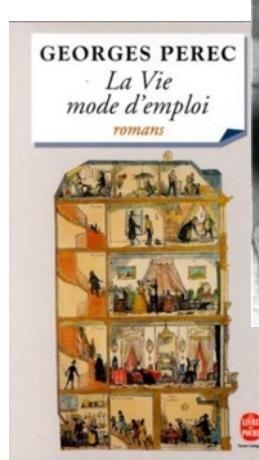
La vie mode d'emploi,

Georges Pérec

Georges Perec
La disparition

Raymond Queneau

Italo Calvino
Le Château des destins croisés, 1969

http://oumupo.org/

Zoi Kapoula · Emmanuelle Volle
Moreno Andreatta · Julien Renoult
Editors

Exploring Transdisciplinarity in Art and Sciences
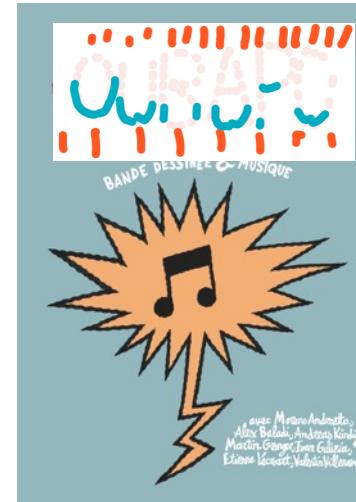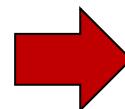
Springer

- M. Andreatta et al., « Music, mathematics and language: chronicles from the Oumupo sandbox », in Kapoula, Z., Volle, E., Renoult, J., Andreatta, M. (Eds.), *Exploring Transdisciplinarity in Art and Sciences*, Springer, 2018

Valentin Villenave

Mike Solomon

Jean-François Piette

Martin Granger

Joseph Boisseau

Moreno Andreatta
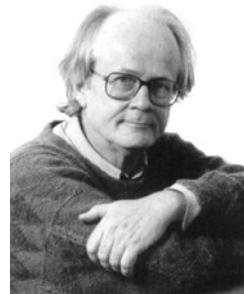
Tom Johnson

A personal contribution to the history of algorithmic music

L. Hiller (1924-1994)

A·M·I·C·V·S
PÉRIODE CLASSIQUE
Domaine Germanique, 1
Johann Joseph Fux
Gradus ad Parnassum (1725)
Texte original intégral
Introduction, traduction et notes par
Jean-Philippe Navarre
MARDAGA
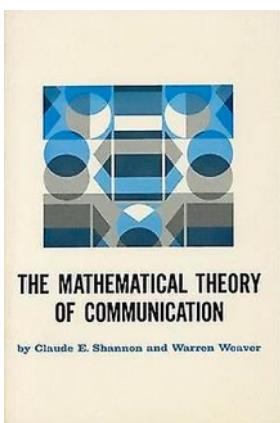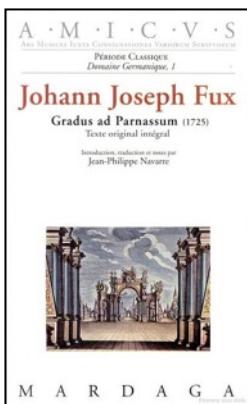
THE MATHEMATICAL THEORY OF COMMUNICATION
by Claude E. Shannon and Warren Weaver

ÉTUDES DE RADIO-TÉLÉVISION
ABRAHAM MOLES
THÉORIE DE L'INFORMATION ET PERCEPTION ESTHÉTIQUE
FLAMMARION

Théories de la composition musicale au XXᵉ siècle
volume 2
Nicolas Donin et Laurent Feneyrou
direction scientifique
SYMÉTRIE
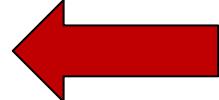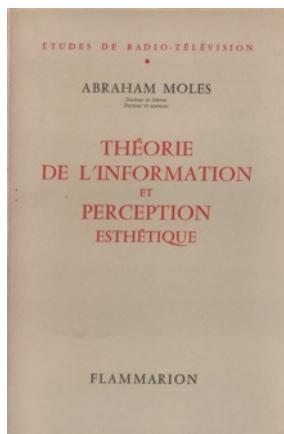
P. Barbaud (1911-1990)
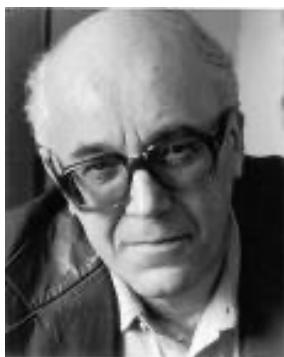
M. Philippot (1925-1996)

Andreatta M., 'Musique algorithmique', In N. Donin et L. Feneyrou (dir.), *Théorie de la composition musicale au XXᵉ siècle*, Symétrie, p. 1239-1268, 2013

Algorithmic processes *vs* algorithmic music

(http://sunsite.univie.ac.at/Mozart/dice/)

Algorithmic processes *vs* algorithmic music

la.la.lab
THE MATHS OF MUSIC

IMAGINARY
open mathematics

HEIDELBERG
LAUREATE FORUM
FOUNDATION

May. 17, 2019 to Dec. 2020

▶ `https://www.imaginary.org/exhibition/la-la-lab-the-mathematics-of-music`

Pink Trombone
Program

The Spectrum of
Sound - Synthesizer
Program

Scale Lab
Program

Tonnetz
Program

Con Espressione!
Program

The Graph
Composer
Program

Show me music
Program

Musical bench
Physical Exhibit / Hands on

Whitney Music Box
Program

Beat Box
Program

NSynth
Physical Exhibit / Hands on

AI Jam
Program

Mind and Music
Jukebox
Program

The Harmonic
Series #3 - 3D
display
Program

The Harmonic
Series #1 - Laser
Physical Exhibit / Hands on
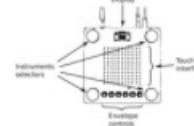
Lissajous figures
Gallery

The Harmonic
Series #2 - 3D
sculptures
Physical Exhibit / Hands on

The Sound of
Sequences
Program

The Spectrum of
Sound - Analyzer
Program

Gerhard Widmer
on Expressive Music
Performance, the
Boesendorfer
CEUS, and a MIDI
Theremin
Film

Pentatonic Scales
Program

Note Compass
Program

JSBach333 -
canone permutativo
al triangolo from
BACH333 - Canon
Composition
Competition 2018
Film
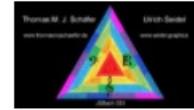
Algebraic
Vibrations
Film

EXHIBITION BOOKLET

La La
Lab
THE MATHEMATICS
OF MUSIC

Raum der 3-notigen Akkorde

Dive into a multitude of topics that visualize the complex interrelations of melody, harmony and mathematics. Push the "play" button in each visualization and experience the effect of parameters.

Each of the animations looks at a certain musical piece or pattern from a special mathematical viewpoint. Aspects of symmetry, both in time and space help to understand musical ideas. If you look at an animation of a Chopin Prelude, it unveils hidden structures and helps you to understand the music better.

Show me music

MOZARTS WÜRFEL

SHEPARD TONE

DUR-MOLL KANON

PACHELBALLS

A permutation song based on a permutational poem

Antes, despues

Como los juegos al llanto
como la sombra a la columna
el perfume dibuja el jazmín
el amante precede al amor
como la caricia a la mano
el amor sobrevive al amante
pero inevitablemente
aunque no haya huella ni presagio

Aunque no haya huella ni presagio
como la caricia a la mano
el perfume dibuja el jazmín
el amante precede al amor
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el amor sobrevive al amante
como los juegos al llanto
como la sombra a la columna

Como la caricia a la mano
aunque no haya huella ni presagio
el amante precede al amor
el perfume dibuja el jazmín
como los juegos al llanto
como la sombra a la columna
el amor sobrevive al amante
pero inevitablemente

Julio Cortázar

PRODUIT DU HASARD

Hypertexte et poésie combinatoire

PUL
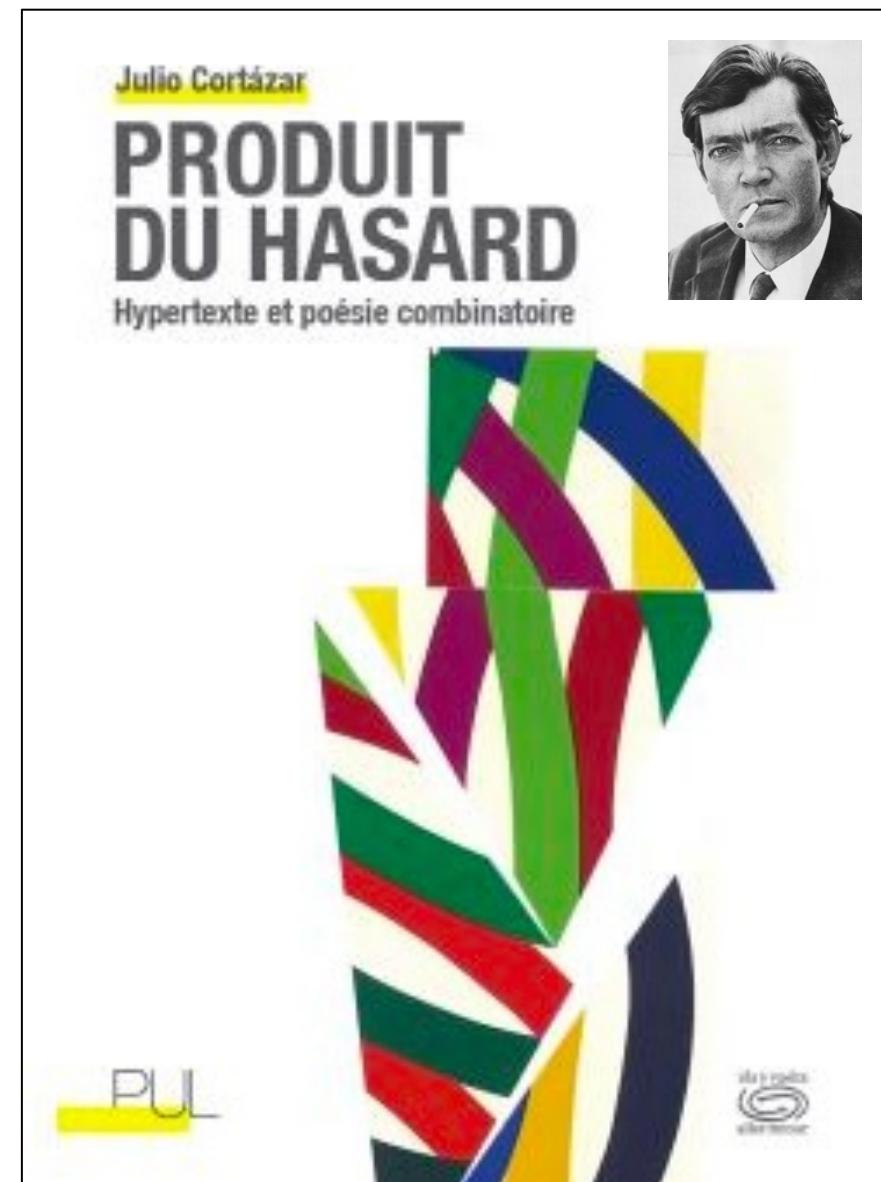
A permutation song based on a permutational poem
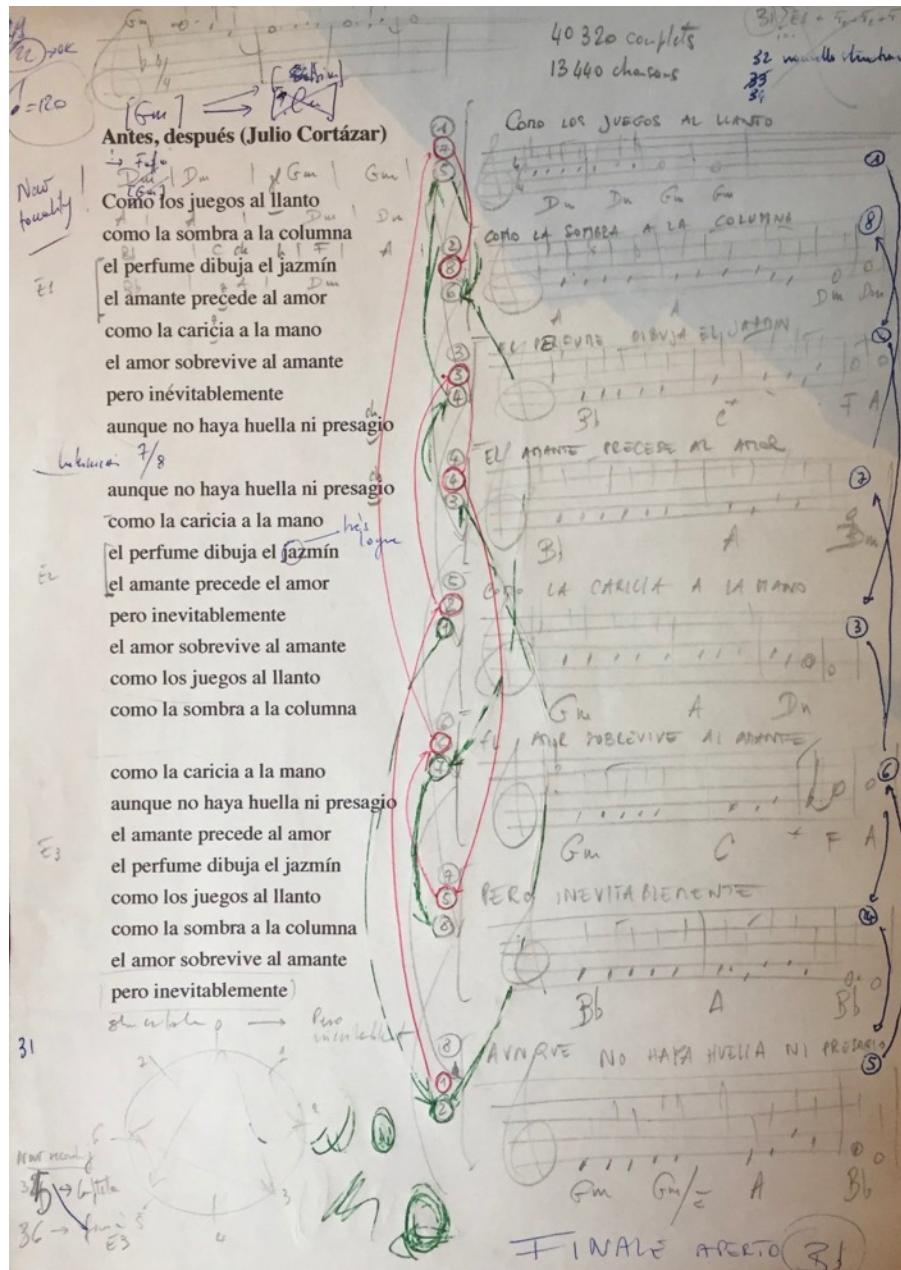
Before, after

Like the games to crying,
like the shadow to the column,
the perfume draws the jasmine,
the lover precedes love,
like the caress to the hand,
love survives the lover,
but inevitably
even if there is no trace or omen

Even if there is no trace or omen
like the caress to the hand,
the perfume draws the jasmine,
the lover precedes love,
but inevitably
love survives the lover,
like the games to crying,
like the shadow to the column,

Like the caress to the hand,
even if there is no trace or omen
the lover precedes love,
the perfume draws the jasmine,
Like the games to crying,
like the shadow to the column,
love survives the lover,
but inevitably

Julio Cortazar

15

Leonhard Euler, the father of graph theory
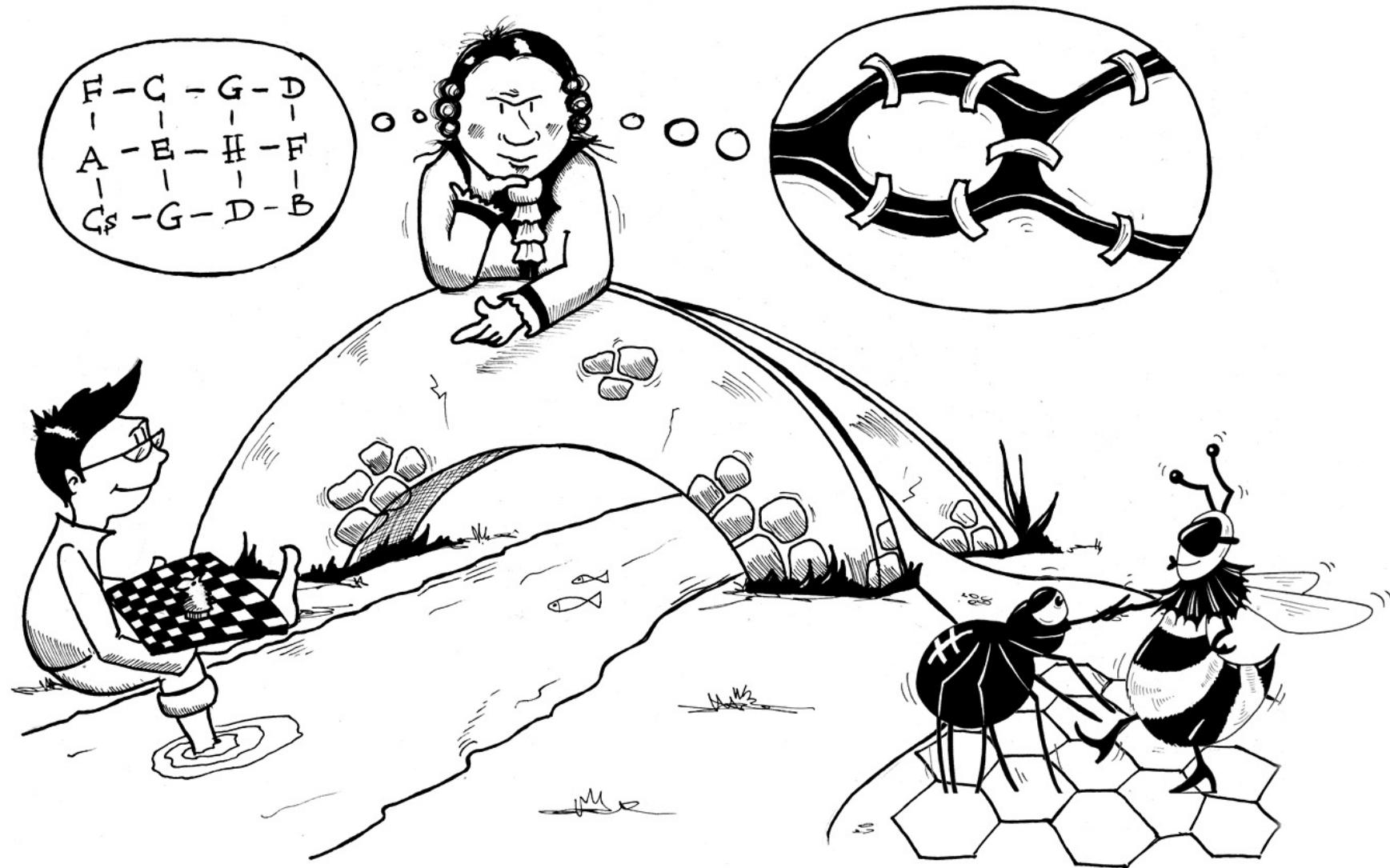
The three main major-minor symmetries
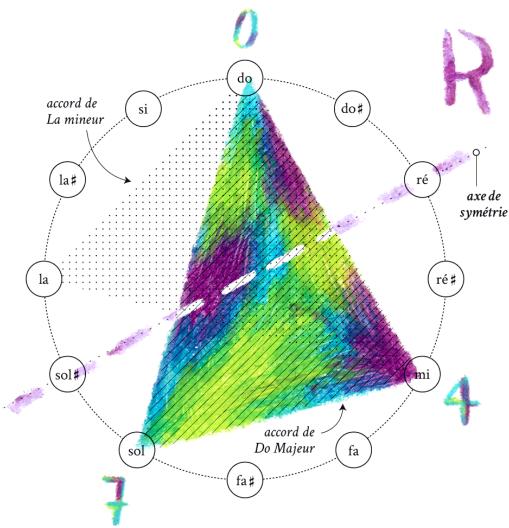
R as RELATIVE

C major
↕
A minor
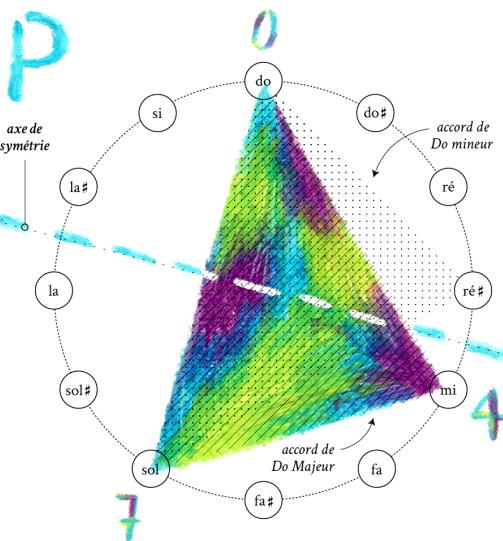
P as PARALLEL

C major
↕
C minor
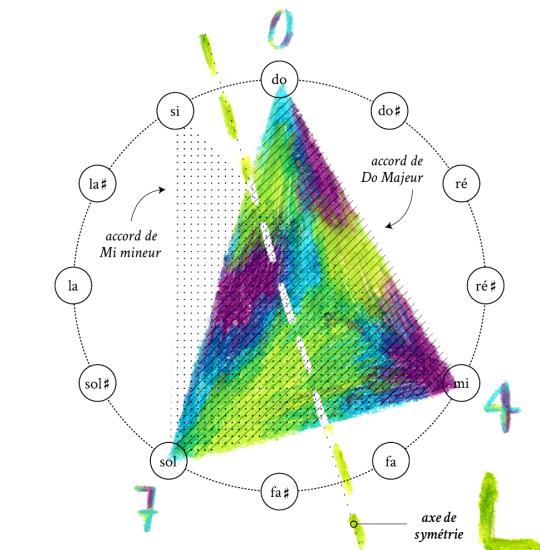
L as LEADING-TONE (EXCHANGE)

C major
↕
E minor

The *Tonnetz* web environment (© SMIR Project)

https://morenoandreatta.com/software/
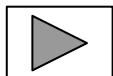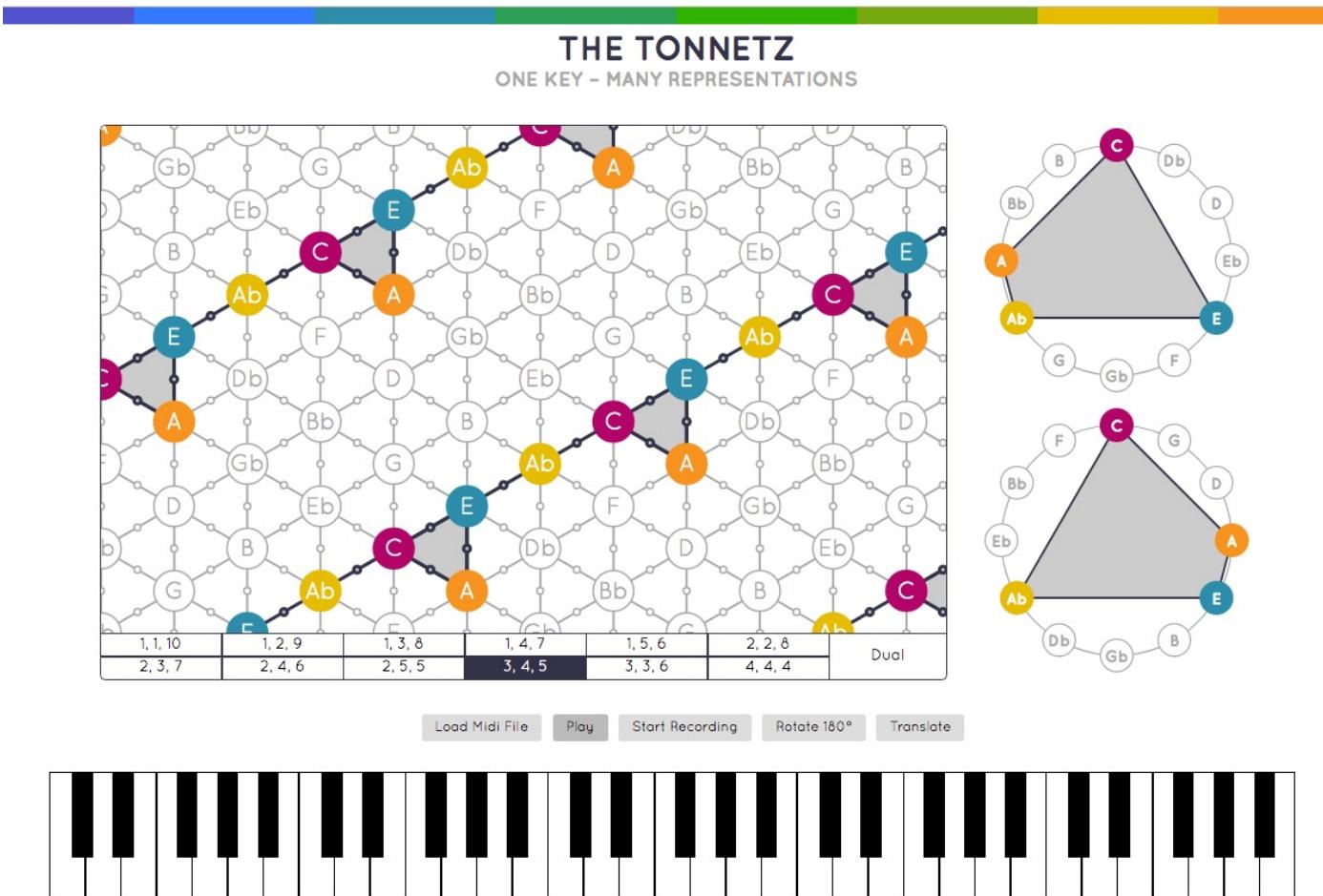
From the *Tonnetz* to the dual one
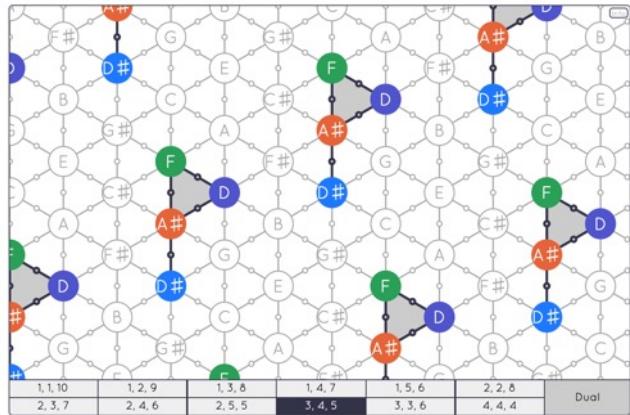
duality
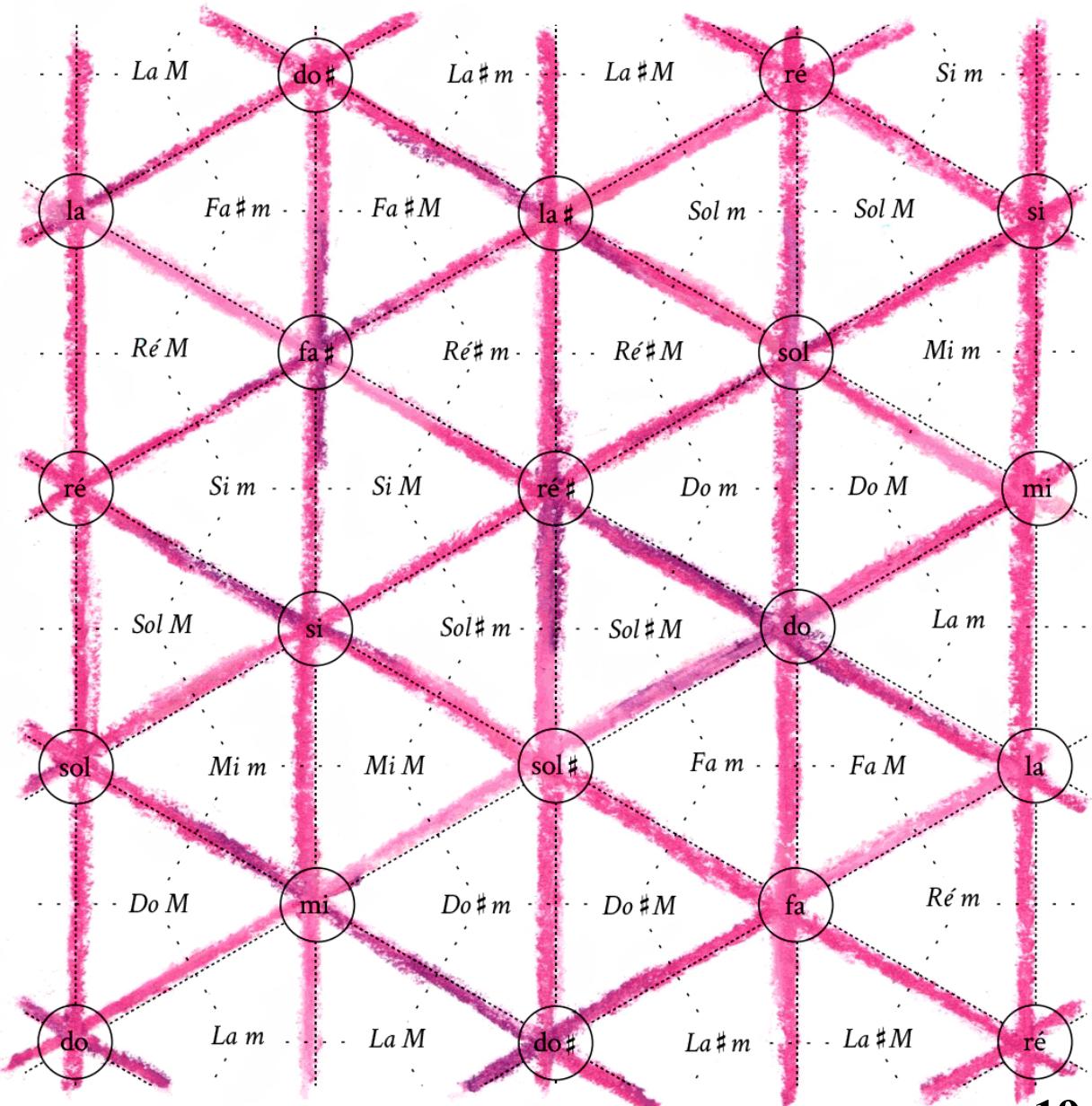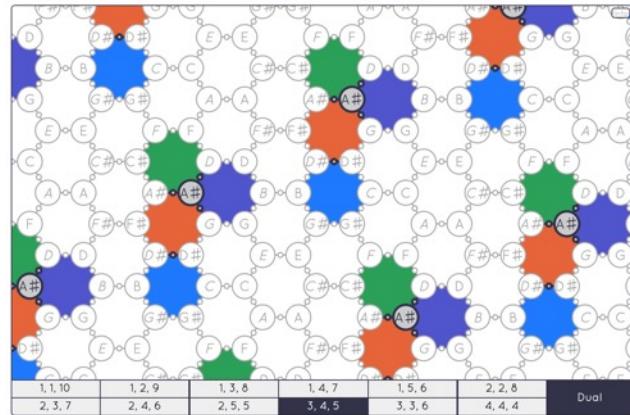
Hamiltonian Cycles in the Tonnetz and Hamiltonian Songs

List of 124 Hamiltonian Cycles (Bigo/Andreatta, February 2016)

1. C-Cm-Ab-Abm-E-Em-G-Gm-Eb-Ebm-B-Bm-D-Dm-Bb-Bbm-F#-F#m-A-C#m-C#-Fm-F-Am--
PLPLPRPLPLPRPLPLPRLPLPLR
2. C-Cm-Ab-Abm-E-Em-G-Gm-Eb-Ebm-B-Bm-D-F#m-F#-Bbm-Bb-Dm-F-Fm-C#-C#m-A-Am--
PLPLPRPLPLPRLPLPLRPLPLPR
3. C-Cm-Ab-Abm-E-Em-Bm-Bbm-Eb-Gm-Bb-Bbm-F#-F#m-D-Dm-F-Fm-C#-C#m-A-Am--
PLPLPRLPLPLRPLPLPRPLPLPR
4. C-Cm-Ab-Abm-E-C#m-C#-Fm-F-Am-A-F#m-F#-Bbm-Bb-Dm-D-Bm-B-Ebm-Eb-Gm-G-Em--
PLPLPRLPLPRPLPLPRPLPLPRL
5. C-Cm-Ab-Abm-E-C#m-A-Am-F-Fm-C#-Bbm-F#-F#m-D-Dm-Bb-Gm-Eb-Ebm-B-Bm-G-Em--
PLPLRL
6. C-Cm-Ab-Abm-B-Bm-G-Gm-Eb-Ebm-F#-F#m-D-Dm-Bb-Bbm-C#-Fm-F-Am-A-C#m-E-Em--
PLPRPLPLPRPLPLPRLPLPLRPL
7. C-Cm-Ab-Abm-B-Bm-G-Gm-Eb-Ebm-F#-Bbm-Bb-Dm-D-F#m-A-Am-F-Fm-C#-C#m-E-Em--
PLPRPLPLPRLPLPLRPLPLPRPL
8. C-Cm-Ab-Bm-G-Em-E-C#m-A-F#m-D-Dm-Bb-Gm-Eb-Ebm-F#-Bbm-C#-Fm-F-Am--
PLPRPLRPRLRLPLRLPRLRLPLR
9. C-Cm-Ab-Abm-B-Ebm-Eb-Gm-G-Bm-D-Dm-Bb-Bbm-F#-F#m-A-Am-F-Fm-C#-C#m-E-Em--
PLPRLPLPLRPLPLPRPLPLPRPL
10. C-Cm-Ab-Abm-B-Ebm-Eb-Gm-Bb-Dm-D-Bm-G-Em-E-C#m-A-F#m-F#-Bbm-C#-Fm-F-Am--
PLPRLPLRLPRLRPRLRLPLRLPLR
11. C-Cm-Ab-Abm-B-Ebm-Eb-Gm-Bb-Dm-F-Fm-C#-Bbm-F#-F#m-D-Bm-G-Em-E-C#m-A-Am--
PLPRLPLRLRPLRLPLRLPRLPR
12. C-Cm-Ab-Fm-F-Am-A-C#m-C#-Bbm-Bb-Dm-D-F#m-F#-Ebm-Eb-Gm-G-Bm-B-Abm-E-Em--
PLRPLPLPRPLPLPRPLPLPRLPL
13. C-Ab-Fm-F-Am-A-F#m-F#-Bbm-C#-C#m-E-Abm-B-Ebm-Eb-Gm-Bb-Dm-D-Bm-G-Em--
PLRPLPRPLRPRLRLPLRLPRLRL
14. C-Cm-Ab-Fm-F-Am-A-F#m-D-Dm-Bb-Gm-Eb-Ebm-F#-Bbm-C#-C#m-E-Abm-B-Bm-G-Em--
PLRPLPRLPLRLPRLRPRLRPLRL
15. C-Cm-Ab-Fm-F-Dm-Bb-Gm-Eb-Ebm-B-Abm-E-Em-G-Bm-D-F#m-F#-Bbm-C#-C#m-A-Am--
PLRPRLRLPLRLPRLPLRLPLPLPR
16. C-Cm-Ab-Fm-C#-C#m-A-Am-F-Dm-Bb-Bbm-F#-F#m-D-Bm-G-Gm-Eb-Ebm-B-Abm-E-Em--
PLRLPL
17. C-Cm-Ab-Fm-C#-C#m-A-F#m-F#-Bbm-Bb-Gm-Eb-Ebm-B-Abm-E-Em-G-Bm-D-Dm-F-Am--
PLRLPLRPLPRLPLRLPRLRPRLR
18. C-Cm-Ab-Fm-C#-C#m-A-F#m-D-Bm-B-Abm-E-Em-G-Gm-Eb-Ebm-F#-Bbm-Bb-Dm-F-Am--
PLRLPLRLRPRLRPRPLPRLPLRLR
19. C-Cm-Ab-Fm-C#-C#m-E-Abm-B-Bm-D-F#m-A-Am-F-Dm-Bb-Bbm-F#-Ebm-Eb-Gm-G-Em--
PLRLPRLRPRLRPLRLPLRPLPRL
20. C-Cm-Ab-Fm-C#-C#m-E-Abm-B-Ebm-Eb-Gm-Bb-Bbm-F#-F#m-A-Am-F-Dm-D-Bm-G-Em--
PLRLPLRLPLRPLPRPLPRRLRL
21. C-Cm-Ab-Fm-C#-Bbm-Bb-Dm-F-Am-A-C#m-E-Abm-B-Bm-D-F#m-F#-Ebm-Eb-Gm-G-Em--
PLRLRPLRLPLRLRPRLPRPLRL
22. C-Cm-Ab-Fm-C#-Bbm-Bb-Gm-Eb-Ebm-F#-F#m-D-Dm-F-Am-A-C#m-E-Abm-B-Bm-G-Em--
PLRLRPRLPRPLPRLPLRLRPLRL
23. C-Cm-Eb-Ebm-B-Bm-G-Gm-Bb-Bbm-F#-F#m-D-Dm-F-Am-A-C#m-C#-Fm-Ab-Abm-E-Em--
PRPLPLPRPLPLPRLPLPLRPLPL
24. C-Cm-Eb-Ebm-B-Bm-G-Gm-Bb-Dm-D-F#m-F#-Bbm-C#-C#m-A-Am-F-Fm-Ab-Abm-E-Em--
PRPLPLPRLPLPLRPLPLPRPLPL
25. C-Cm-Eb-Ebm-B-Bm-D-F#m-F#-Bbm-C#-Fm-Ab-Abm-E-C#m-A-Am-F-Dm-Bb-Gm-G-Em--
PRPLPRLPLRLRPLRLPLRLPRPL
26. C-Cm-Eb-Ebm-B-Abm-Ab-Fm-C#-Bbm-F#-F#m-D-Bm-G-Gm-Bb-Dm-F-Am-A-C#m-E-Em--
PRPLRPRLRLPLRLPRLRLPLRPL
27. C-Cm-Eb-Ebm-F#-F#m-A-C#m-E-Em-G-Gm-Bb-Bbm-C#-Fm-Ab-Abm-B-Bm-D-Dm-F-Am--
PRPRPRLR
28. C-Cm-Eb-Ebm-F#-Bbm-C#-C#m-E-Em-G-Gm-Bb-Dm-F-Fm-Ab-Abm-B-Bm-D-F#m-A-Am--
PRPRLRPR
29. C-Cm-Eb-Ebm-F#-Bbm-C#-Fm-Ab-Abm-B-Bm-D-F#m-A-C#m-E-Em-G-Gm-Bb-Dm-F-Am--
PRPRLRLR
30. C-Cm-Eb-Gm-G-Bm-B-Ebm-F#-F#m-D-Dm-Bb-Bbm-C#-C#m-A-Am-F-Fm-Ab-Abm-E-Em--
PRRLPLPLRPLPLPRPLPLPRLPL
31. C-Cm-Eb-Gm-G-Bm-D-F#m-F#-Ebm-B-Abm-Ab-Fm-C#-Bbm-Bb-Dm-F-Am-A-C#m-E-Em--
PRLPLRLPRLRPRLRPLRLPLRPL
32. C-Cm-Eb-Gm-G-Bm-D-F#m-A-Am-F-Dm-Bb-Bbm-F#-Ebm-B-Abm-Ab-Fm-C#-C#m-E-Em--
PRLPLRLRPLRLRLRPRLPRPL
33. C-Cm-Eb-Gm-G-Em-E-Abm-Ab-Fm-C#-C#m-A-F#m-D-Bm-B-Ebm-F#-Bbm-Bb-Dm-F-Am--
PRLPRPLRPLRLRLRPLRLRLPLRLR
34. C-Cm-Eb-Gm-Bb-Bbm-F#-Ebm-B-Bm-G-Em-E-Abm-Ab-Fm-C#-C#m-A-F#m-D-Dm-F-Am--
PRLRPLRLRLRPLPRLPLRLRPLR
35. C-Cm-Eb-Gm-Bb-Bbm-C#-C#m-E-Em-G-Bm-D-Dm-F-Fm-Ab-Abm-B-Ebm-F#-F#m-A-Am--
PRLRPRPR
36. C-Cm-Eb-Gm-Bb-Bbm-C#-Fm-Ab-Abm-E-C#m-A-Am-F-Dm-D-F#m-F#-Ebm-B-Bm-G-Em--
PRLRPRLRPLRLRLPLRLPRLPRL
37. C-Cm-Eb-Gm-Bb-Bbm-C#-Fm-Ab-Abm-B-Ebm-F#-F#m-A-C#m-E-Em-G-Bm-D-Dm-F-Am--
PRLR
38. C-Cm-Eb-Gm-Bb-Dm-D-F#m-A-Am-F-Fm-Ab-Abm-E-C#m-C#-Bbm-F#-Ebm-B-Bm-G-Em--
PRLRLPLRPLPRPLRPRLRLPLRL
39. C-Cm-Eb-Gm-Bb-Dm-D-F#m-A-C#m-C#-Bbm-F#-Ebm-B-Bm-G-Em-E-Abm-Ab-Fm-F-Am--
PRLRLPLRLPRLRLPLRPLPRPLR
40. C-Cm-Eb-Gm-Bb-Dm-F-Fm-Ab-Abm-B-Ebm-F#-Bbm-C#-C#m-E-Em-G-Bm-D-F#m-A-Am--
PRLRLRPR
41. C-Em-E-Abm-Ab-Cm-Eb-Ebm-B-Bm-G-Gm-Bb-Bbm-F#-F#m-D-Dm-F-Fm-C#-C#m-A-Am--
LPLPLRPLPLPRPLPLPRPLPLPR
42. C-Em-E-Abm-Ab-Cm-Eb-Gm-G-Bm-B-Ebm-F#-Bbm-Bb-Dm-D-F#m-A-C#m-C#-Fm-F-Am--
LPLPLR
43. C-Em-E-Abm-Ab-Fm-F-Am-A-C#m-C#-Bbm-Bb-Dm-D-F#m-F#-Ebm-B-Bm-G-Gm-Eb-Cm--
LPLPRPLPLPRPLPLPRLPLPLRP
44. C-Em-E-Abm-Ab-Fm-F-Am-A-C#m-C#-Bbm-F#-F#m-D-Dm-Bb-Gm-G-Bm-B-Ebm-Eb-Cm--
LPLPRPLPLPRLPLPLRPLPLPRP
45. C-Em-E-Abm-Fm-C#-C#m-A-Am-F-Dm-D-F#m-F#-Bbm-Bb-Gm-G-Bm-B-Ebm-Eb-Cm--
LPLPRLPLPLRPLPLPRPLPLPRP
46. C-Em-E-Abm-B-Bm-G-Gm-Eb-Ebm-F#-F#m-D-Dm-Bb-Bbm-C#-C#m-A-Am-F-Fm-Ab-Cm--
LPLRPLPLPRPLPLPRPLPLPRLP
47. C-Em-E-Abm-B-Bm-G-Gm-Bb-Bbm-F#-Ebm-Eb-Cm-Ab-Fm-C#-C#m-A-F#m-D-Dm-F-Am--
LPLRPLPRPLRPRLRLPLRLPRLR
48. C-Em-E-Abm-B-Bm-G-Gm-Bb-Dm-D-F#m-A-C#m-C#-Bbm-F#-Ebm-Eb-Cm-Ab-Fm-F-Am--
LPLRPLPRLPLRLPRLRPRLRPLR
49. C-Em-E-Abm-B-Ebm-Eb-Gm-G-Bm-D-F#m-F#-Bbm-Bb-Dm-F-Am-A-C#m-C#-Fm-Ab-Cm--
LPLRLP
50. C-Em-E-Abm-B-Ebm-Eb-Cm-Ab-Fm-F-Dm-Bb-Gm-G-Bm-D-F#m-F#-Bbm-C#-C#m-A-Am--
LPLRLPRLRPRLRPLRLPLRPLPR
51. C-Em-E-Abm-B-Ebm-Eb-Cm-Ab-Fm-C#-C#m-A-F#m-F#-Bbm-Bb-Gm-G-Bm-D-Dm-F-Am--
LPLRLPRLRLPLRPLPRPLRPRLR
52. C-Em-E-Abm-B-Ebm-Eb-Cm-Ab-Fm-F-Am-A-F#m-D-Dm-F-Bm-G-Gm-Eb-Cm-Ab-Fm-F-Dm-Bb-Bbm-C#-C#m-A-Am--
LPLRLRPLRLPLRLRPRLPRPLR
53. C-Em-E-Abm-B-Ebm-F#-F#m-A-C#m-C#-Bbm-Bb-Dm-D-Bm-G-Gm-Eb-Cm-Ab-Fm-F-Am--
LPLRLRPRLPRPLPRLPLRLRPLR

Hamiltonian Cycles in the Tonnetz and Hamiltonian Songs

54. C-Em-E-C#m-C#-Fm-F-Am-A-F#m-F#-Bbm-Bb-Dm-D-Bm-G-Gm-Eb-Ebm-B-Abm-Ab-Cm--
LPRPLPLPRPLPLPRLPLPLPLPLRPLP

55. C-Em-E-C#m-C#-Fm-F-Am-A-F#m-D-Dm-Bb-Bbm-F#-Ebm-Eb-Gm-G-Bm-B-Abm-Ab-Cm--
LPRPLPLPRLPLPLRPLPLPRPLP

56. C-Em-E-C#m-C#-Fm-F-Dm-Bb-Bbm-F#-Ebm-B-Abm-Ab-Cm-Eb-Gm-G-Bm-D-F#m-A-Am--
LPRPLPRLPLRLRPLRLPLRLRPR

57. C-Em-E-C#m-C#-Fm-Ab-Abm-B-Ebm-F#-Bbm-Bb-Dm-F-Am-A-F#m-D-Bm-G-Gm-Eb-Cm--
LPRPLRPRLRLPLRLPRLRLPLRP

58. C-Em-E-C#m-A-Am-F-Fm-C#-Bbm-Bb-Dm-D-F#m-F#-Ebm-Eb-Gm-G-Bm-B-Abm-Ab-Cm--
LPRLPLPLRPLPLPRPLPLPRPLP

59. C-Em-E-C#m-A-Am-F-Dm-Bb-Bbm-C#-Fm-Ab-Abm-B-Ebm-F#-F#m-D-Bm-G-Gm-Eb-Cm--
LPRLPLRLPRLRPRLRPLRLPLRP

60. C-Em-A-Am-F-Fm-D-Bm-G-Bm-G-Bm-D-F#m-F#-Bbm-C#-Fm-Ab-Abm-B-Ebm-Eb-Cm--
LPRLPLRLRPLRLRPLRLRPRLPRP

61. C-Em-E-C#m-A-F#m-F#-Ebm-B-Abm-Ab-Cm-Eb-Gm-G-Bm-D-Dm-Bb-Bbm-C#-Fm-F-Am--
LPRLRPRLRPLRLPLRPLPRLPLR

62. C-Em-A-F#m-D-Dm-Bb-Bbm-F#-Ebm-Eb-Cm-Eb-Ebm-F#-Bbm-C#-Fm-F-Am--
LPRLRLPLRPLPRPLRPRLRLPLR

63. C-Em-G-Gm-Eb-Ebm-B-Bm-D-Dm-Bb-Bbm-F#-F#m-A-Am-F-Fm-C#-C#m-E-Abm-Ab-Cm--
LRPLPLPRPLPLPRPLPLPRLPLP

64. C-Em-G-Gm-Eb-Ebm-F#-F#m-D-Bm-B-Abm-E-C#m-A-Am-F-Dm-Bb-Bbm-C#-Fm-Ab-Cm--
LRPLPRPLRPRLRLPLRLPLRLRLP

65. C-Em-G-Gm-Eb-Ebm-F#-Bbm-Bb-Dm-F-Am-A-F#m-D-Bm-B-Abm-E-C#m-C#-Fm-F-Am--
LRPLPRLPLRLPRLRPRLRPLRLP

66. C-Em-G-Gm-Eb-Cm-Ab-Abm-E-C#m-C#-Fm-F-Dm-Bb-Bbm-F#-Ebm-B-Bm-D-F#m-A-Am--
LRPLRLPLRPLPRLPLRLPRLRPR

67. C-Em-G-Gm-Eb-Cm-Ab-Abm-E-C#m-A-F#m-F#-Ebm-B-Bm-D-Dm-Bb-Bbm-C#-Fm-F-Am--
LRPLRLPLRLRPRLPRPLPRLPLR

68. C-Em-G-Gm-Bb-Bbm-C#-C#m-E-Abm-B-Bm-D-Dm-F-Fm-Ab-Cm-Eb-Ebm-F#-F#m-A-Am--
LRPRPRPR

69. C-Em-G-Gm-Bb-Bbm-C#-Fm-Ab-Cm-Eb-Ebm-F#-F#m-A-C#m-E-Abm-B-Bm-D-Dm-F-Am--
LRPRPRLR

70. C-Em-G-Gm-Bb-Dm-D-Bm-B-Ebm-Eb-Cm-Ab-Abm-E-C#m-A-F#m-F#-Bbm-C#-Fm-F-Am--
LRPRLPRPLPRLPLRLRPLRLPLR

71. C-Em-G-Gm-Bb-Dm-F-Fm-C#-Bbm-F#-F#m-D-Bm-B-Ebm-Eb-Cm-Ab-Abm-E-C#m-A-Am--
LRPRLRPLRLRLPRLPRLPLRLPR

72. C-Em-G-Gm-Bb-Dm-F-Fm-Ab-Cm-Eb-Ebm-F#-Bbm-C#-C#m-E-Abm-B-Bm-D-F#m-A-Am--
LRPR

73. C-Em-G-Gm-Bb-Dm-F-Am-A-C#m-E-Abm-Ab-Fm-C#-Bbm-F#-F#m-D-Bm-B-Ebm-Eb-Cm--
LRPRLRLPLRLPRLRLPLRPLRP

74. C-Em-G-Bm-B-Ebm-Eb-Gm-Bb-Dm-D-F#m-F#-Bbm-C#-Fm-F-Am-A-C#m-E-Abm-Ab-Cm--
LRLPLP

75. C-Em-G-Bm-B-Ebm-F#-F#m-D-Dm-F-Fm-C#-Bbm-Bb-Gm-Eb-Cm-Ab-Abm-E-C#m-A-Am--
LRLPLRPLPRPLRPLRLRLPLRLPR

76. C-Em-G-Bm-B-Ebm-F#-F#m-D-Dm-F-Am-A-C#m-E-Abm-Ab-Fm-C#-Bbm-Bb-Gm-Eb-Cm--
LRLPLRPLRPLPLRLPRLRPLRLP

77. C-Em-G-Bm-B-Ebm-F#-Bbm-Bb-Gm-Eb-Cm-Ab-Abm-E-C#m-C#-Fm-F-Dm-D-F#m-A-Am--
LRLPLRLPRLRLPLRPLPRPLRPR

78. C-Em-G-Bm-B-Ebm-F#-Bbm-C#-C#m-E-Abm-Ab-Fm-F-Am-A-F#m-D-Dm-Bb-Gm-Eb-Cm--
LRLPLRLRPRLPRPLPLRLRLRP

79. C-Em-G-Bm-B-Abm-E-C#m-C#-Bbm-F#-Ebm-Eb-Gm-Bb-Dm-D-F#m-A-Am-F-Fm-Ab-Cm--
LRLPRLRLPRLRPLRLPLRPLPRLP

80. C-Em-G-Bm-B-Abm-E-C#m-A-Am-F-Dm-D-F#m-F#-Ebm-Eb-Gm-Bb-Bbm-C#-Fm-Ab-Cm--
LRLPRLRLPLRPLPRPLRPRLRLP

81. C-Em-G-Bm-D-Dm-Bb-Gm-Eb-Ebm-B-Abm-E-C#m-C#-Bbm-F#-F#m-A-Am-F-Fm-Ab-Cm--
LRLRPLRLPLRLRPRLPRPLPRLP

82. C-Em-G-Bm-D-Dm-F-Fm-Ab-Cm-Eb-Gm-Bb-Bbm-C#-C#m-E-Abm-B-Ebm-F#-F#m-A-Am--
LRLRPRPR

83. C-Em-G-Bm-D-Dm-F-Am-A-F#m-F#-Bbm-Bb-Gm-Eb-Ebm-B-Abm-E-C#m-C#-Fm-Ab-Cm--
LRLRPRLPRPLPRLPLRLRPLRLP

84. C-Em-G-Bm-D-F#m-A-C#m-E-Abm-B-Ebm-F#-Bbm-C#-Fm-Ab-Cm-Eb-Gm-Bb-Dm-F-Am--
LR

85. C-Am-A-C#m-C#-Fm-F-Dm-D-F#m-F#-Bbm-Bb-Gm-G-Bm-B-Ebm-Eb-Cm-Ab-Abm-E-Em--
RPLPLPRPLPLPRPLPLPRLPLPL

86. C-Am-A-C#m-C#-Fm-F-Dm-D-F#m-F#-Bbm-Bb-Gm-Eb-Ebm-B-Bm-G-Em-E-Abm-Ab-Cm--
RPLPLPRPLPLPRLPLPLPLRPLPLP

87. C-Am-A-C#m-C#-Fm-F-Dm-Bb-Bbm-F#-F#m-D-Bm-B-Ebm-Eb-Gm-G-Em-E-Abm-Ab-Cm--
RPLPLPRLPLPLRPLPLPRPLPLP

88. C-Am-A-C#m-C#-Bbm-Bb-Dm-F-Fm-Ab-Cm-Eb-Gm-G-Bm-D-F#m-F#-Ebm-B-Abm-E-Em--
RPLPRPLRPLRPLRLPLRLPRLRLPL

89. C-Am-A-C#m-C#-Bbm-F#-F#m-D-Bm-G-Gm-Bb-Dm-F-Fm-Ab-Cm-Eb-Ebm-B-Abm-E-Em--
RPLPRLPLRLPRLRPLRLPLRLPL

90. C-Am-A-C#m-C#-Bbm-F#-F#m-D-Bm-G-Em-E-Abm-B-Ebm-Eb-Gm-Bb-Dm-F-Fm-Ab-Cm--
RPLPRLPLRLRLPLRLPLRLRPRLP

91. C-Am-A-C#m-E-Em-G-Bm-D-F#m-F#-Bbm-C#-Fm-F-Dm-Bb-Gm-Eb-Ebm-B-Abm-Ab-Cm--
RPLRPRLRLPLRLPRLRLPLRPLP

92. C-Am-A-C#m-E-Abm-Ab-Cm-Eb-Ebm-B-Bm-D-F#m-F#-Bbm-C#-Fm-F-Dm-Bb-Gm-G-Em--
RPLRLPLRPLPRLPLRLPRLRPRL

93. C-Am-A-C#m-E-Abm-Ab-Cm-Eb-Gm-Bb-Bbm-C#-Fm-F-Dm-D-F#m-F#-Ebm-B-Bm-G-Em--
RPLRLPLRLRPRLPRPLPRLPLRL

94. C-Am-A-F#m-F#-Ebm-Eb-Cm-Ab-Fm-F-Dm-D-Bm-B-Abm-E-C#m-C#-Bbm-Bb-Gm-G-Em--
RPRPRPRL

95. C-Am-A-F#m-F#-Ebm-B-Abm-Ab-Fm-F-Dm-D-Bm-G-Em-E-C#m-C#-Bbm-Bb-Gm-Eb-Cm--
RPRPRLRP

96. C-Am-A-F#m-F#-Ebm-B-Abm-E-C#m-C#-Bbm-Bb-Gm-Eb-Cm-Ab-Fm-F-Dm-D-Bm-G-Em--
RPRPRLRL

97. C-Am-A-F#m-D-Dm-F-Fm-C#-C#m-E-Abm-Ab-Cm-Eb-Gm-Bb-Bbm-F#-Ebm-B-Bm-G-Em--
RPRLPRPLPRLPLRLRPLRLPLRL

98. C-Am-A-F#m-D-Bm-B-Ebm-F#-Bbm-Bb-Dm-F-Fm-C#-C#m-E-Abm-Ab-Cm-Eb-Gm-G-Em--
RPRLRPLRLPLRPLPRLPLRLPRL

99. C-Am-A-F#m-D-Bm-B-Abm-Ab-Fm-F-Dm-Bb-Gm-G-Em-E-C#m-C#-Bbm-F#-Ebm-Eb-Cm--
RPRLRPRP

100. C-Am-A-F#m-D-Bm-B-Abm-E-C#m-C#-Bbm-F#-Ebm-Eb-Cm-Ab-Fm-F-Dm-Bb-Gm-G-Em-
-RPRL

101. C-Am-A-F#m-D-Bm-G-Gm-Eb-Cm-Ab-Abm-B-Ebm-F#-Bbm-Bb-Dm-F-Fm-C#-C#m-E-Em-
-RPRLRLPLRLPRLRLPLRPLPRPL

102. C-Am-A-F#m-D-Bm-G-Em-E-C#m-C#-Bbm-F#-Ebm-B-Abm-Ab-Fm-F-Dm-Bb-Gm-Eb-Cm-
-RPRLRLRP

103. C-Am-F-Fm-C#-C#m-A-F#m-F#-Bbm-Bb-Dm-D-Bm-B-Ebm-Eb-Gm-G-Em-E-Abm-Ab-Cm-
-RLPLPLRPLPLPRPLPLPRPLPLP

104. C-Am-F-Fm-C#-C#m-A-F#m-D-Dm-Bb-Bbm-F#-Ebm-B-Bm-G-Gm-Eb-Cm-Ab-Abm-E-Em-
-RLPLPL

105. C-Am-F-Fm-C#-Bbm-Bb-Dm-D-Bm-B-Ebm-F#-F#m-A-C#m-E-Abm-Ab-Cm-Eb-Gm-G-Em-
-RLPLRPLPRPLRPRLRLPLRLPRL

106. C-Am-F-Fm-C#-Bbm-Bb-Dm-D-Bm-G-Gm-Eb-Cm-Ab-Abm-B-Ebm-F#-F#m-A-C#m-E-Em-
-RLPLRPLPRLPLRLPRLRPRLRPL

107. C-Am-F-Fm-C#-Bbm-F#-F#m-A-C#m-E-Em-G-Bm-D-Dm-Bb-Gm-Eb-Ebm-B-Abm-Ab-Cm-
-RLPLRLPRLRPRLRPLRLPLRPLP

108. C-Am-F-Fm-C#-Bbm-F#-F#m-A-C#m-E-Abm-Ab-Cm-Eb-Ebm-B-Bm-D-Dm-Bb-Gm-G-Em-
 -RLPLRLPRLRLPLRPLPRPLRPRL
109. C-Am-F-Fm-C#-Bbm-F#-Ebm-Eb-Gm-Bb-Dm-D-F#m-A-C#m-E-Em-G-Bm-B-Abm-Ab-Cm-
 -RLPLRLRPLRLPLRLPLRLRPRLPRPLP
110. C-Am-F-Fm-C#-Bbm-F#-Ebm-Eb-Cm-Ab-Abm-B-Bm-G-Gm-Bb-Dm-D-F#m-A-C#m-E-Em-
 -RLPLRLRPRLPRPLPRLPLRLRPL
111. C-Am-F-Fm-Ab-Abm-E-Em-G-Bm-B-Ebm-F#-Bbm-C#-C#m-A-F#m-D-Dm-Bb-Gm-Eb-Cm-
 -RLPRPLPRLPLRLRPLRLPLRLRP
112. C-Am-F-Fm-Ab-Cm-Eb-Ebm-F#-Bbm-C#-C#m-A-F#m-D-Dm-Bb-Gm-G-Bm-B-Abm-E-Em-
 -RLPRLRPRLRPLRLPLRPLPRLPL
113. C-Am-F-Fm-Ab-Cm-Eb-Gm-G-Bm-D-Dm-Bb-Bbm-C#-C#m-A-F#m-F#-Ebm-B-Abm-E-Em-
 -RLPRLRLPLRPLPRPLRPRLRLPL
114. C-Am-F-Dm-D-F#m-A-C#m-C#-Fm-Ab-Abm-E-Em-G-Bm-B-Ebm-F#-Bbm-Bb-Gm-Eb-Cm-
 -RLRPLRLPLRPLPLRLRPLRP
115. C-Am-F-Dm-D-F#m-A-C#m-C#-Fm-Ab-Cm-Eb-Ebm-F#-Bbm-Bb-Gm-G-Bm-B-Abm-E-Em-
 -RLRPLRLPLRLRPRLPRPLRPLPL
116. C-Am-F-Dm-D-Bm-B-Abm-Ab-Fm-C#-Bbm-Bb-Gm-G-Em-E-C#m-A-F#m-F#-Ebm-Eb-Cm-
 -RLRPRPRP
117. C-Am-F-Dm-D-Bm-B-Abm-E-C#m-A-F#m-F#-Ebm-Eb-Cm-Ab-Fm-C#-Bbm-Bb-Gm-G-Em-
 -RLRPRPRL
118. C-Am-F-Dm-D-Bm-G-Gm-Bb-Bbm-F#-F#m-A-C#m-C#-Fm-Ab-Cm-Eb-Ebm-B-Abm-E-Em-
 -RLRPRLPRPLPRLPLRLRPLRLPL
119. C-Am-F-Dm-D-Bm-G-Em-E-Abm-B-Ebm-Eb-Gm-Bb-Bbm-F#-F#m-A-C#m-C#-Fm-Ab-Cm-
 -RLRPRLRPLRLPLRLPRLPRLPLRLP
120. C-Am-F-Dm-D-Bm-G-Em-E-C#m-A-F#m-F#-Ebm-B-Abm-Ab-Fm-C#-Bbm-Bb-Gm-Eb-Cm-
 -RLRP
121. C-Am-F-Dm-Bb-Bbm-F#-Ebm-Eb-Gm-G-Em-E-Abm-B-Bm-D-F#m-A-C#m-C#-Fm-Ab-Cm-
 -RLRLPLRPLPRPLRPRLPLRLRLP
122. C-Am-F-Dm-Bb-Bbm-F#-Ebm-B-Bm-D-F#m-A-C#m-C#-Fm-Ab-Abm-E-Em-G-Gm-Eb-Cm-
 -RLRLPLRLPRLRLPLRPLPRPLRP
123. C-Am-F-Dm-Bb-Gm-G-Em-E-C#m-A-F#m-D-Bm-B-Abm-Ab-Fm-C#-Bbm-F#-Ebm-Eb-Cm-
 -RLRLRPRP
124. C-Am-F-Dm-Bb-Gm-Eb-Cm-Ab-Fm-C#-Bbm-F#-Ebm-B-Abm-E-C#m-A-F#m-D-Bm-G-Em-
 -RL

Beethoven and the Hypersphere (and the Tonnetz)
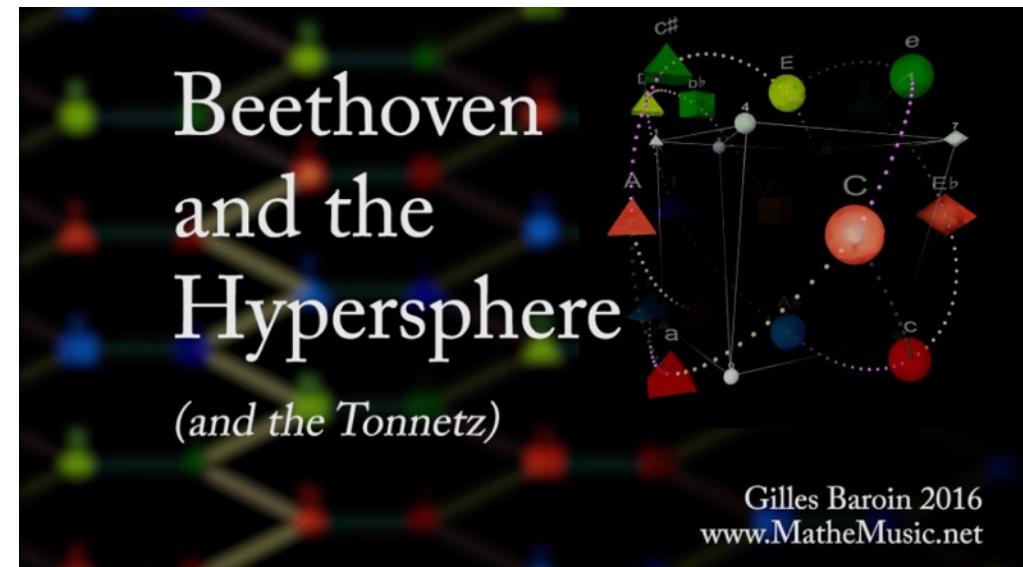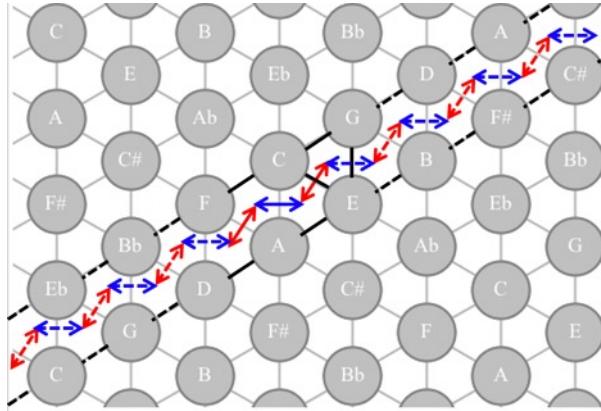
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108. C-Am-F-Fm-C#-Bbm-F#-F#m-A-C#m-E-Abm-Ab-Cm-Eb-Ebm-B-Bm-D-Dm-Bb-Gm-G-Em-
    -RLPLRLPRLRLPLRPLPRPLRPRL
109. C-Am-F-Fm-C#-Bbm-F#-Ebm-Eb-Gm-Bb-Dm-D-F#m-A-C#m-E-Em-G-Bm-B-Abm-Ab-Cm-
    -RLPLRLRPLRLRLPLRLRPRLPRPLP
110. C-Am-F-Fm-C#-Bbm-F#-Ebm-Eb-Cm-Ab-Abm-B-Bm-G-Gm-Bb-Dm-D-F#m-A-C#m-E-Em-
    -RLPLRLRPRLPRPLPRLPLRLRPL
111. C-Am-F-Fm-Ab-Abm-E-Em-G-Bm-B-Ebm-F#-Bbm-C#-C#m-A-F#m-D-Dm-Bb-Gm-Eb-Cm-
    -RLRPLPRLPLRLRPLRLPLRLRP
112. C-Am-F-Fm-Ab-Cm-Eb-Ebm-F#-Bbm-C#-C#m-A-F#m-D-Dm-Bb-Gm-G-Bm-B-Abm-E-Em-
    -RLPRLRPRLRPLRLPLRPLRLPL
113. C-Am-F-Fm-Ab-Cm-Eb-Gm-G-Bm-D-Dm-Bb-Bbm-C#-C#m-A-F#m-F#-Ebm-B-Abm-E-Em-
    -RLPRLRLPLRPLPRPLRPRLRLPL
114. C-Am-F-Dm-D-F#m-A-C#m-C#-Fm-Ab-Abm-E-Em-G-Bm-B-Ebm-F#-Bbm-Bb-Gm-Eb-Cm-
    -RLRPLRLPLRPLPRLPLRLPRLRP
115. C-Am-F-Dm-D-F#m-A-C#m-C#-Fm-Ab-Cm-Eb-Ebm-F#-Bbm-Bb-Gm-G-Bm-B-Abm-E-Em-
    -RLRPLRLPLRLRPRLPRPLPRLPL
116. C-Am-F-Dm-D-Bm-B-Abm-Ab-Fm-C#-Bbm-Bb-Gm-G-Em-E-C#m-A-F#m-F#-Ebm-Eb-Cm-
    -RLRPRPRP
117. C-Am-F-Dm-D-Bm-B-Abm-E-C#m-A-F#m-F#-Ebm-Eb-Cm-Ab-Fm-C#-Bbm-Bb-Gm-G-Em-
    -RLRPRPRL
118. C-Am-F-Dm-D-Bm-G-Gm-Bb-Bbm-F#-F#m-A-C#m-C#-Fm-Ab-Cm-Eb-Ebm-B-Abm-E-Em-
    -RLRPRLPRPLPRLPLRLRPLRLPL
119. C-Am-F-Dm-D-Gm-E-Em-B-Ebm-Eb-Gm-Bb-Bbm-F#-F#m-A-C#m-C#-Fm-Ab-Cm-
    -RLRPRLRPLRLPLRPLPRLPLRLP
120. C-Am-F-Dm-D-Bm-G-Em-E-C#m-A-F#m-F#-Ebm-B-Abm-Ab-Fm-C#-Bbm-Bb-Gm-Eb-Cm-
    -RLRP
121. C-Am-F-Dm-Bb-Bbm-F#-Ebm-Eb-Gm-G-Em-E-Abm-B-Bm-D-F#m-A-C#m-C#-Fm-Ab-Cm-
    -RLRLPLRPLPRPLRPRLRLPLRLP
122. C-Am-F-Dm-Bb-Bbm-F#-Ebm-B-Bm-D-F#m-A-C#m-C#-Fm-Ab-Abm-E-Em-G-Gm-Eb-Cm-
    -RLRLPLRLPRLRLPLRPLPRPLRP
123. C-Am-F-Dm-Bb-Gm-G-Em-E-C#m-A-F#m-D-Bm-B-Abm-Ab-Fm-C#-Bbm-F#-Ebm-Eb-Cm-
    -RLRLRPRP
124. C-Am-F-Dm-Bb-Gm-Eb-Cm-Ab-Fm-C#-Bbm-F#-Ebm-B-Abm-E-C#m-A-F#m-D-Bm-G-Em-
    -RL

41. C-Em-E-Abm-Ab-Cm-Eb-Ebm-B-Bm-G-Gm-Bb-Bbm-F#-F#m-D-Dm-F-Fm-C#-C#m-A-Am--
LPLPLRPLPLPRPLPLPRPLPLPR

42. C-Em-E-Abm-Ab-Cm-Eb-Gm-G-Bm-B-Ebm-F#-Bbm-Bb-Dm-D-F#m-A-C#m-C#-Fm-F-Am--
LPLPLR

43. C-Em-E-Abm-Ab-Fm-F-Am-A-C#m-C#-Bbm-Bb-Dm-D-F#m-F#-Ebm-B-Bm-G-Gm-Eb-Cm--
LPLPRPLPLPRPLPLPRLPLPLRP

→

L P L P L R …
P L P L R L …
L P L R L P …
P L R L P L …
L R L P L P …
R L P L P L …

Luzi

Hamiltonian Song on *Spinnenn*Tonnetz
2014

Lyrics by Mario Luzi
Music and Vocals by Moreno Andreatta
Graphics and Animation by Gilles Baroin
SpinnenTonnetz by Gilles Baroin, Hugò Seress
Original "Chicken Wire" graph by J Douthett, P Steinbach

https://morenoandreatta.com/musicmaths/

min. 1'02"
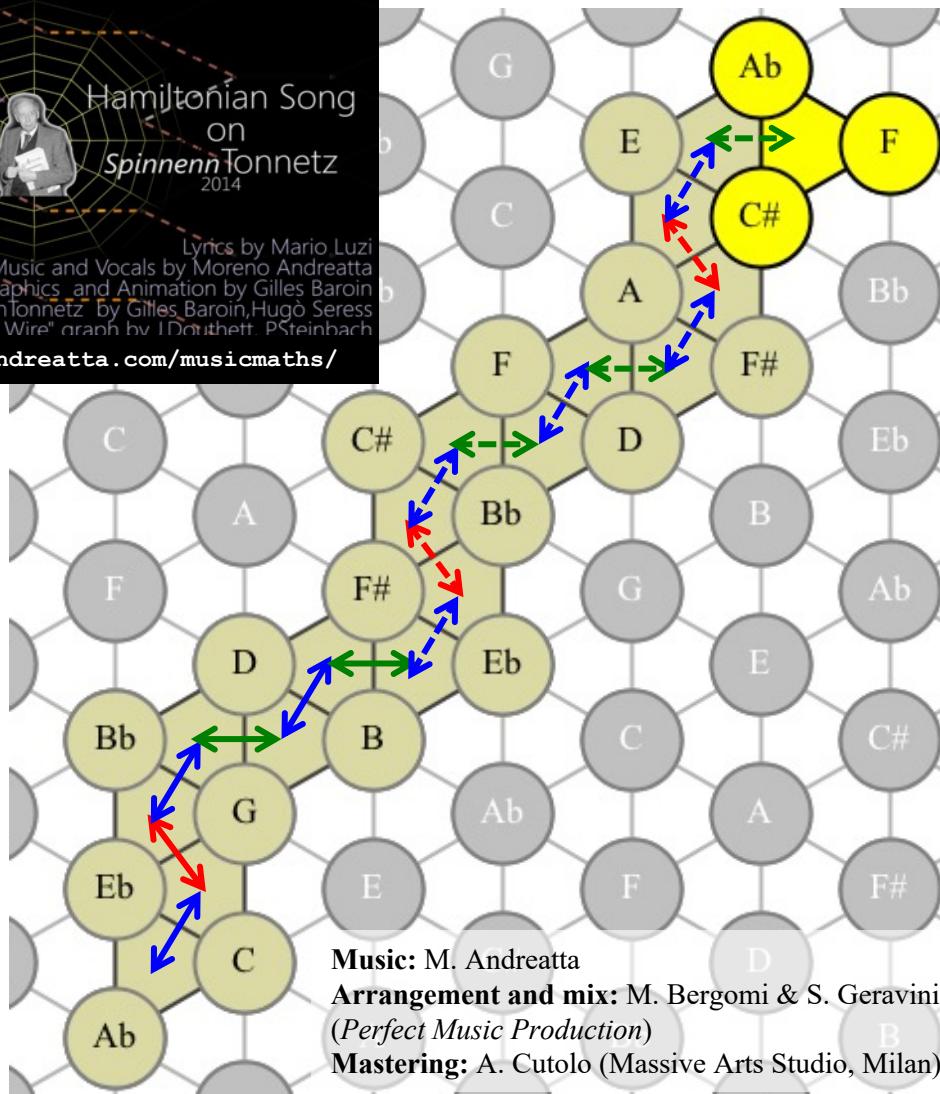
La sera non è più la tua canzone
**(Mario Luzi, 1945, in *Poesie sparse*)**

La sera non è più la tua canzone,
è questa roccia d'ombra traforata
dai lumi e dalle voci senza fine,
la quiete d'una cosa già pensata.

Ah questa luce viva e chiara viene
solo da te, sei tu così vicina
al vero d'una cosa conosciuta,
per nome hai una parola ch'è passata
nell'intimo del cuore e s'è perduta.

Caduto è più che un segno della vita,
riposi, dal viaggio sei tornata
dentro di te, sei scesa in questa pura
sostanza così tua, così romita
nel silenzio dell'essere, (compiuta).

L'aria tace ed il tempo dietro a te
si leva come un'arida montagna
dove vaga il tuo spirito e si perde,
un vento raro scivola e ristagna.

Music: M. Andreatta
Arrangement and mix: M. Bergomi & S. Geravini
(*Perfect Music Production*)
Mastering: A. Cutolo (Massive Arts Studio, Milan)

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Aprile (d'après Gabriele D'Annunzio)
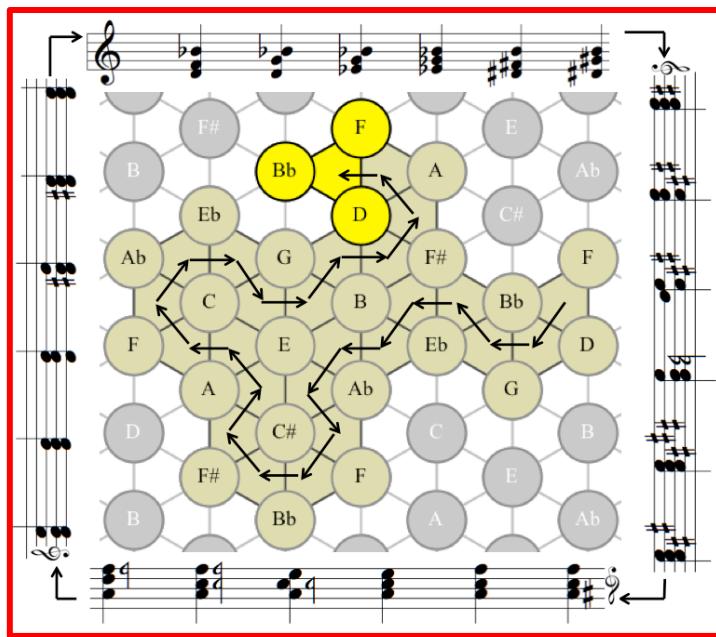
$$Do \leftarrow do_m \leftarrow Sol\# \leftarrow fa_m \leftarrow Fa \leftarrow la_m \leftarrow La \leftarrow fa\#_m \leftarrow Fa\# \leftarrow sib_m \leftarrow Do\# \leftarrow do\#_m$$
$$mi_m \rightarrow Sol \rightarrow si_m \rightarrow Ré \rightarrow ré_m \rightarrow Sib \rightarrow sol_m \rightarrow Mib \rightarrow mib_m \rightarrow Si \rightarrow sol\#_m \rightarrow Mi$$

$$Do \rightarrow mi_m \rightarrow Mi \rightarrow sol\#_m \rightarrow Si \rightarrow ré\#_m \rightarrow Re\# \rightarrow do_m \rightarrow Lab \rightarrow fa_m \rightarrow Do\# \rightarrow do\#_m$$
$$la_m \leftarrow Fa \leftarrow ré_m \leftarrow Ré \leftarrow si_m \leftarrow Sol \leftarrow sol_m \leftarrow Sib \leftarrow sib_m \leftarrow Fa\# \leftarrow fa\#_m \leftarrow La$$

$$Mi \leftarrow mi_m \leftarrow Do \leftarrow la_m \leftarrow Fa \leftarrow fa_m \leftarrow Reb \leftarrow sib_m \leftarrow Fa\# \leftarrow mib_m \leftarrow Mib \leftarrow do_m$$
$$do\#_m \rightarrow La \rightarrow fa\#_m \rightarrow Ré \rightarrow ré_m \rightarrow Sib \rightarrow sol_m \rightarrow Sol \rightarrow si_m \rightarrow Si \rightarrow sol\#_m \rightarrow Sol\#$$
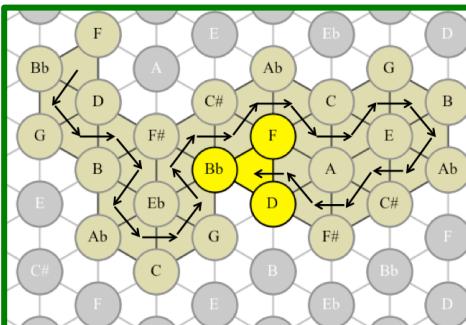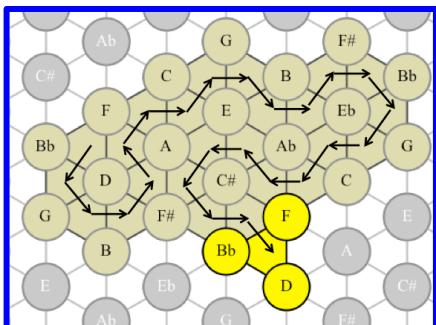
The Gunner's Hamiltonian Dream (a *OuMuPo* experience around Pink-Floyd)

The Gunner's dream (R. Waters, 1983 / M. Andreatta, 2018)

(C)
C   C+
Floating down through the clouds
Am   F
Memories come rushing up to meet me now.
        Fm
In the space between the heavens
                C#   C#m
and in the corner of some foreign field
        A   F+   Bbm
I had a dream.
F#   F#m   D Dm
I had a dream.
Bb
Good-bye Max.
D+
Good-bye Ma.
Ebm   B
After the service when you're walking slowly to the car
        Bm   G
And the silver in her hair shines in the cold November air

Gm
You hear the tolling bell
        Eb
And touch the silk in your lapel
G+   Em   E   G#m
And as the tear drops rise to meet the comfort of the band
G#   Cm
You take her frail hand
(C)
And hold on to the dream.

The three Hamiltonian Cycles (C_M = C, C_m = Cm, C_{aug} = C+)

C-->C+-->Am-->F-->Fm-->C#-->C#m-->A-->F+-->Bbm-->F#-->F#m-->D-->Dm-->Bb-->D+-->Ebm-->B-->Bm-->
-->G-->Gm-->Eb-->G+-->Em-->E-->G#m-->G#-->Cm-->C

C-->C+-->Am-->F-->Fm-->C#-->C#m-->A-->F+-->F#m-->F#-->Bbm-->Bb-->Dm-->D-->D+-->Ebm-->B-->Bm-->
-->G-->Gm-->Eb-->G+-->Em-->E-->G#m-->G#-->Cm-->C

C-->C+-->Am-->F-->Fm-->C#-->C#m-->A-->F+-->F#m-->D-->Dm-->Bb-->Bbm-->F#-->D+-->Ebm-->B-->Bm-->
-->G-->Gm-->Eb-->G+-->Cm-->G#-->G#m-->E-->Em-->C

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Thank you for your attention!