

From music to mathematics and from mathematics to music: some aspects of ‘mathemusical’ research

Ciclo Pomeriggi UniCAmente Matematici
6 novembre 2024

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CNRS / IRMA / Université de Strasbourg

CNRS / IRCAM / Sorbonne Université

<http://repmus.ircam.fr/moreno/smir>

Some examples of PhD on **maths** / **music** / **computer science**

- **Christophe Weis**, *Geometric Models of Harmony as Tools for Computer Assisted Composition and Improvisation*, PhD in **music research**, Hochschule für Musik Karlsruhe (supervised by Marlon Schumacher and Moreno Andreatta).
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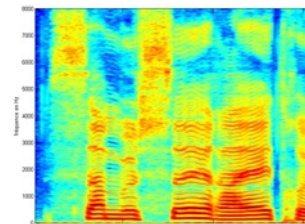
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The SMIR Project: Structural Music Information Research

Signal and audio-based Approaches

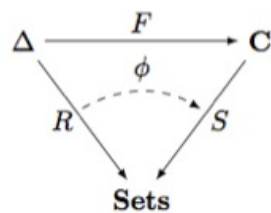
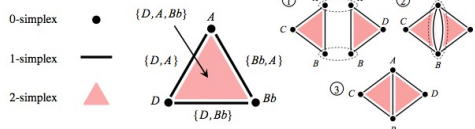
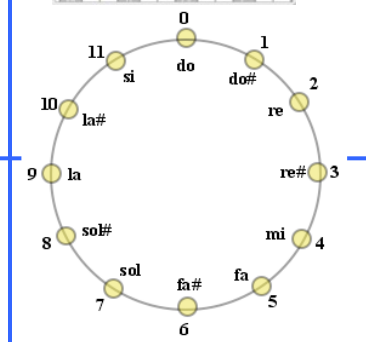
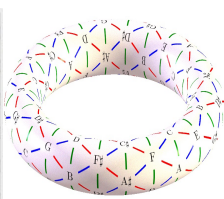
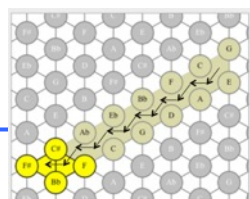


Algebraic models



Topological models

Categorical models

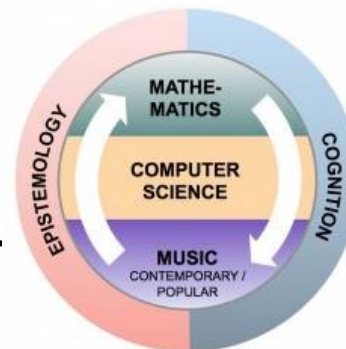


Computational models

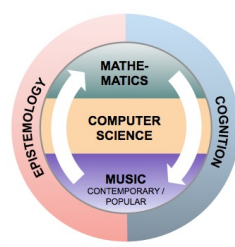
Cognitive models

Structural Symbolic Music Information Research

<http://repmus.ircam.fr/moreno/smir>



The double movement of a 'mathemusical' activity



MATHEMATICS

Mathematical statement

generalisation

General theorem

formalisation

COMPUTER SCIENCE

application

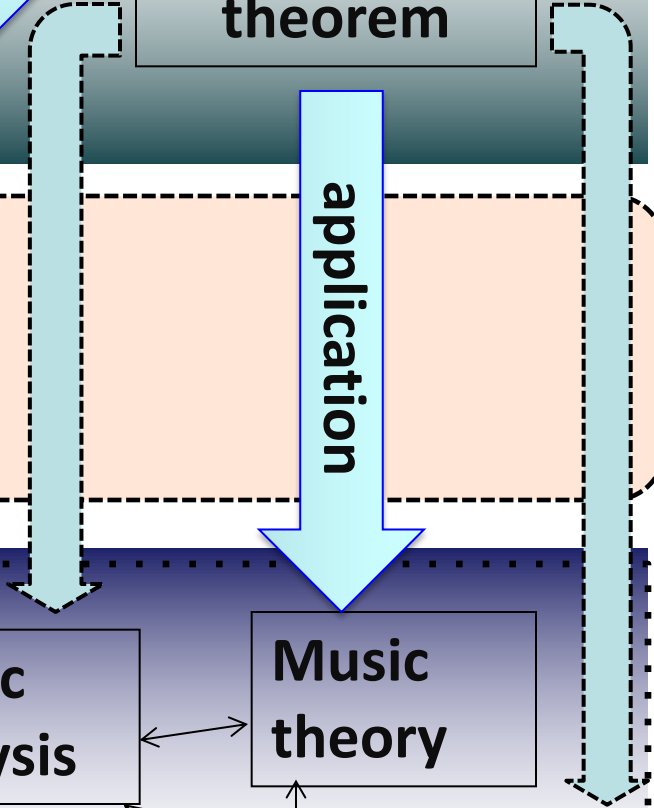
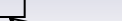
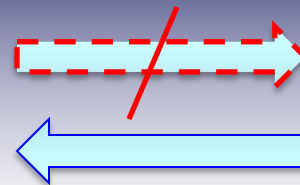
MUSIC

Musical problem

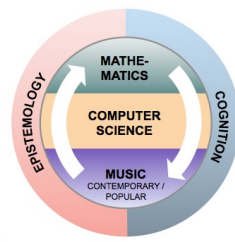
Music analysis

Music theory

Composition



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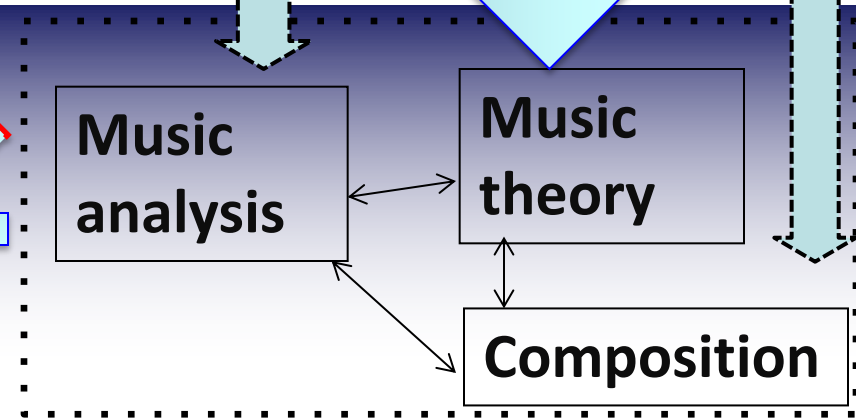
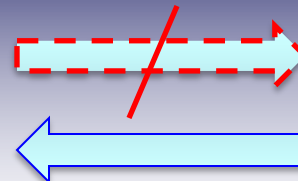
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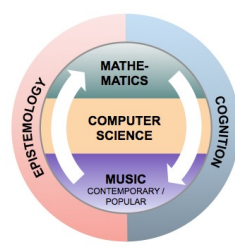
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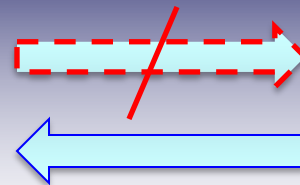
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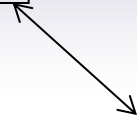
Musical problem



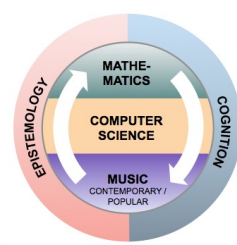
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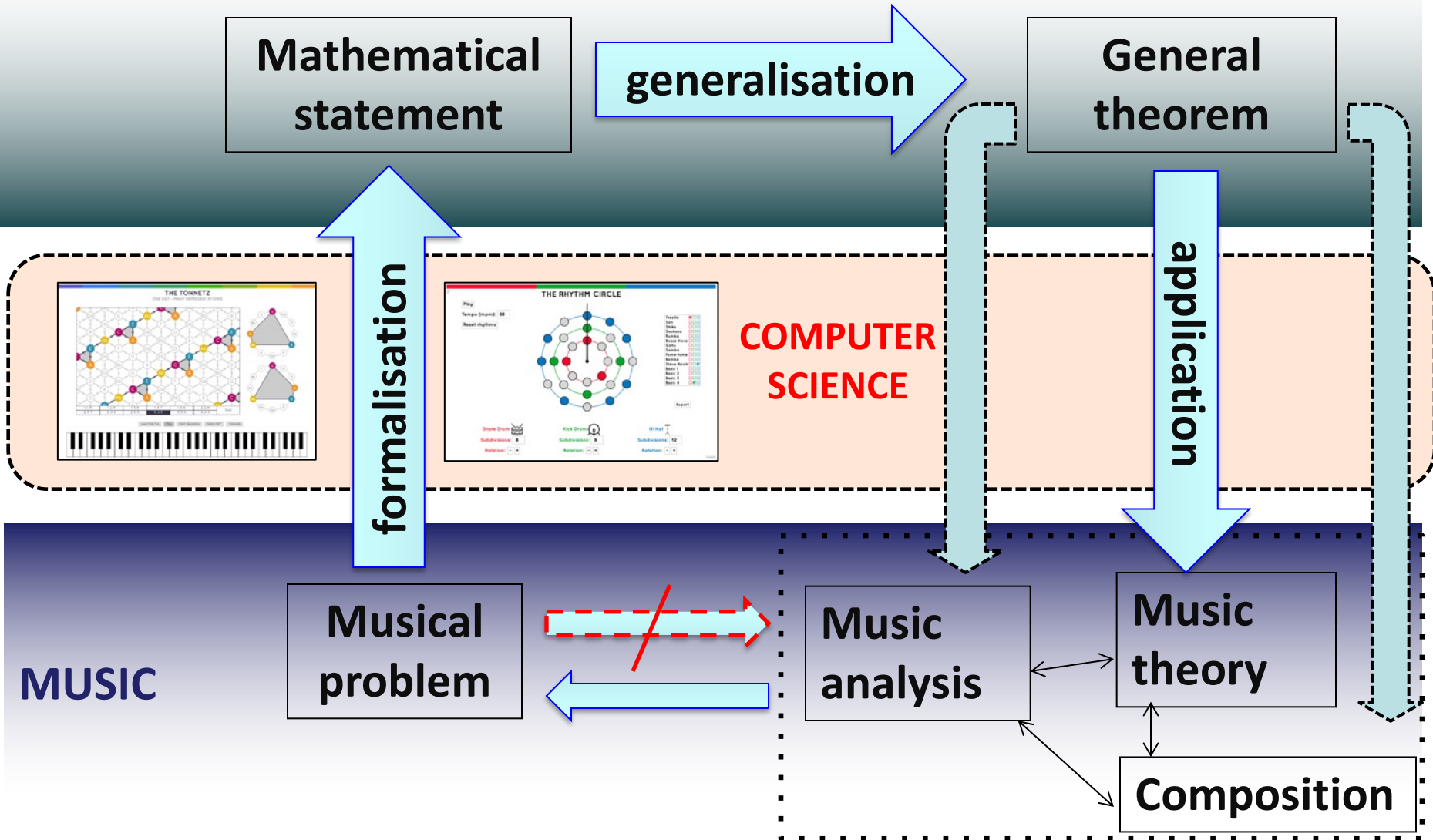
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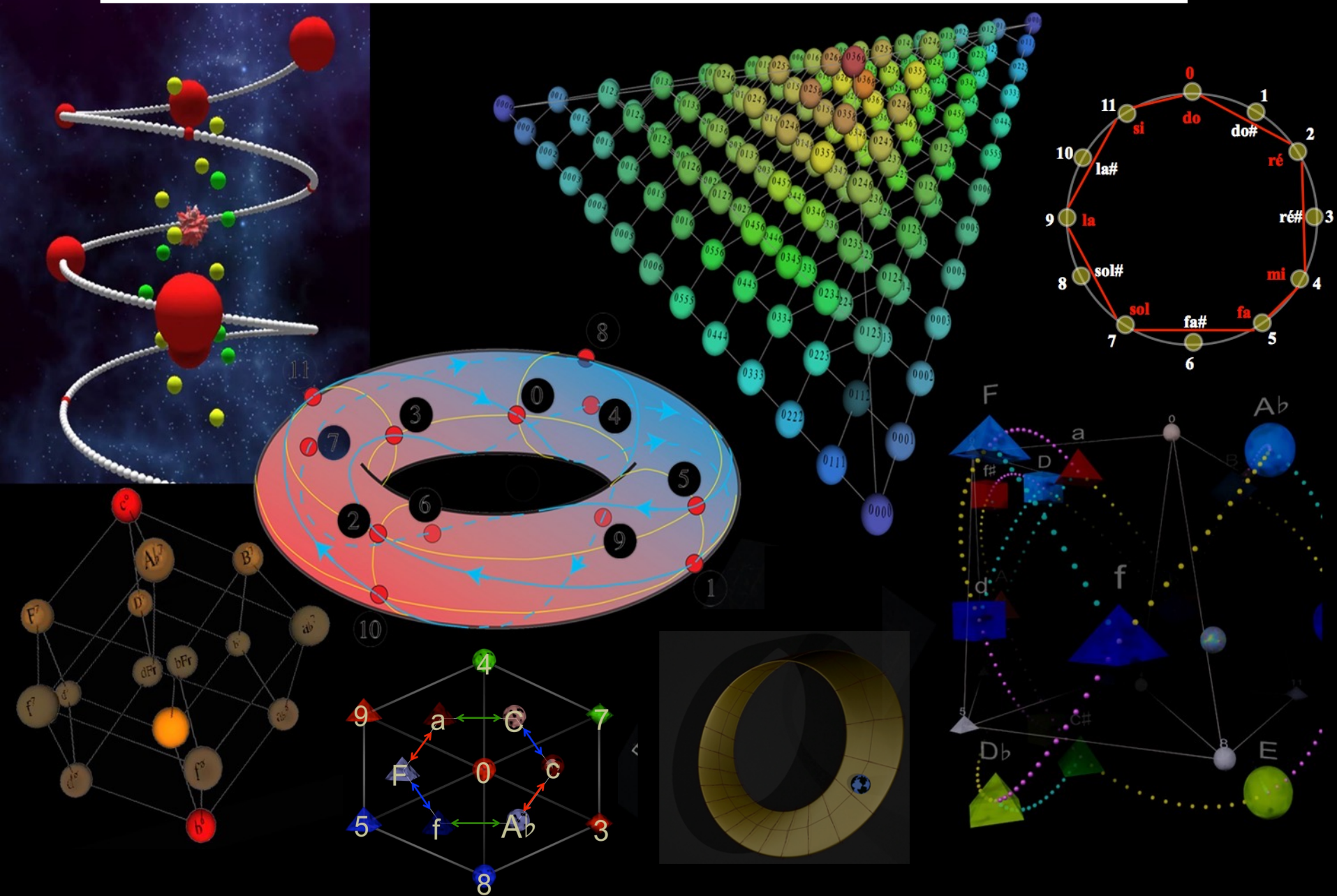
Music analysis

Music theory

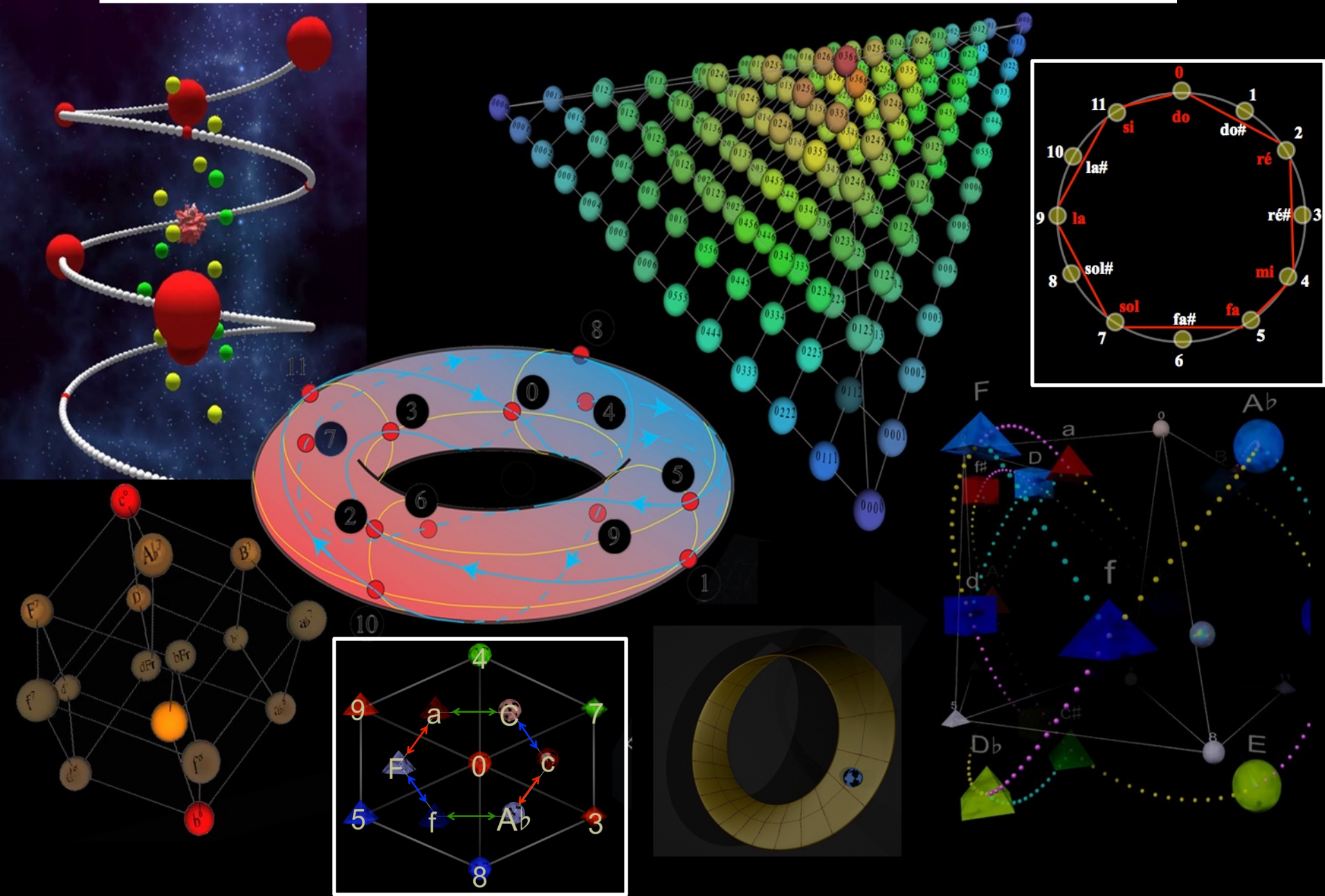
Composition



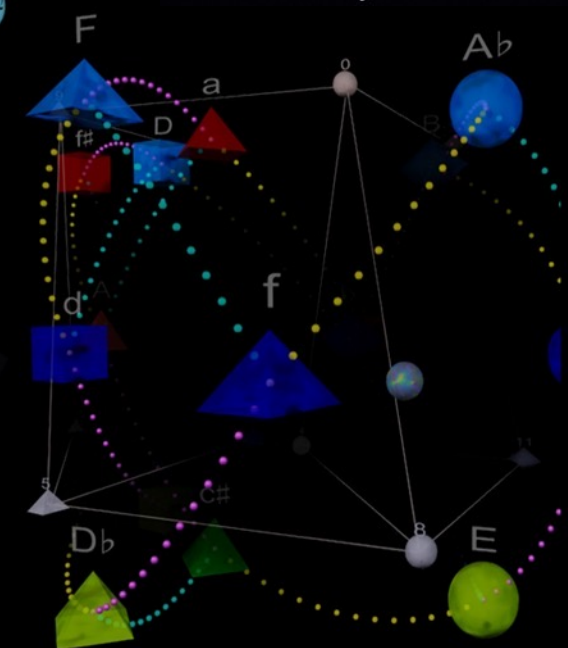
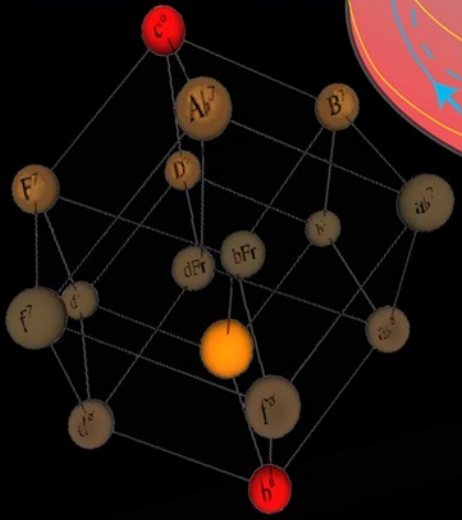
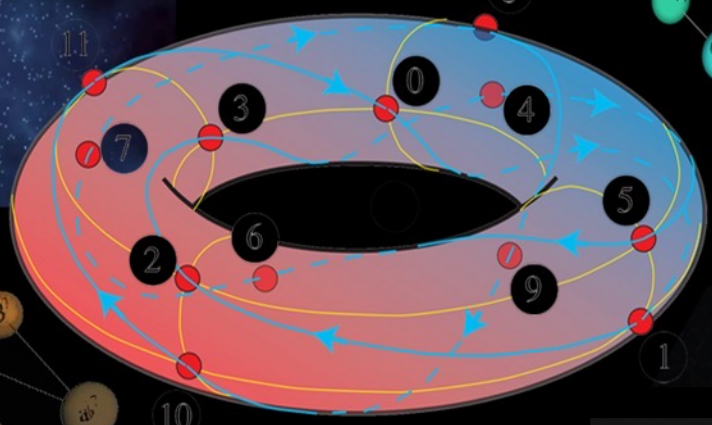
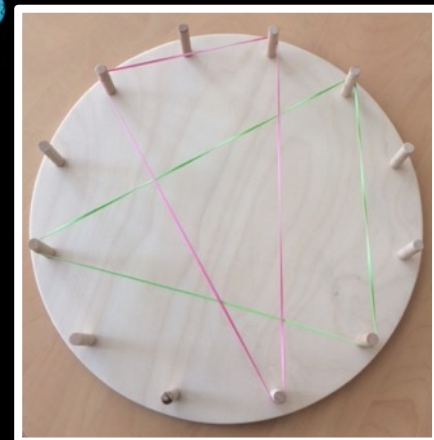
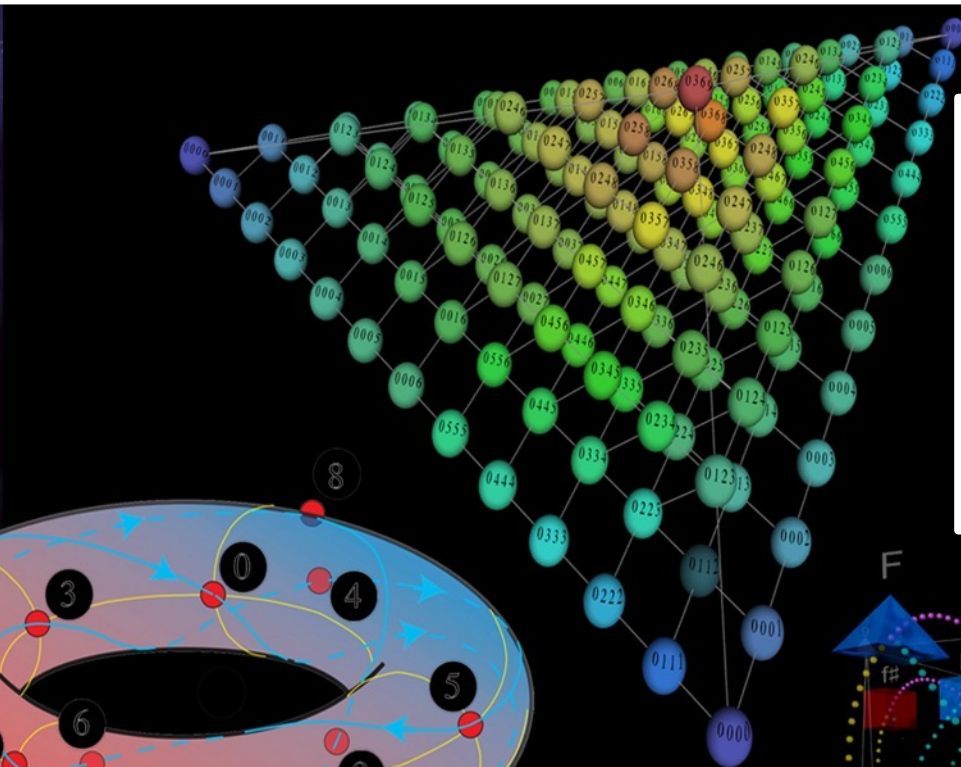
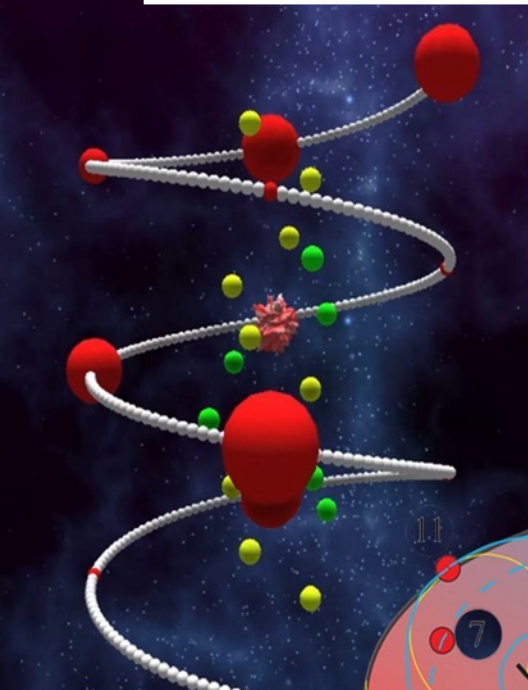
The galaxy of mathematical models at the service of music



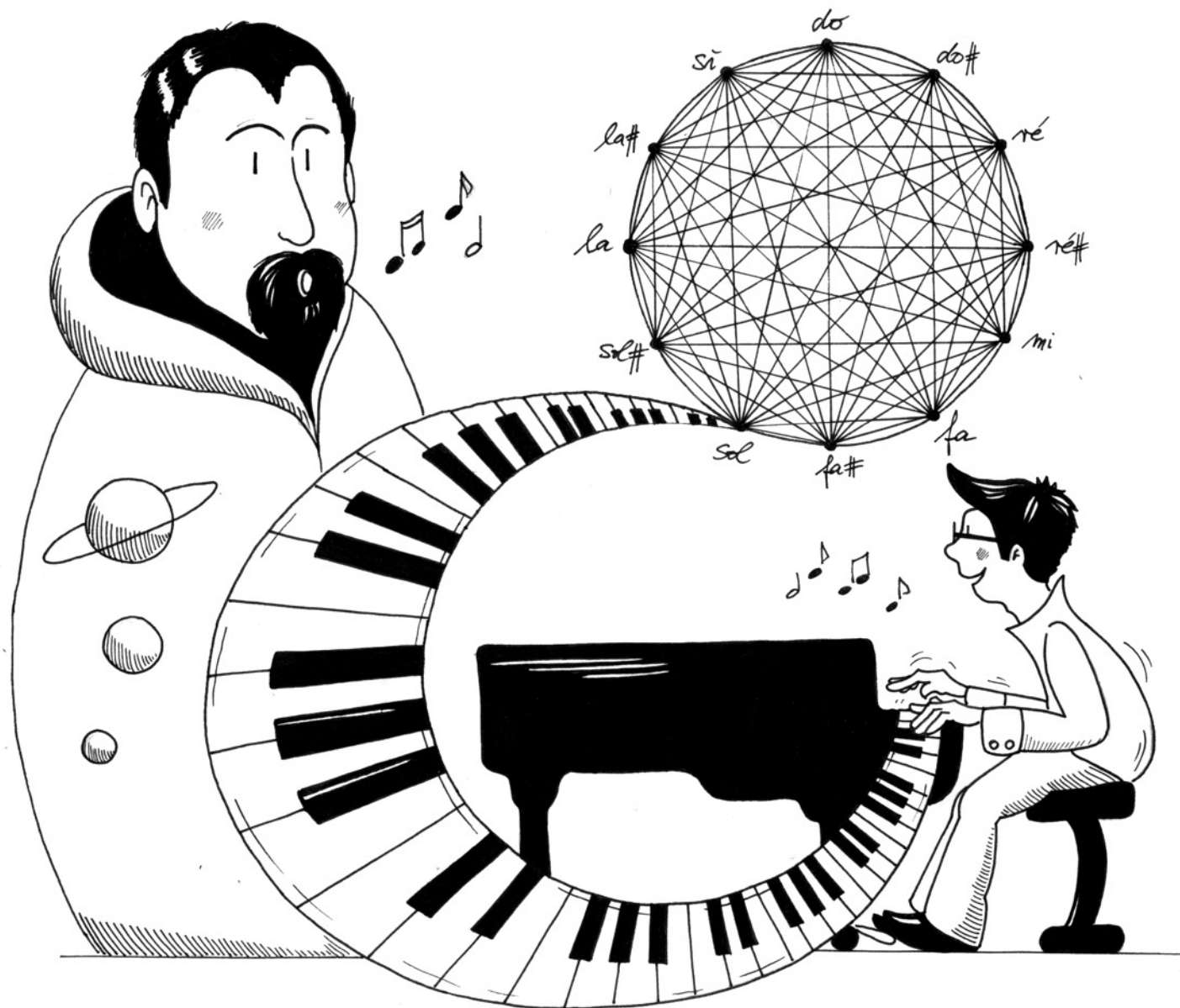
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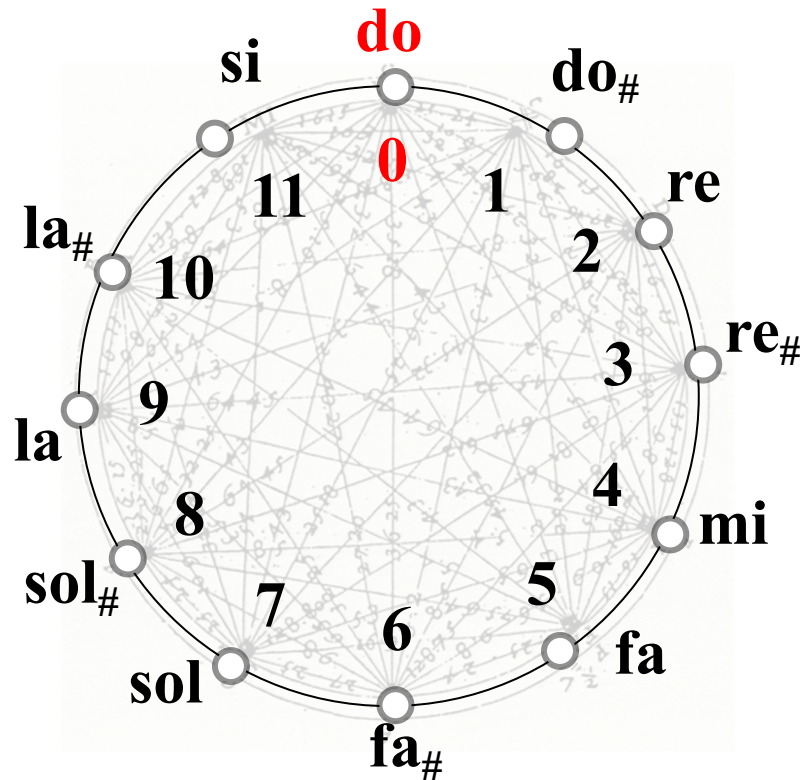
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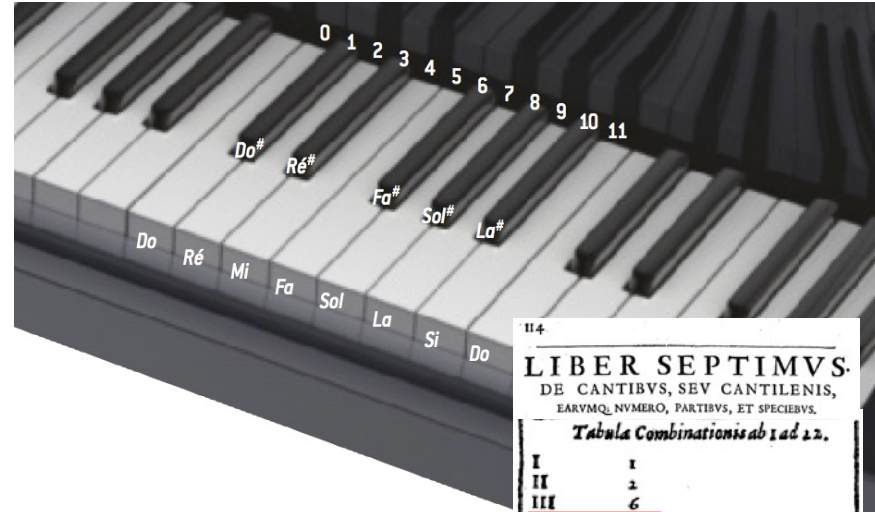
Marin Mersenne, the father of combinatorics



The circular representation of the pitch space



Harmonicorum Libri XII, 1648

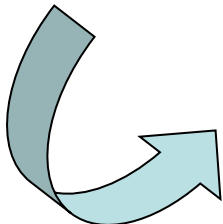
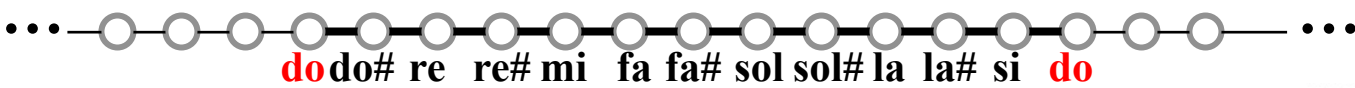


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LIBER SEPTIMVS.
DE CANTIBVS, SEV CANTILENIS,
EARVMQ; NVMERO, PARTIBVS, ET SPECIEBVS.

Tabula Combinationis ab 1 ad 12.

I	1
II	2
III	6
IV	24
V	120
VI	720
VII	5040
VIII	40320
IX	362880
X	3628800
XI	39916800
XII	479001600
XIII	6127020800
XIV	8718192000
XV	1307674368000
XVI	20922789888000
XVII	335687418096000
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XXI	510909421717094400000
XXII	114000727776076800000

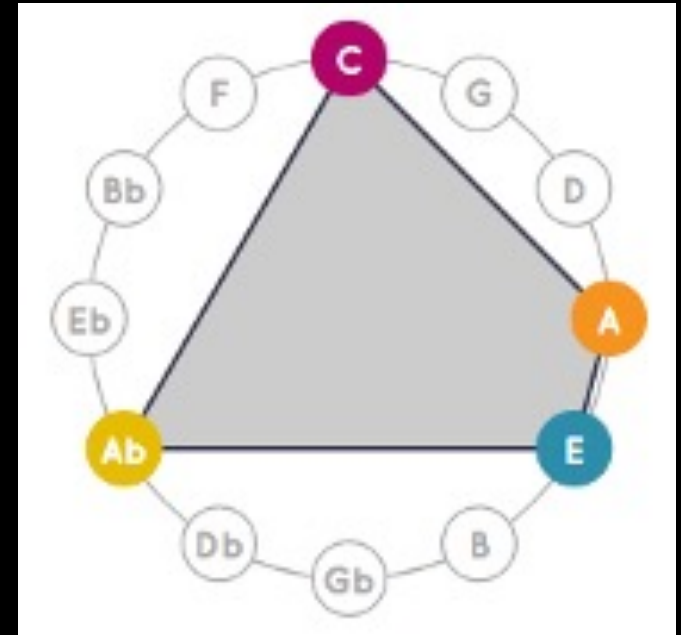
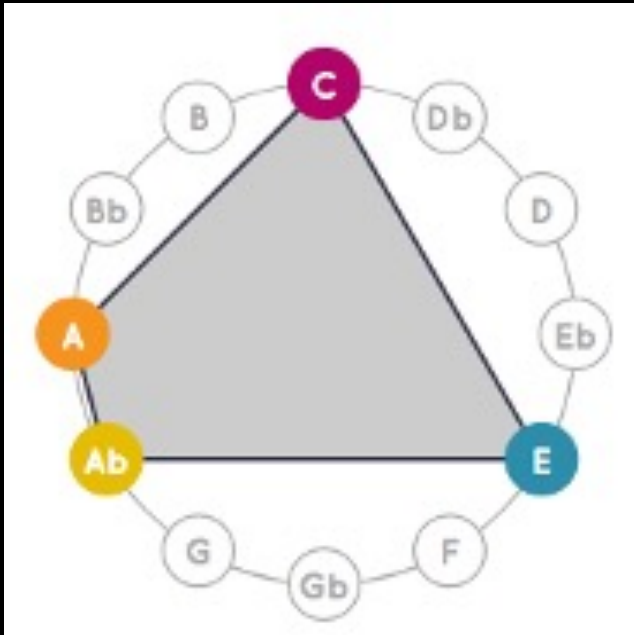


Varietas, seu Combinatio quatuor notarum.

The circular representation

<https://thetonnnetz.com/>

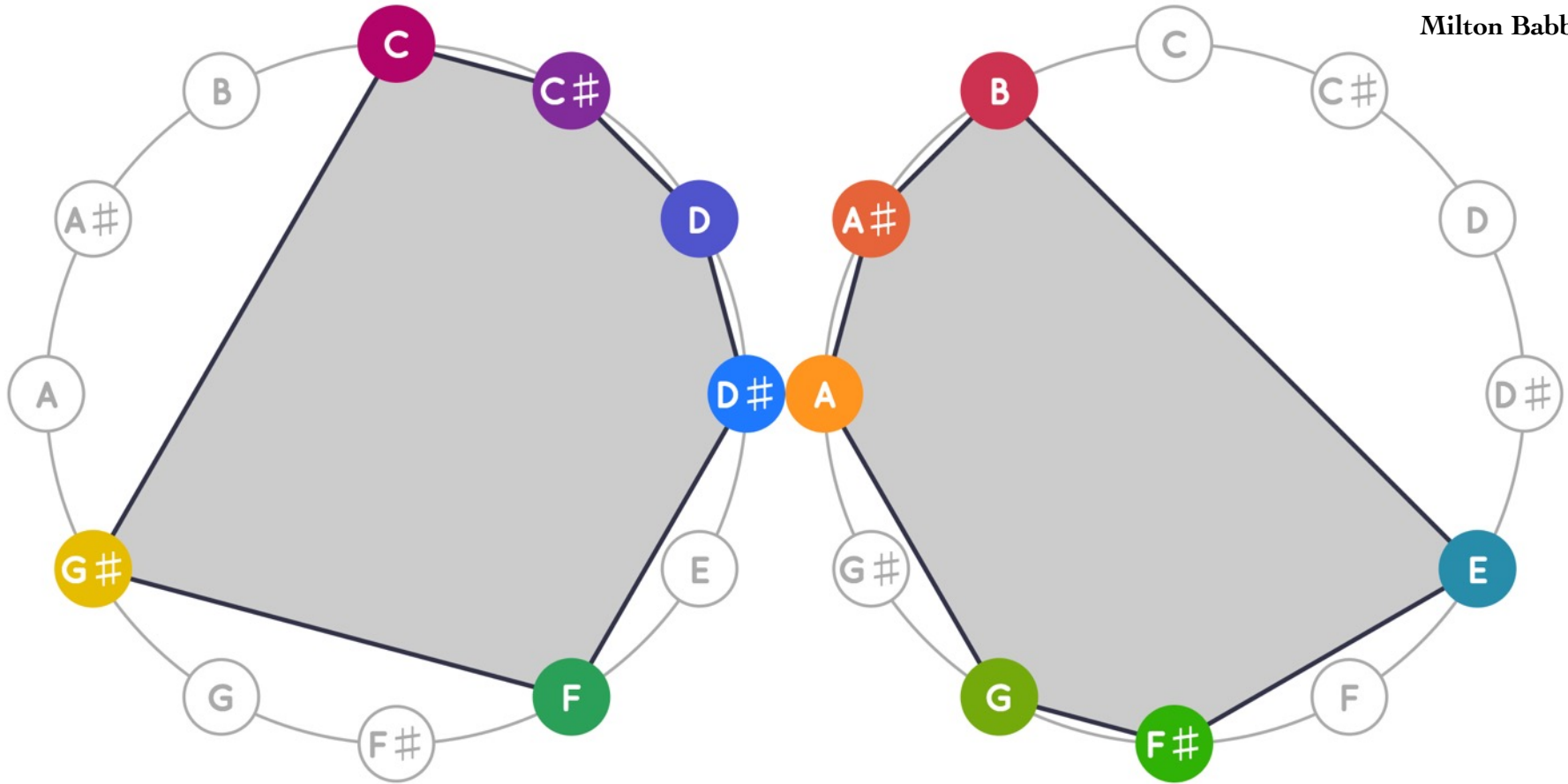
DEMO



A historical example of “mathemusical” problem



Milton Babbitt

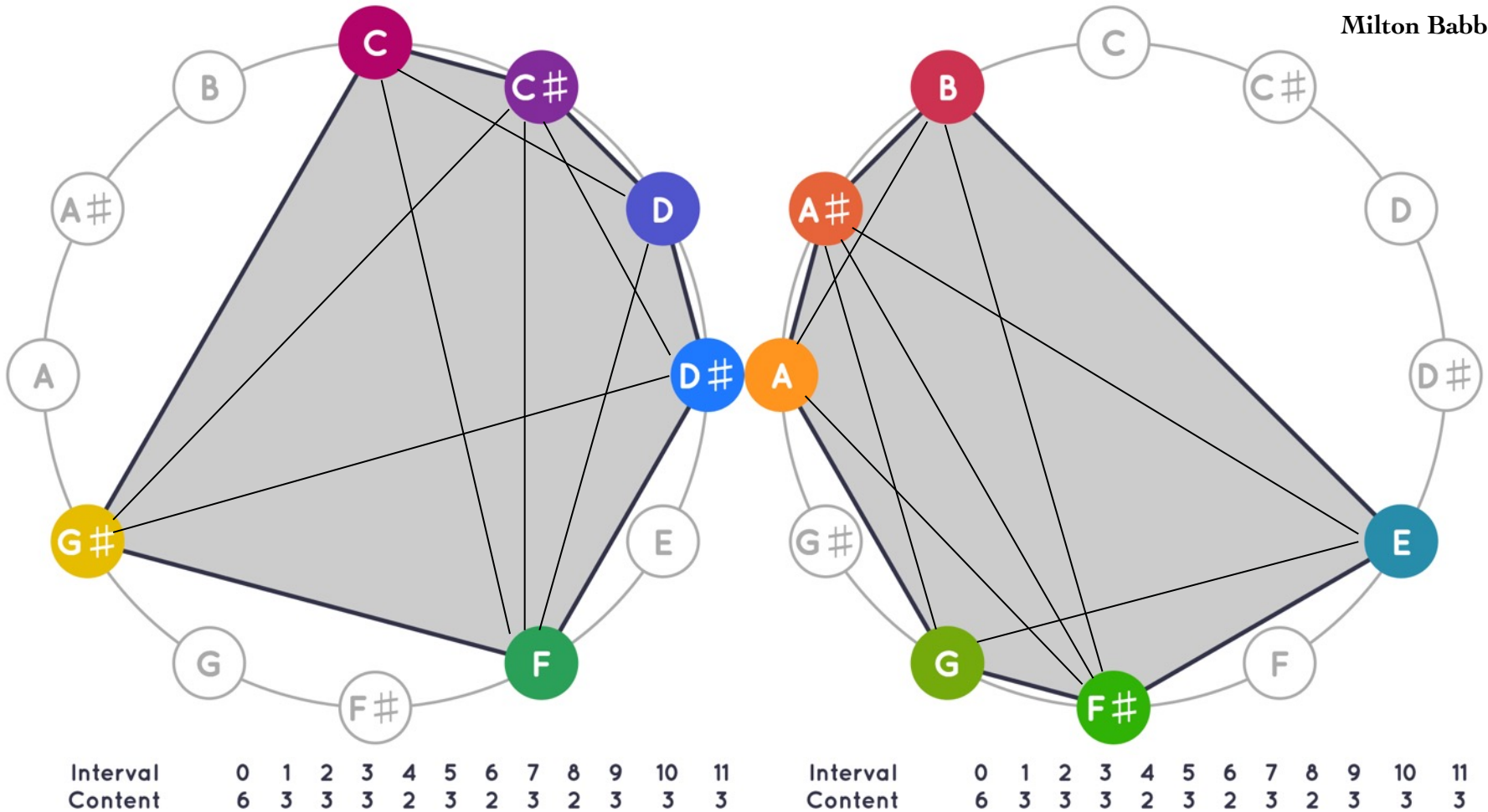


Which is the relation between these two chords?

A historical example of “mathemusal” problem



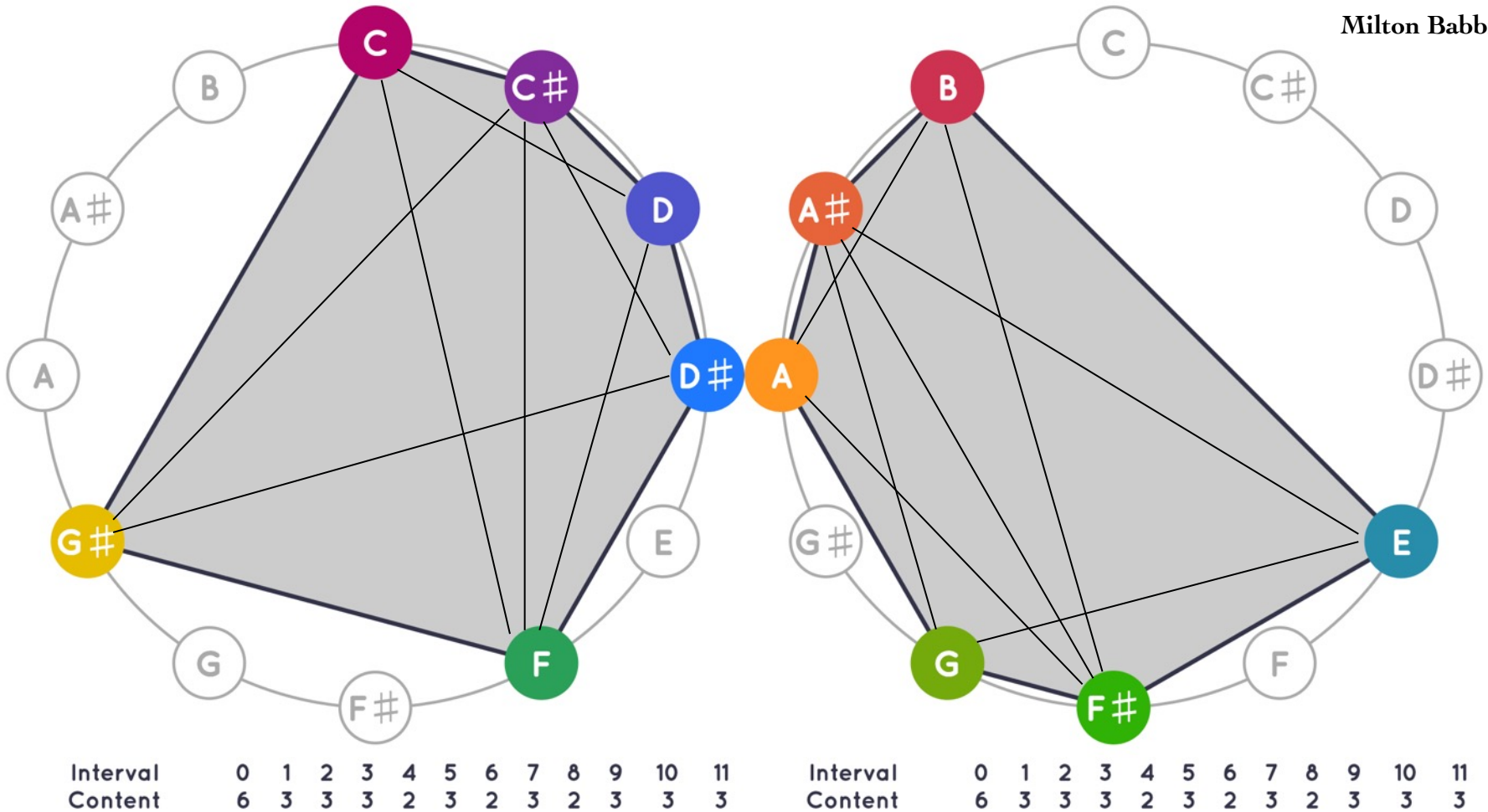
Milton Babbitt



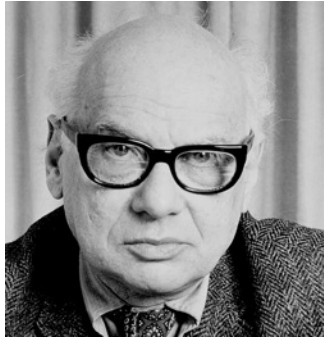
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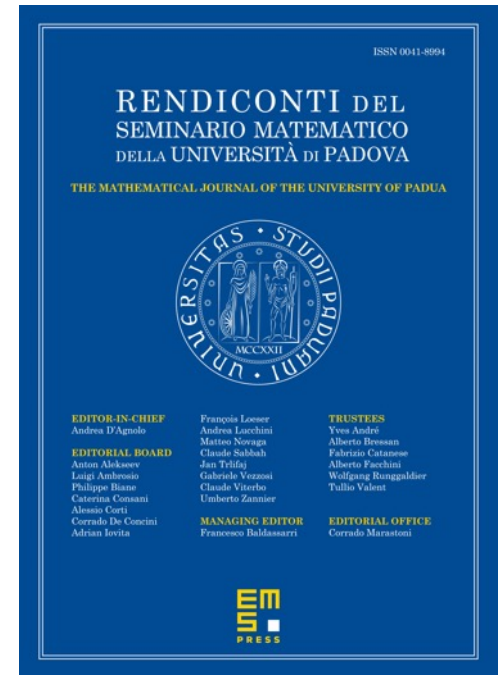
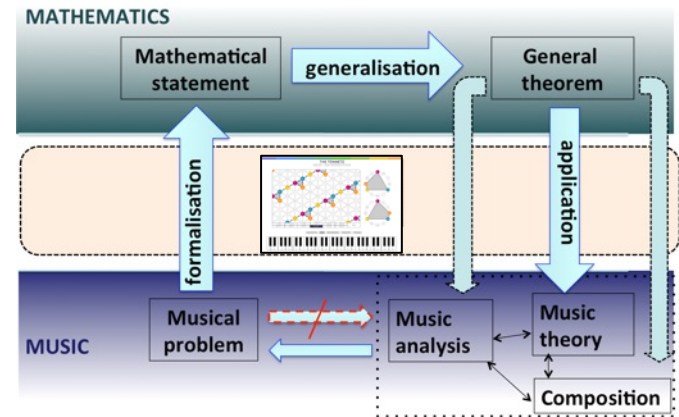
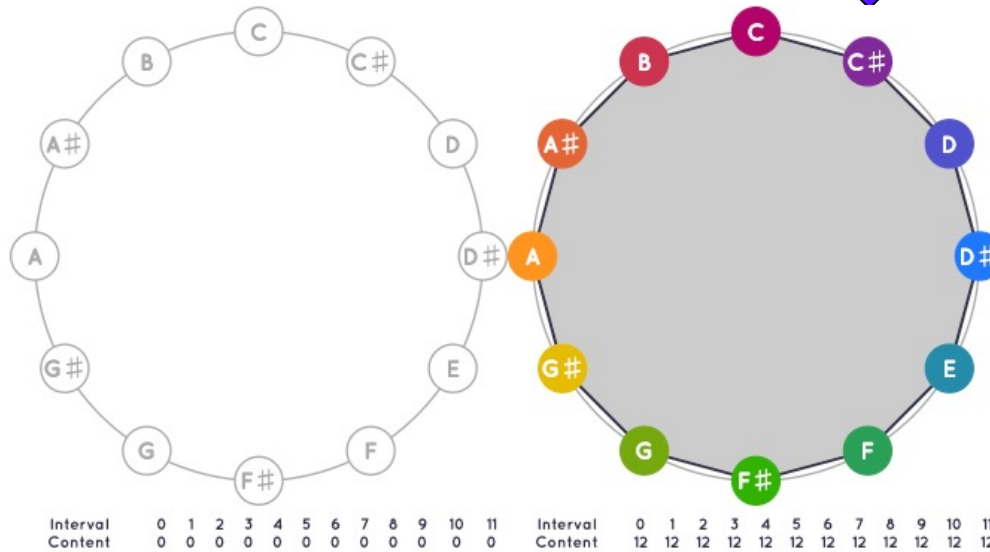
Milton Babbitt



A generalization of Babbitt's Hexachord Theorem



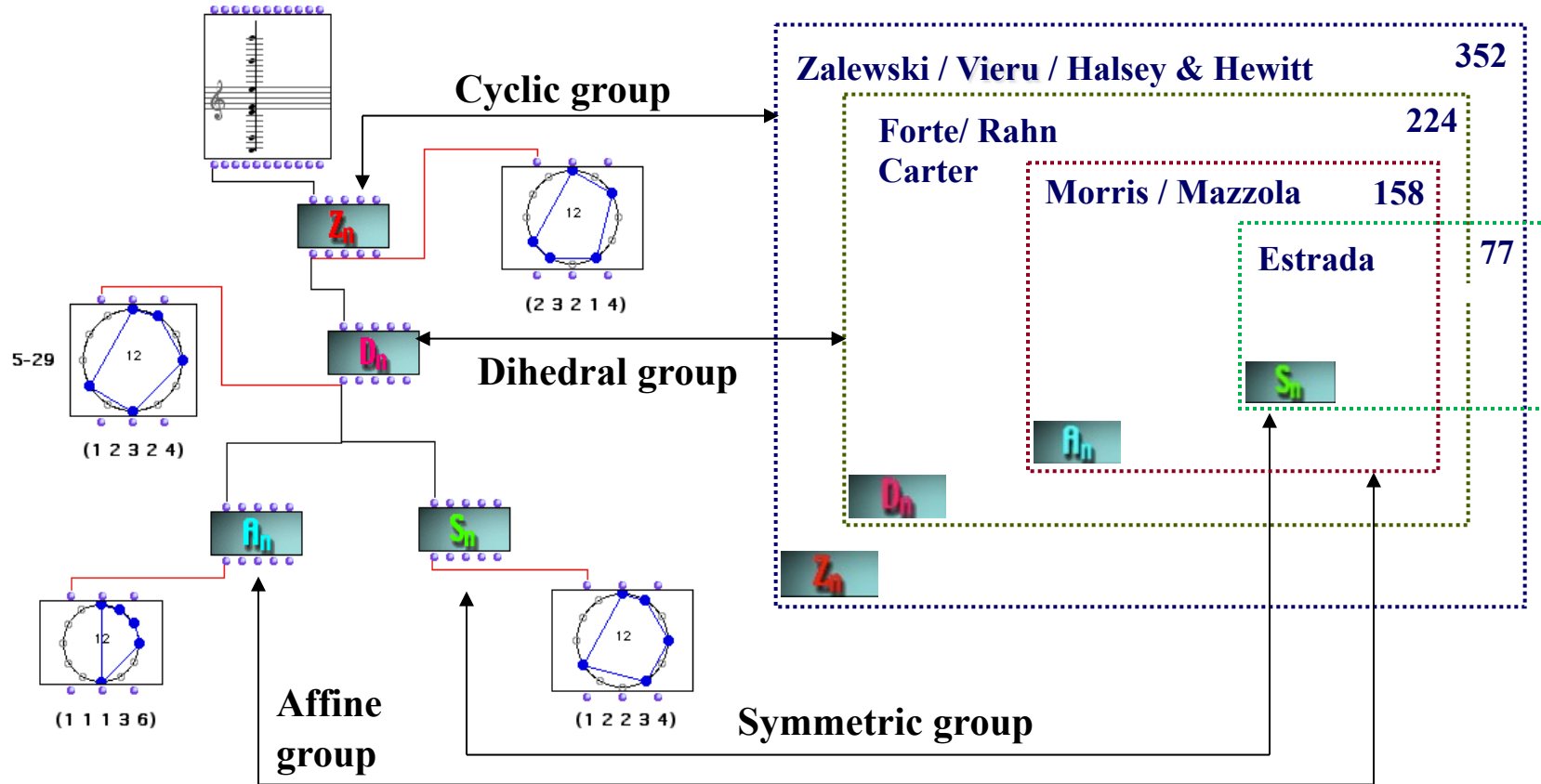
M. Babbitt



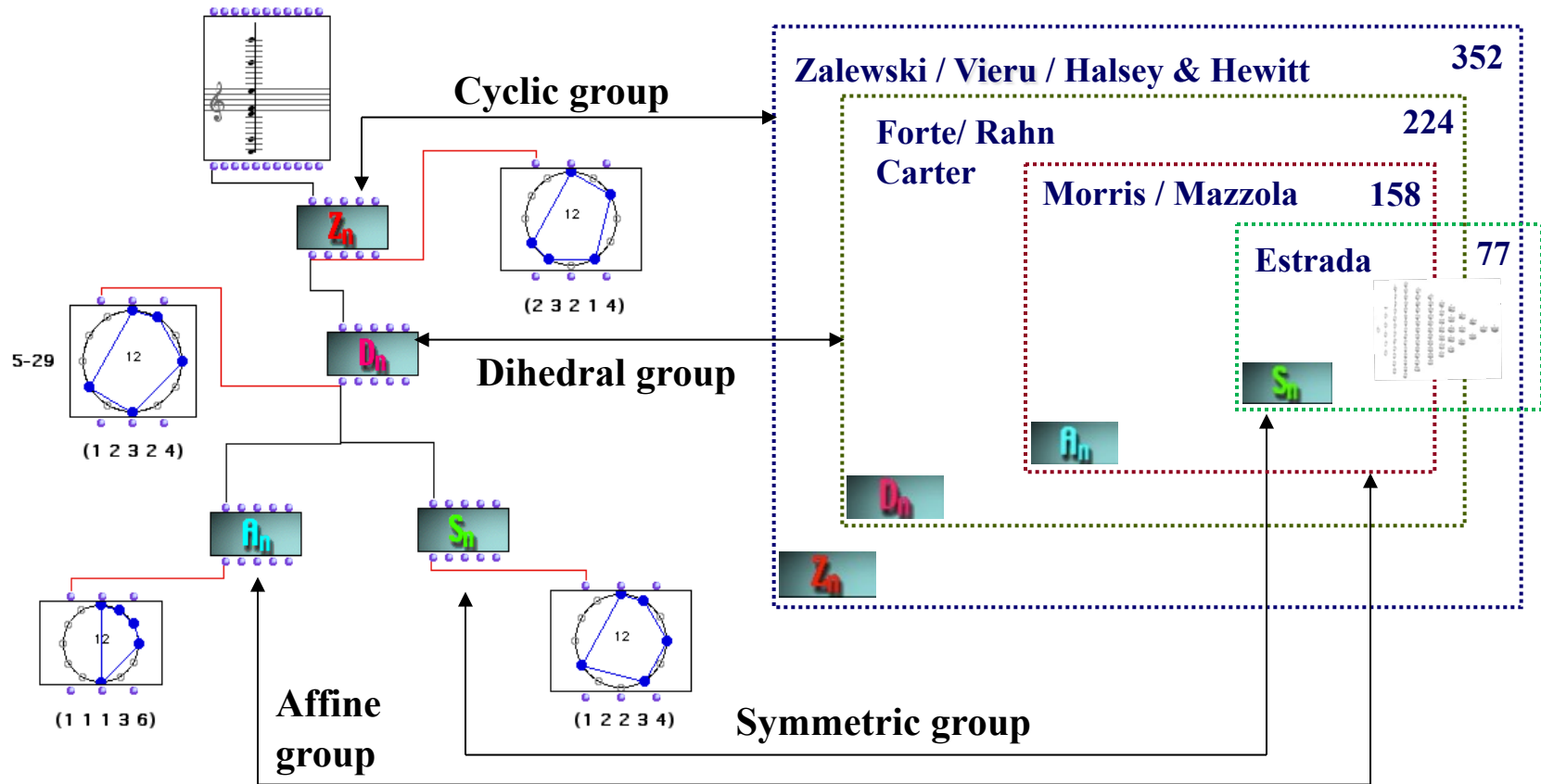
→ <https://guichaoua.gitlab.io/web-hexachord/hexachordTheorem>

→ M. Andreatta et al., "New hexachordal theorems in metric spaces with a probability measure", Rendiconti Univ Padova, 2023

Algebraic Combinatorics and Music Enumeration



Algebraic Combinatorics and Music Enumeration



	1	2	3	4	5	6	7	8	9	10	11	12
Z_n	1	6	19	43	66	80	66	43	19	6	1	1
D_n	1	6	12	29	38	50	38	29	12	6	1	1
A_n	1	5	9	21	25	34	25	21	9	5	1	1
S_n	1	6	12	15	13	11	7	5	3	2	1	1

77

S_n

Algebraic Combinatorics and Music Enumeration



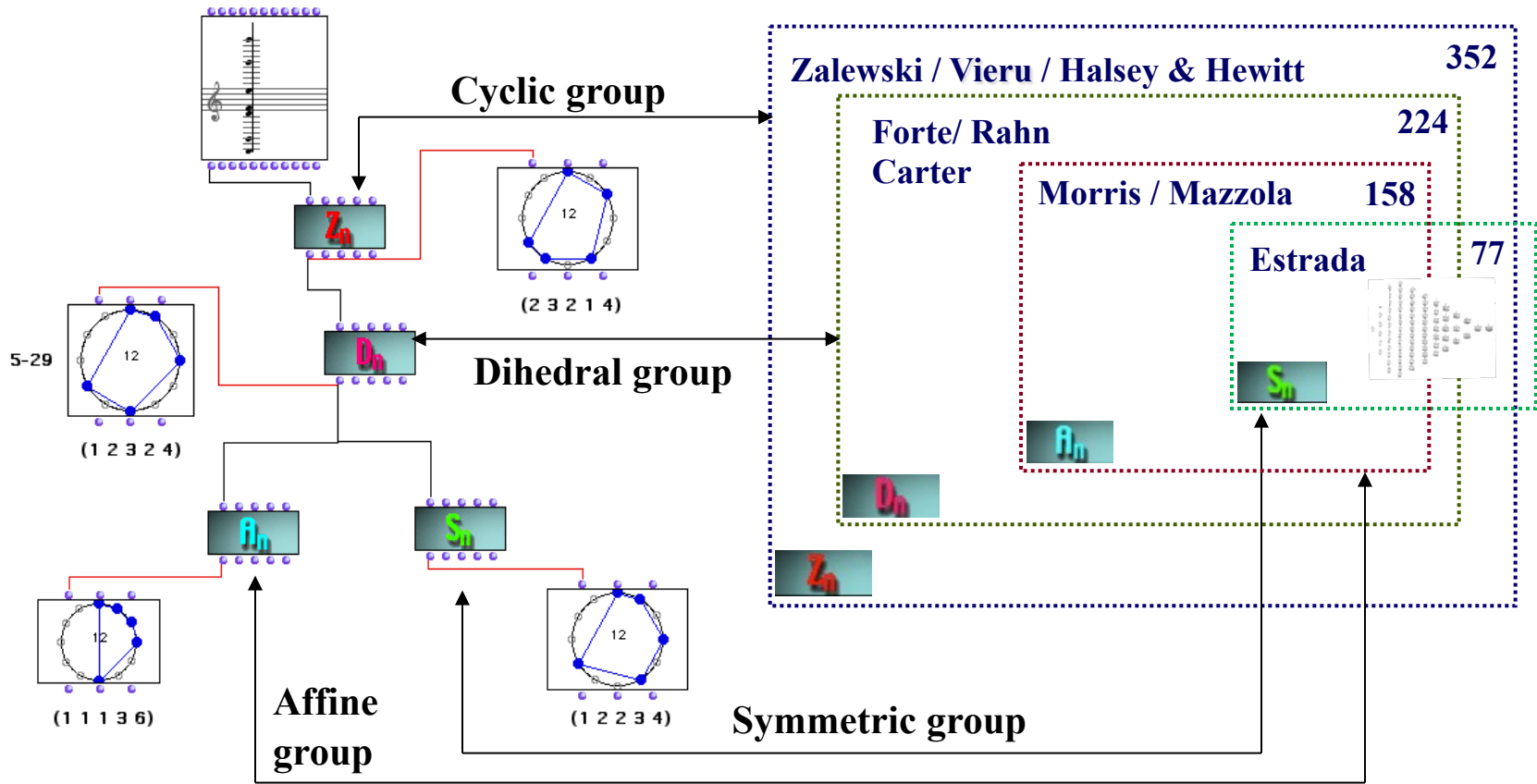
F. Klein



W. Burnside



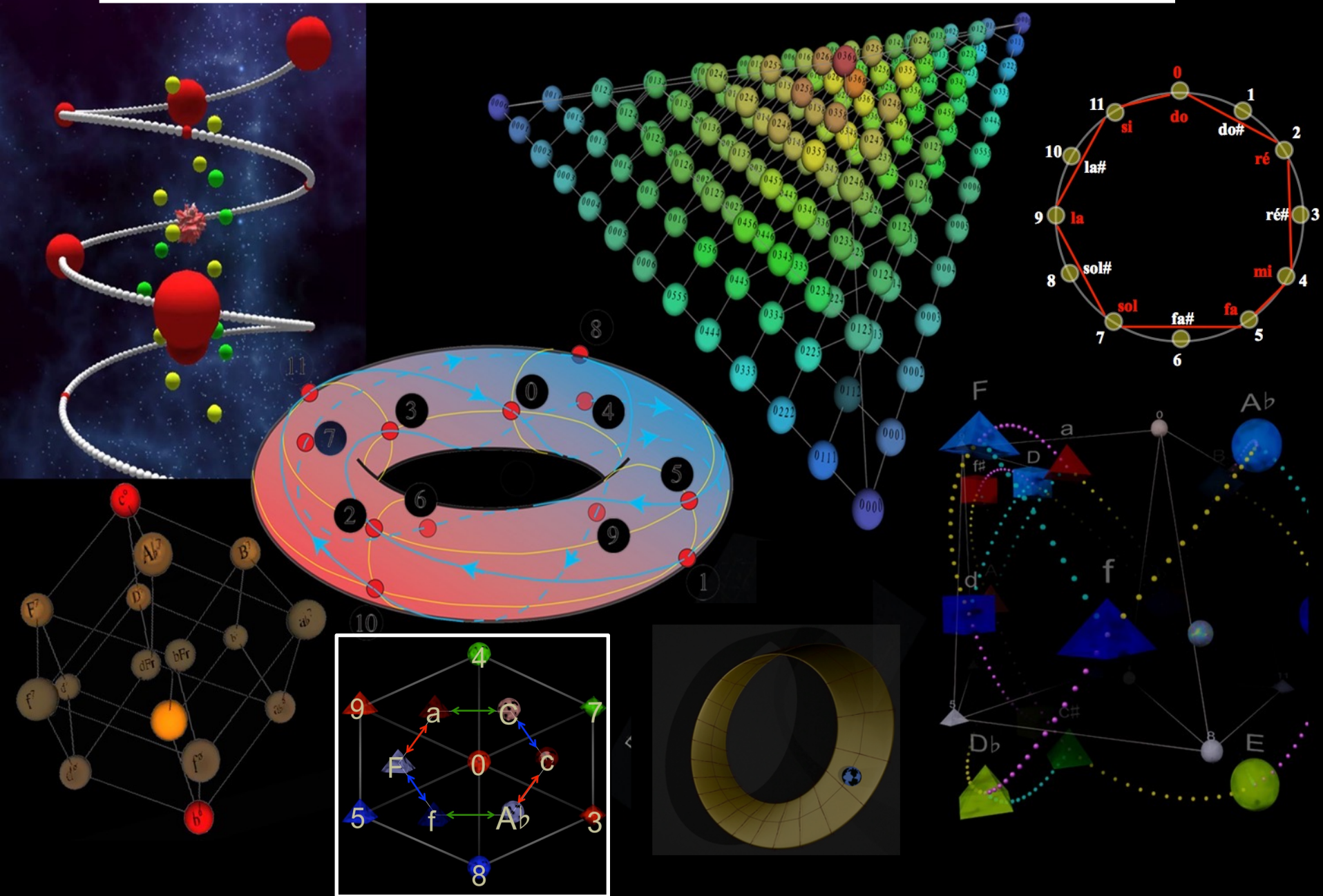
G. Polya



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Z_n	1	6	19	43	66	80	66	43	19	6	1	1
D_n	1	6	12	29	38	50	38	29	12	6	1	1
A_n	1	5	9	21	25	34	25	21	9	5	1	1
S_n	1	6	12	15	13	11	7	5	3	2	1	1

77

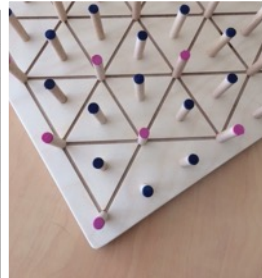
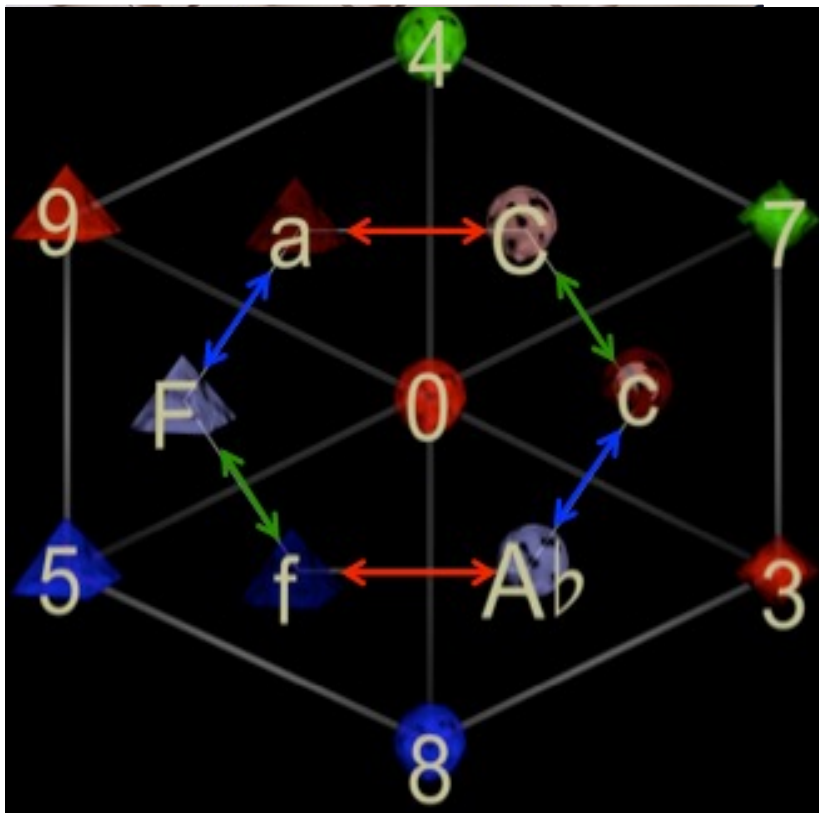
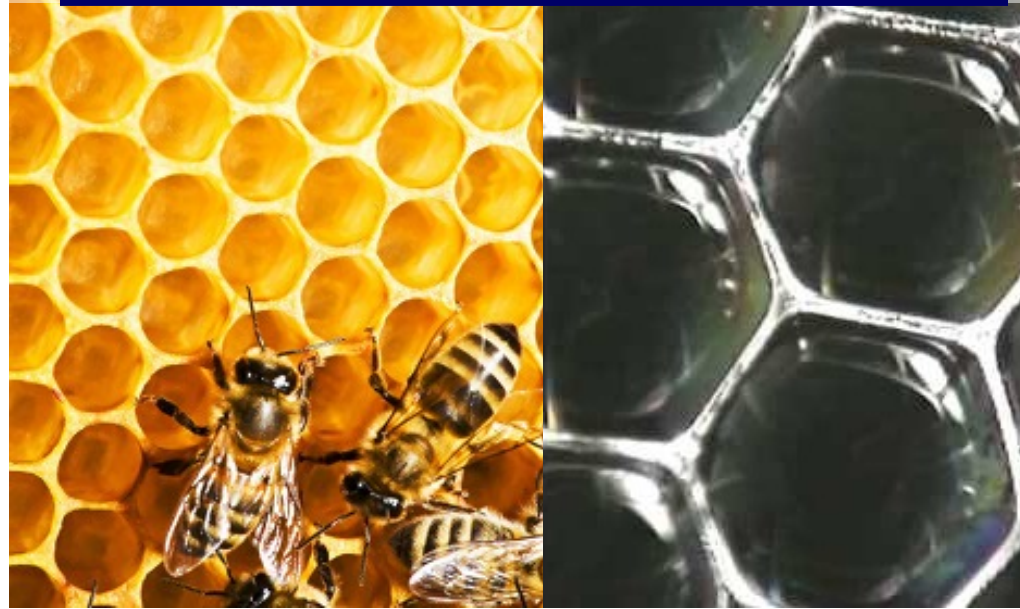
The galaxy of mathematical models at the service of music



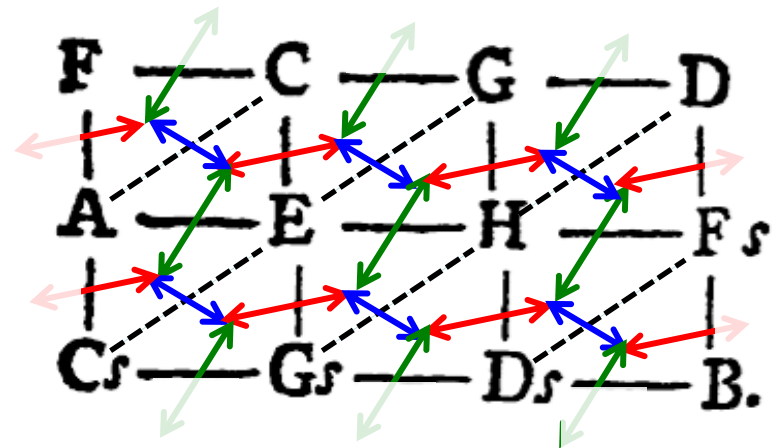
Leonhard Euler, the father of graph theory



The Tonnetz (or 'honeycomb' hexagonal tiling)

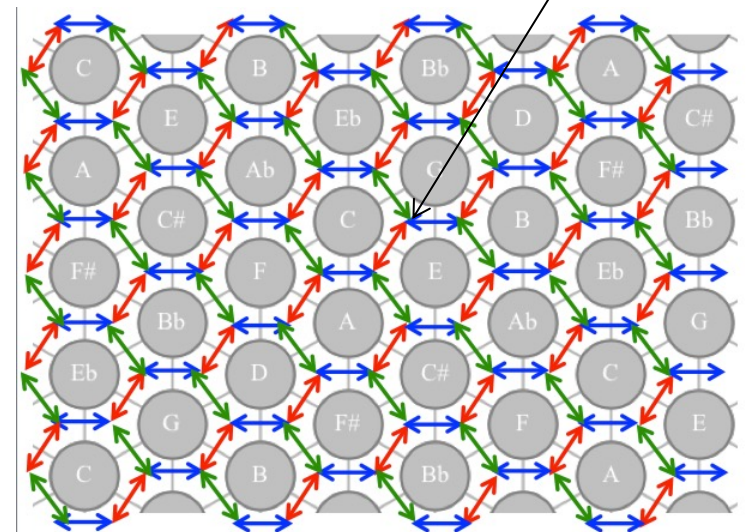
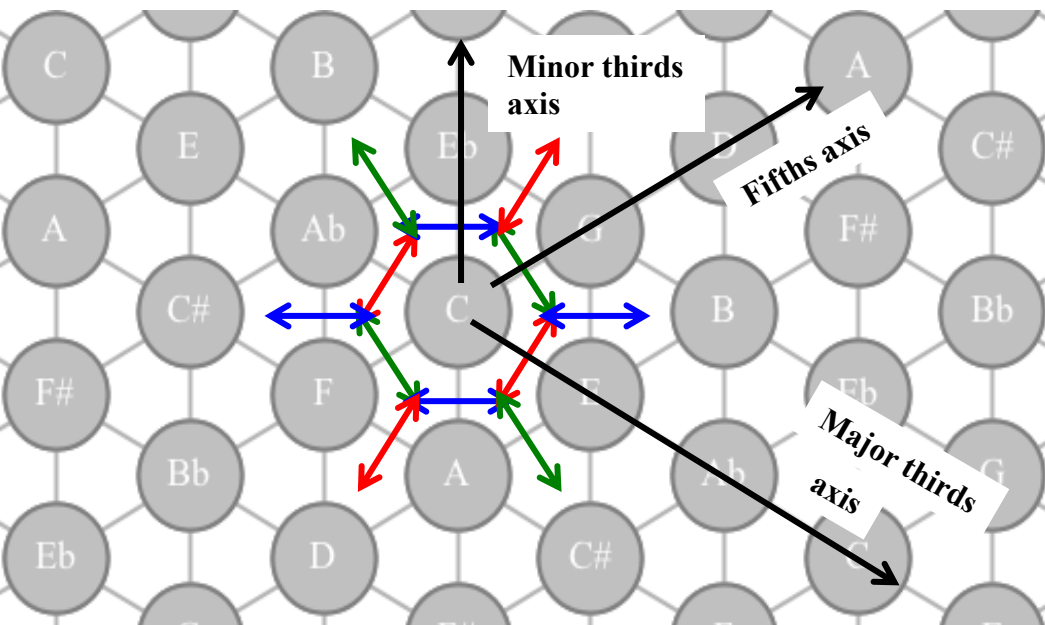
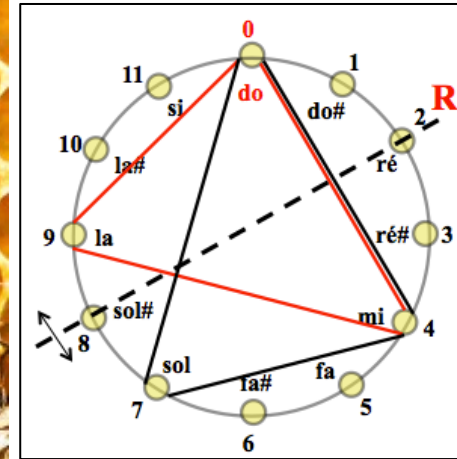
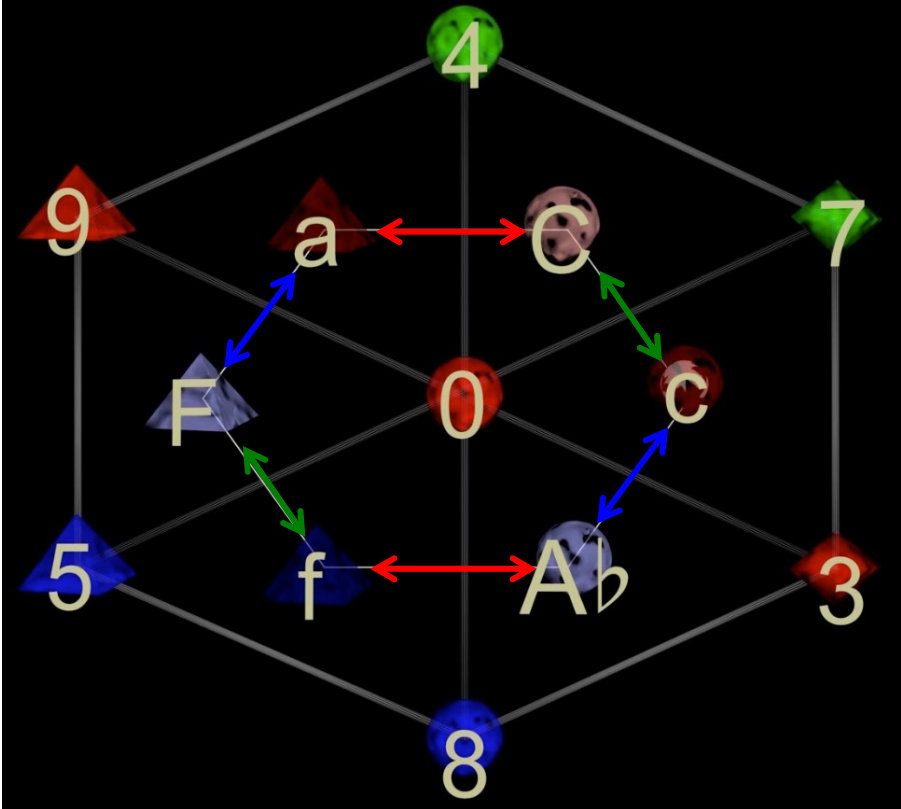


Leonhard Euler

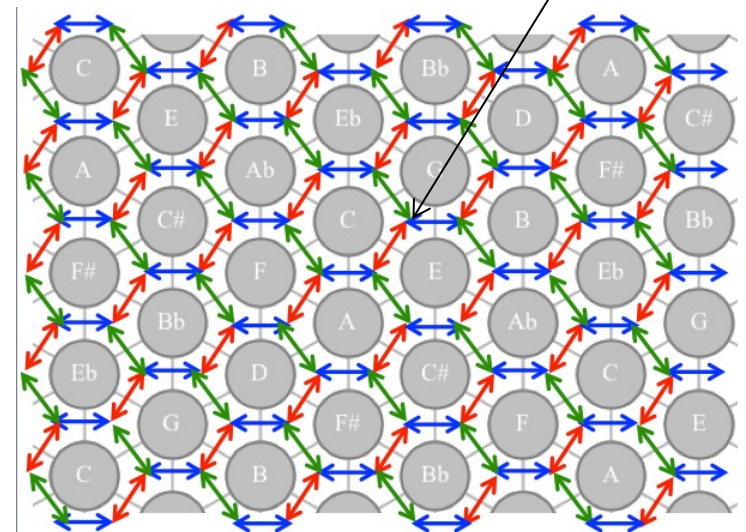
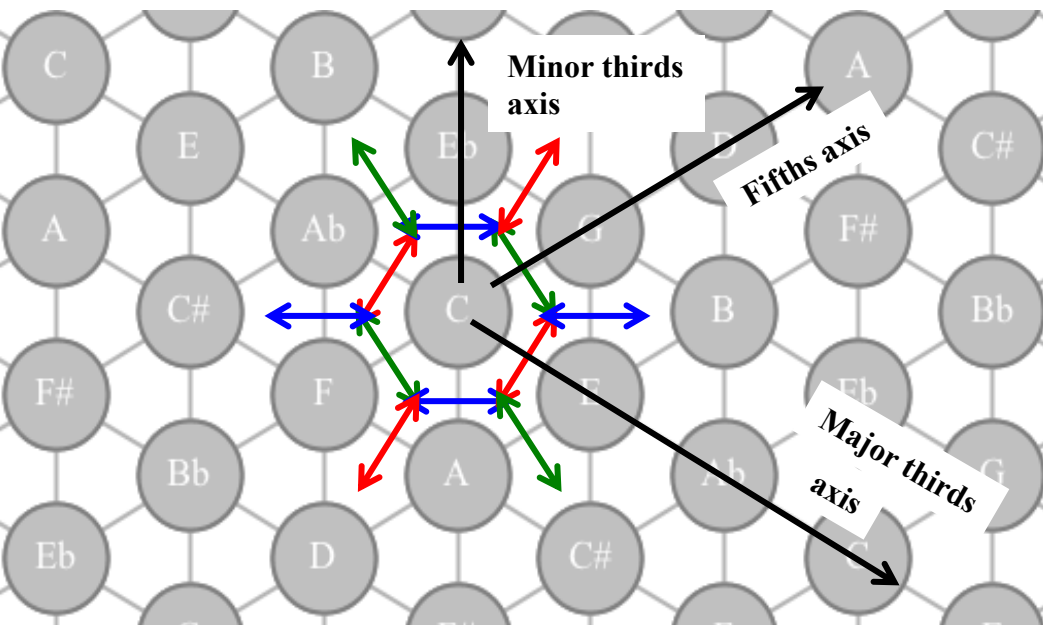
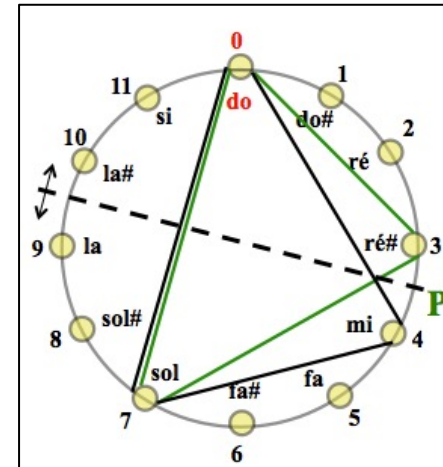
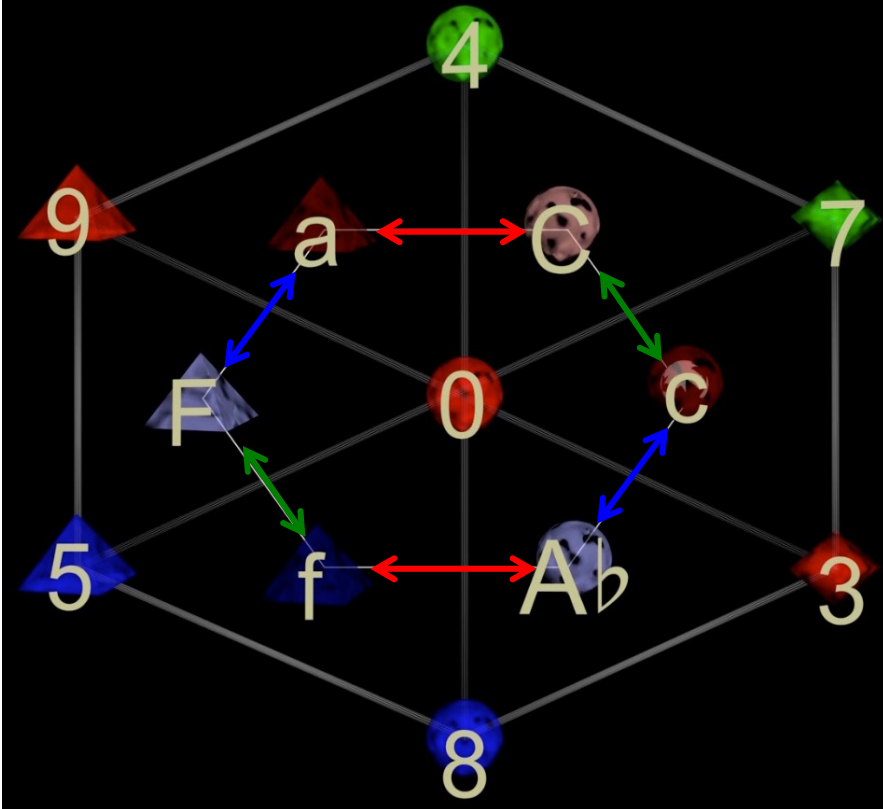


Speculum Musicum (1773)

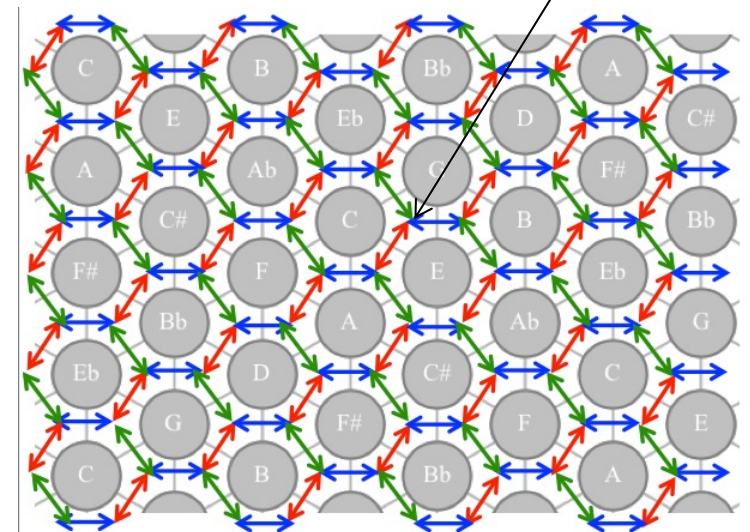
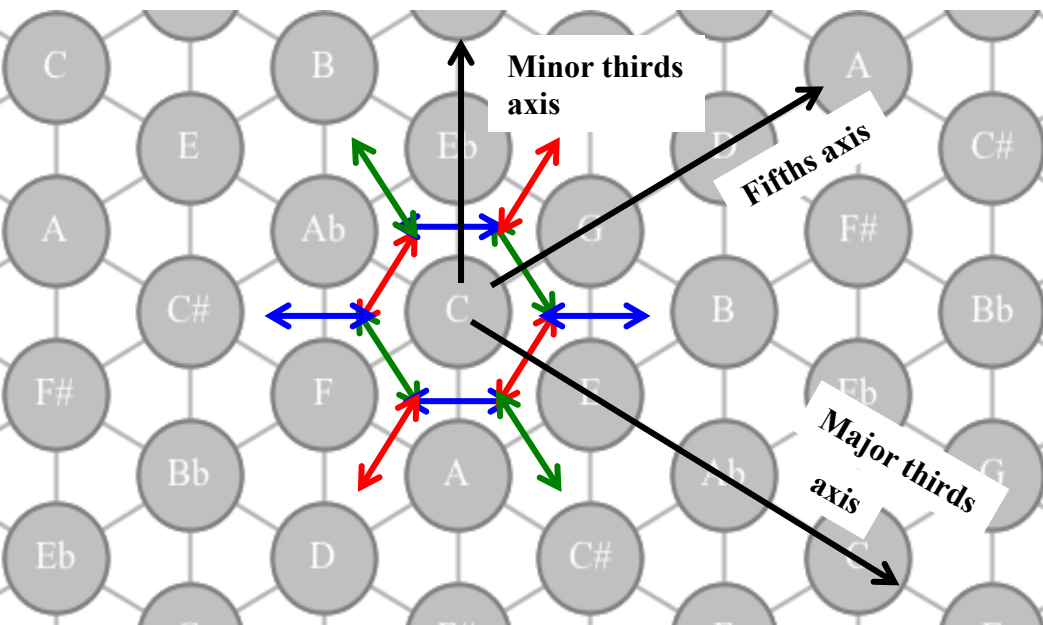
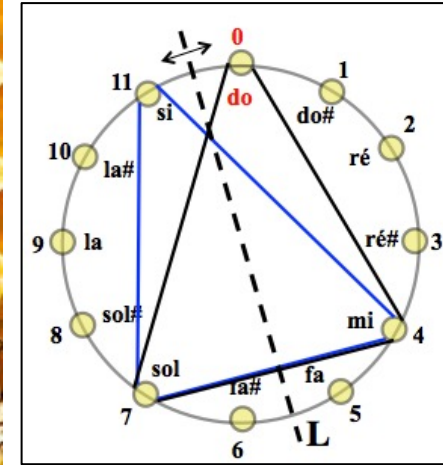
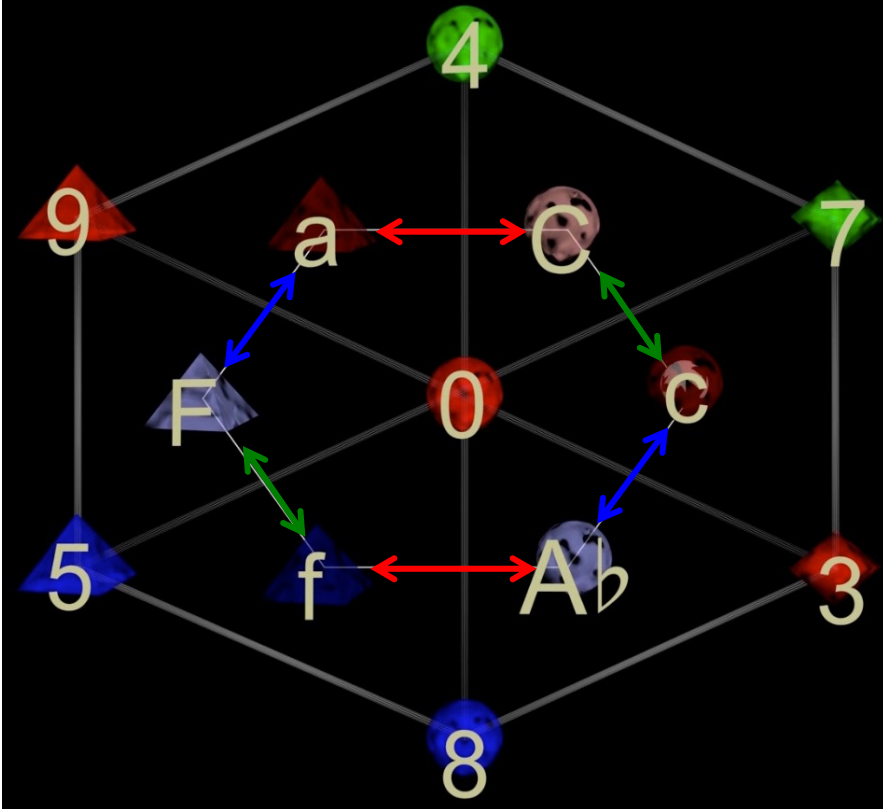
The Tonnetz (or 'honeycomb' hexagonal tiling)



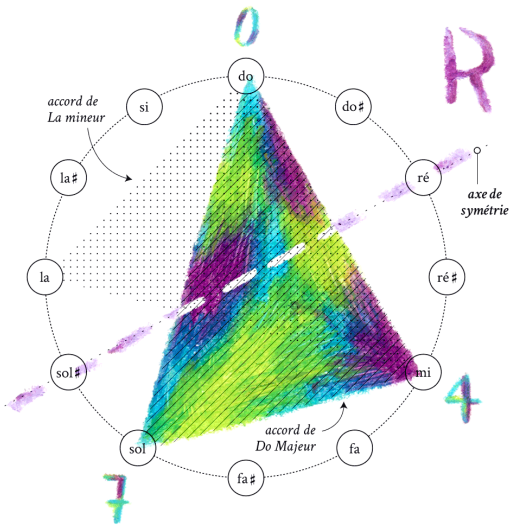
The Tonnetz (or 'honeycomb' hexagonal tiling)



The Tonnetz (or 'honeycomb' hexagonal tiling)

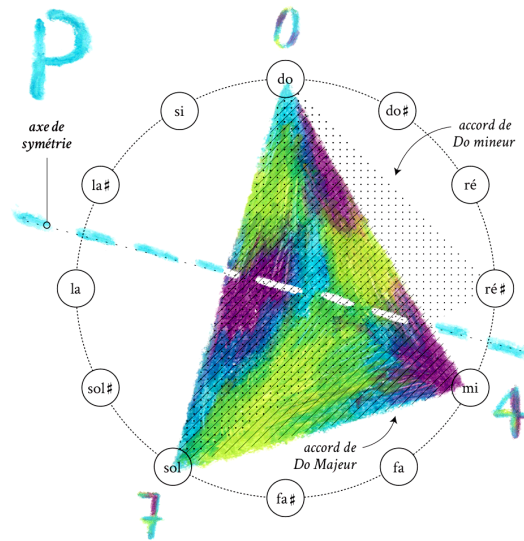


The three main major-minor symmetries



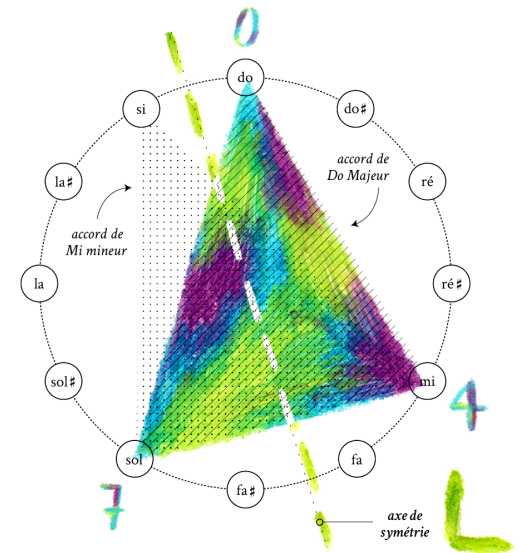
R as RELATIVE

C major
 ↑↓
A minor



P as PARALLEL

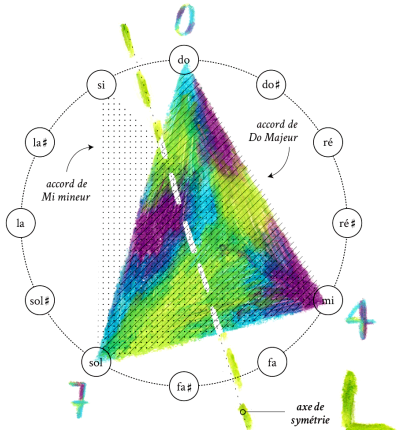
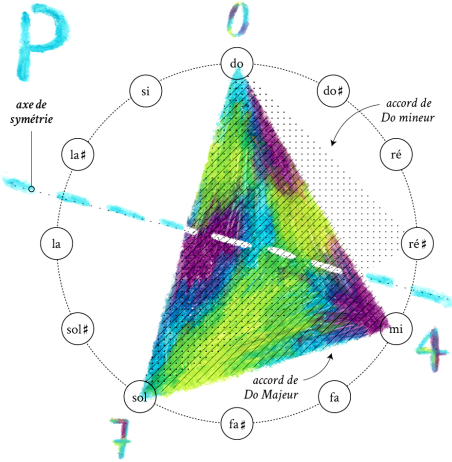
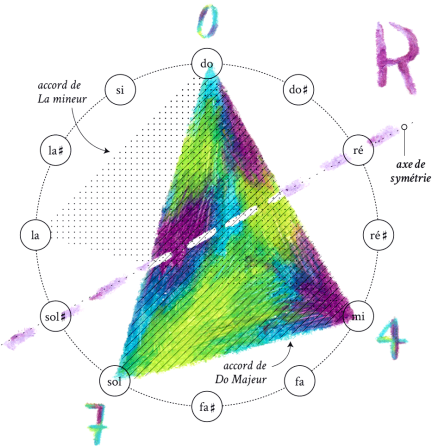
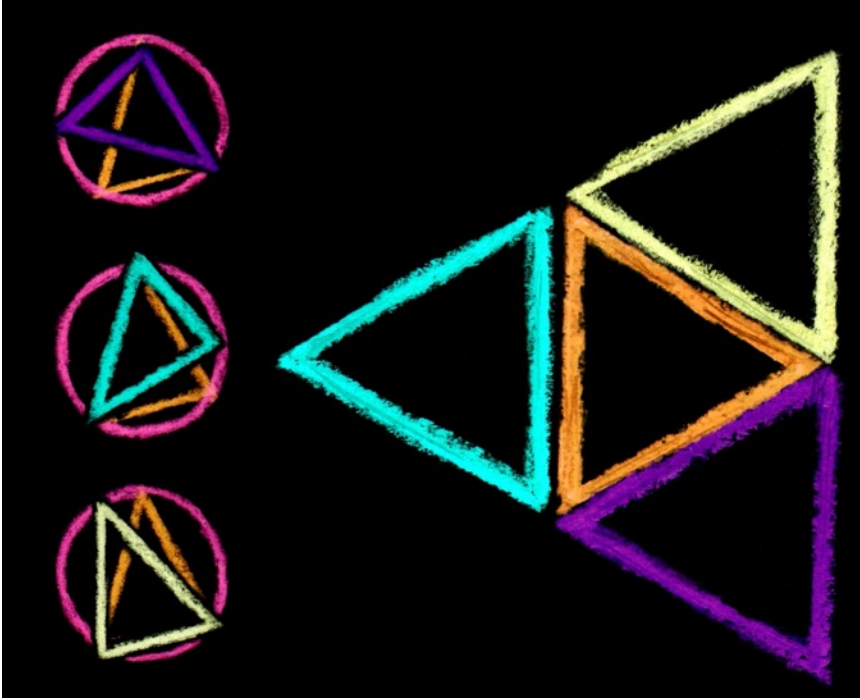
C major
 ↑↓
C minor



L as LEADING-TONE (EXCHANGE)

C major
 ↑↓
E minor

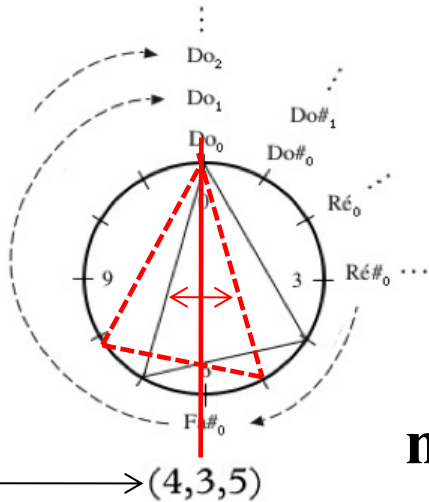
The Tonnetz and the three main major-minor symmetries



The *Tonnetz* as a simplicial complex

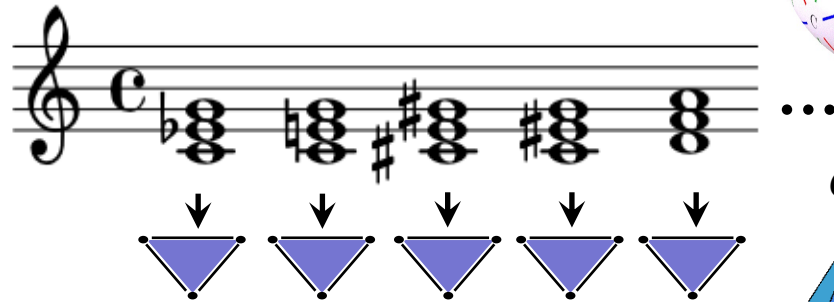
L. Bigo, *Représentation symboliques musicales et calcul spatial*, PhD, Ircam / LACL, 2013

- Assembling chords related by some equivalence relation
 - Equivalence up to transposition/inversion:

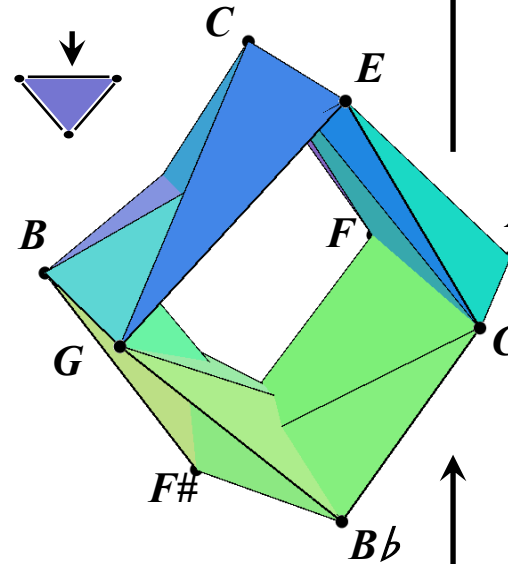
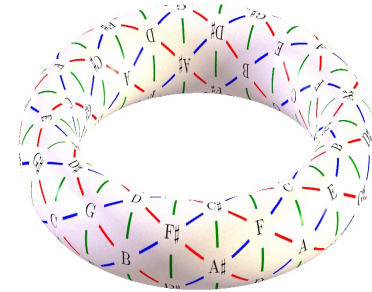


Intervallic structure

(4,3,5)

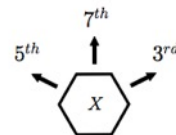
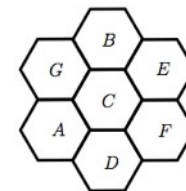
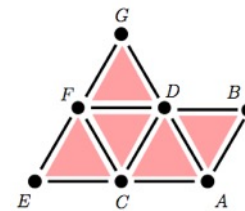
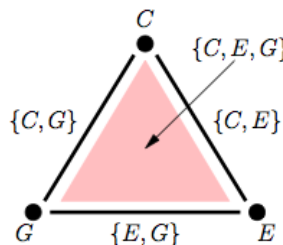


major/minor triads

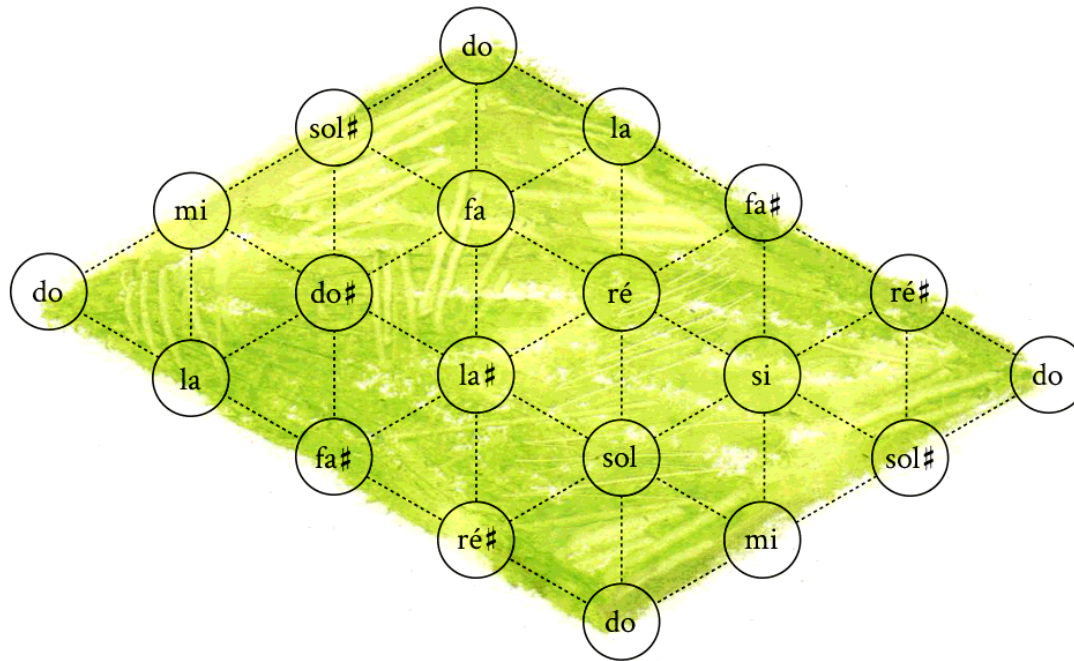


0-cell ● note
1-cell — 2-note chord

2-cell ▲ 3-note chord
3-cell ▽ 4-note chord



The topological structure of the *Tonnetz*



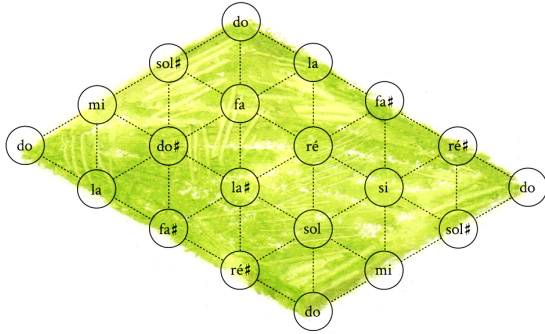
The topological structure of the *Tonnetz*



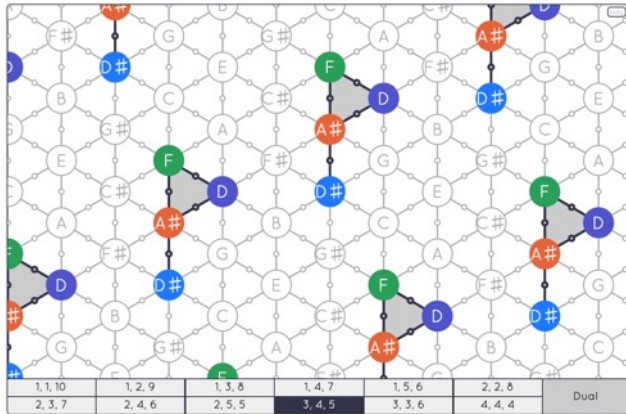
The topological structure of the *Tonnetz*



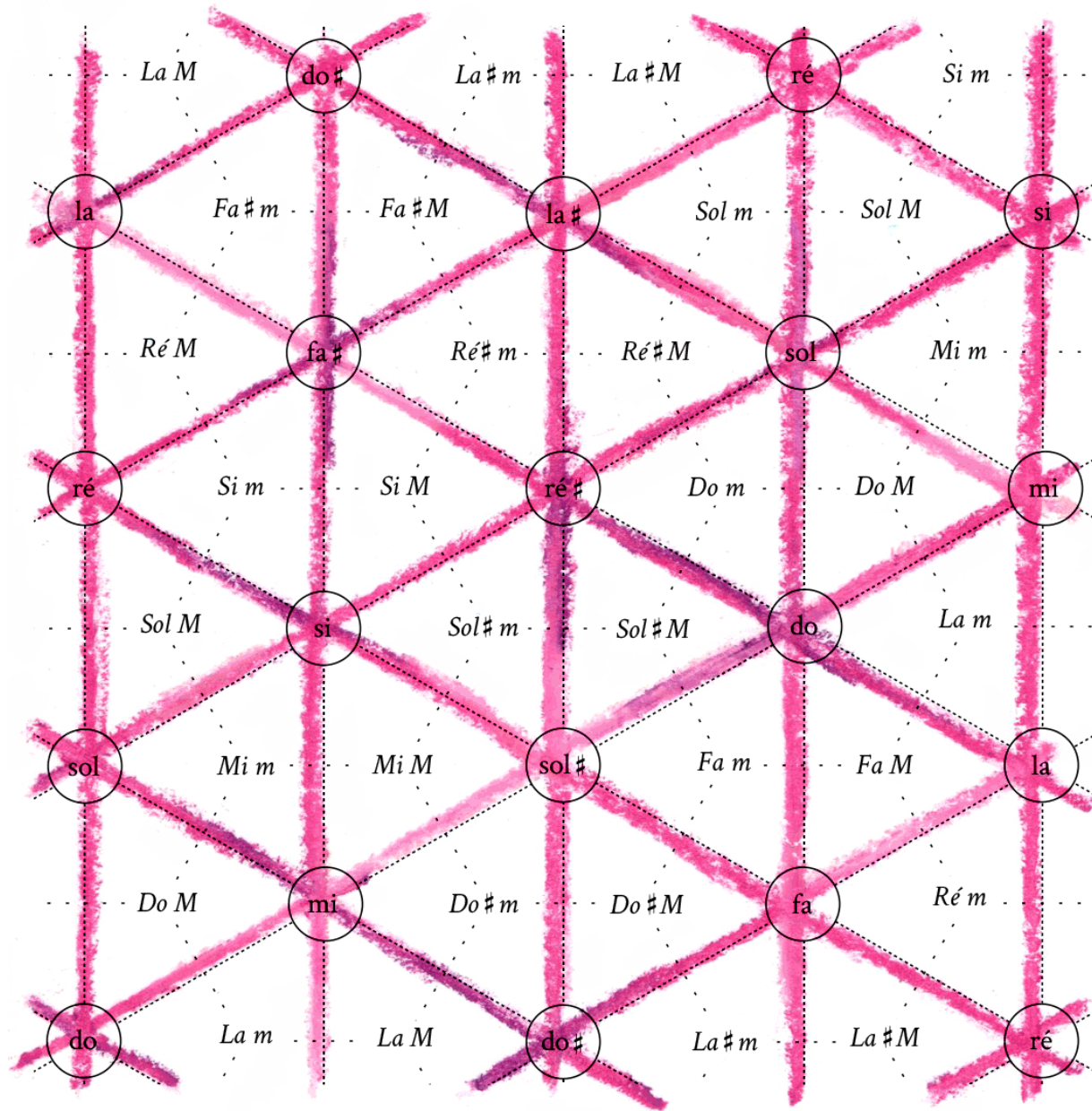
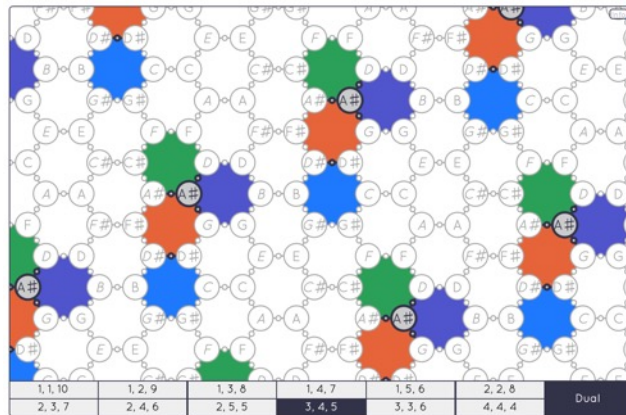
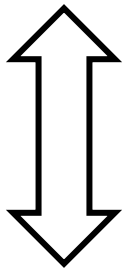
The topological structure of the *Tonnetz*



From the *Tonnetz* to the dual one



duality



A song based on Hamiltonian Cycles in the Tonnetz

41. C-Em-E-Abm-Ab-Cm-Eb-Ebm-B-Bm-G-Gm-Bb-Bbm-F#-F#m-D-Dm-F-Fm-C#-C#m-A-Am--
LPLPLRPLPRPLPLPRPLPLPR
42. C-Em-E-Abm-Ab-Cm-Eb-Gm-G-Bm-B-Ebm-F#-Bbm-Bb-Dm-D-F#m-A-C#m-C#-Fm-F-Am--
LPLPLR
43. C-Em-E-Abm-Ab-Fm-F-Am-A-C#m-C#-Bbm-Bb-Dm-D-F#m-F#-Ebm-B-Bm-G-Gm-Eb-Cm--
LPLPRPLPLPRPLPLRPLPLRP



L P L P L R ...
 P L P L R L ...
 L P L R L P ...
 P L R L P L ...

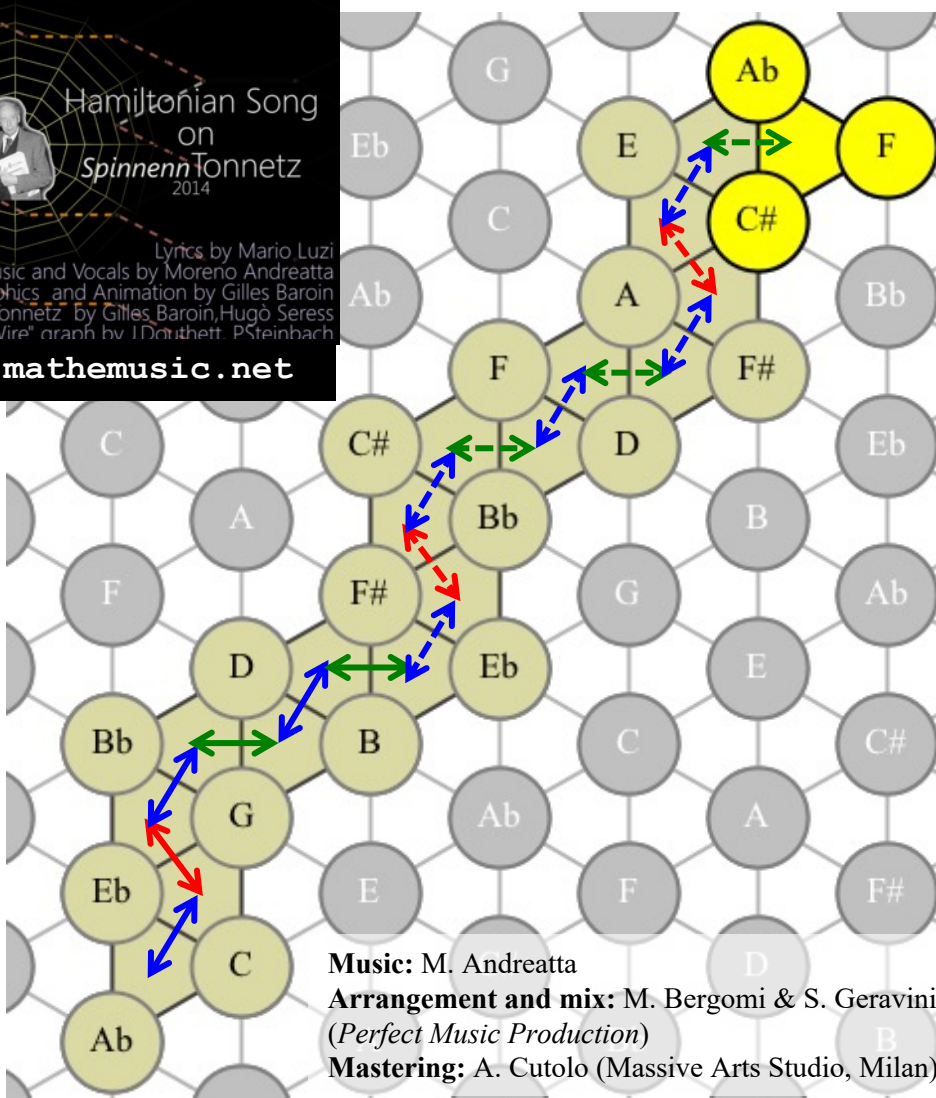
L R L P L P ...



R L P L P L ...



min. 1'02"



Music: M. Andreatta
 Arrangement and mix: M. Bergomi & S. Geravini
 (Perfect Music Production)
 Mastering: A. Cutolo (Massive Arts Studio, Milan)

La sera non è più la tua canzone
 (Mario Luzi, 1945, in *Poesie sparse*)

La sera non è più la tua canzone,
 è questa roccia d'ombra traforata
 dai lumi e dalle voci senza fine,
 la quiete d'una cosa già pensata.

Ah questa luce viva e chiara viene
 solo da te, sei tu così vicina
 al vero d'una cosa conosciuta,
 per nome hai una parola ch'è passata
 nell'intimo del cuore e s'è perduta.

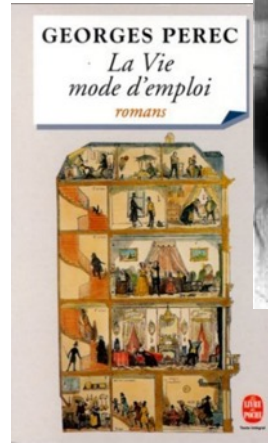
Caduto è più che un segno della vita,
 riposi, dal viaggio sei tornata
 dentro di te, sei scesa in questa pura
 sostanza così tua, così romita
 nel silenzio dell'essere, (compiuta).

L'aria tace ed il tempo dietro a te
 si leva come un'arida montagna
 dove vaga il tuo spirito e si perde,
 un vento raro scivola e ristagna.

Rules and constraints in the artistic process



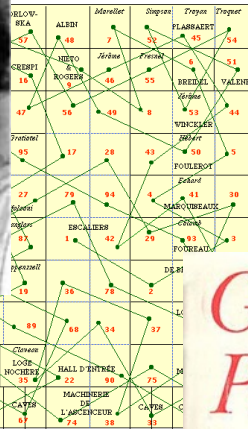
Cent mille milliards de poèmes, 1961



La vie mode d'emploi,

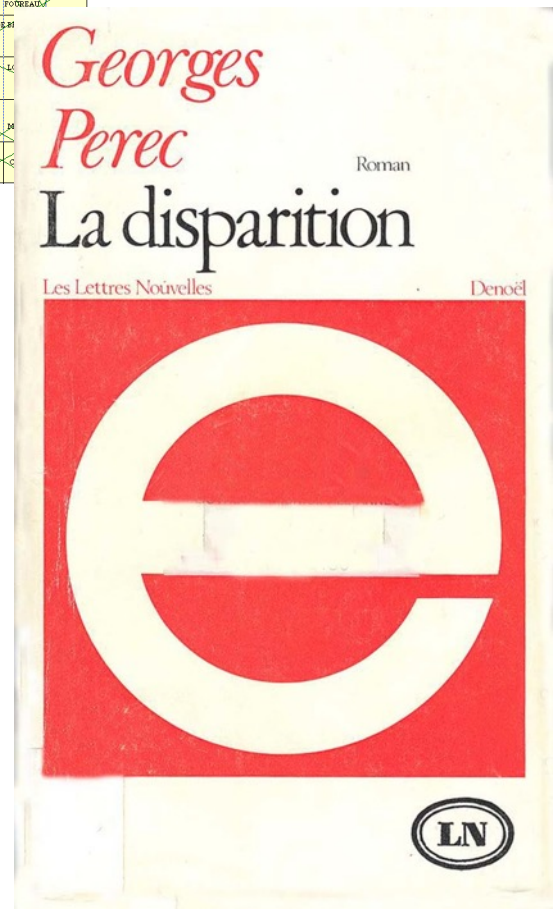


Georges Perec



OuLiPo

OuLiPo (Ouvroir de Littérature Potentielle)



Raymond Queneau



Italo Calvino

Il castello dei destini incrociati, 1969

The circular representation for rhythmic structures

CLAPPING MUSIC

FOR TWO PERFORMERS

$\text{♩} = 144 - 168$

CLAP 1
CLAP 2

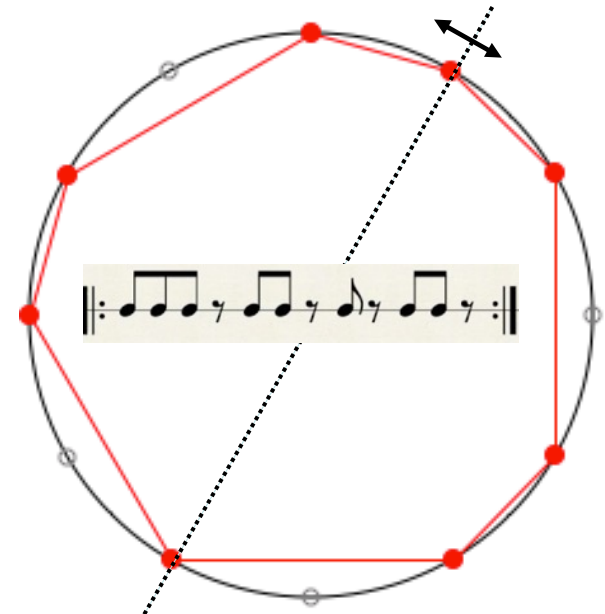
Repeat bar ①, then end.

The performance begins and ends with both performers in unison at bar ①. The number of repeats of each bar should be fixed at twelve repeats per bar. Since the first performer's part does not change, it is up to the second performer to move from one bar to the next. The second performer should try to keep his or her downbeat where it is written, i.e. on the first beat of each measure (not on the first beat of the group of three claps), so that his downbeat always falls on a new beat of his or her changing pattern.

The choice of a particular clapping sound, i.e. with cupped or flat hands is left up to the performers. whichever timbre is chosen, both performers should try to get the same one so that their two parts will blend to produce one overall resulting pattern.

Clapping Music de Steve Reich (1972)

New York 12/72
re-copied 1/78



THE RHYTHM CIRCLE

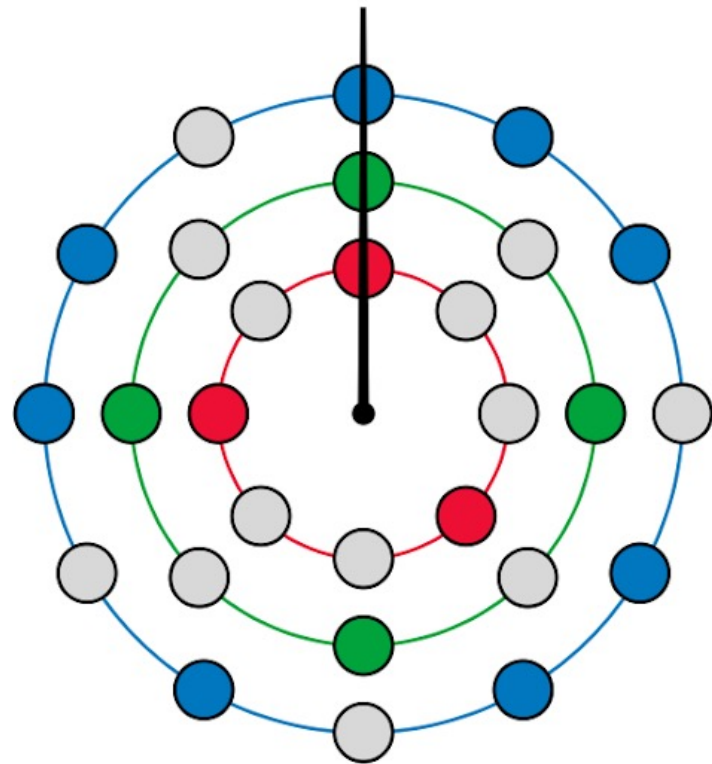
Play

Tempo (mpm): 30

Reset rhythms




→ www.youtube.com/@mathemusique



Tresillo	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
Son	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
Shiko	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
Soukous	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
Rumba	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
Bossa Nova	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
Gahu	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
Samba	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
Fume-fume	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
Bembé	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
Steve Reich	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>
Basic 1	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
Basic 2	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
Basic 3	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
Basic 4	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>

Export

Snare Drum 


Subdivisions:

Rotation:

Kick Drum 

Subdivisions:

Rotation:

Hi Hat 

Subdivisions:

Rotation:

→ <https://rhythm-circle.com>



Paul Lascabettes



MATHÉMUSIQUE

<https://www.mathemusique.fr/>

THE TONNETZ
ONE KEY - MANY REPRESENTATIONS

The screenshot shows the 'THE TONNETZ' interface. It features a large grid of notes (A, B, C, D, E, F, G, Ab, Bb, Cb, Db, Eb, Fb, Gb) with a path highlighted in various colors. To the right are two triangular diagrams representing the same tonal relationships. Below the grid is a piano keyboard. At the bottom, there are controls: 'Load Mid File', 'Play', 'Start Recording', 'Rotate 180°', and 'Transpose'. A table at the bottom left shows numerical sequences: 1,1,10; 2,3,7; 1,2,9; 2,4,6; 1,3,8; 2,5,5; 1,4,7; 3,4,5; 1,5,6; 3,3,6; 2,2,8; 4,4,4; and 'Dual'.

THE TONNETZ

(C. Guichaoua & M. Andreatta)

➔ <https://thetonnnetz.com>

THE RHYTHM CIRCLE

Play
Tempo (mpm): 30
Reset rhythms

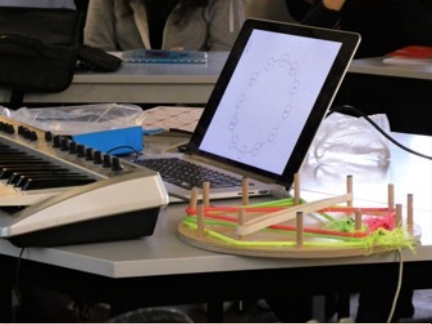
The screenshot shows the 'THE RHYTHM CIRCLE' interface. It features a circular diagram with colored dots (blue, green, red, grey) arranged in concentric circles. To the right is a list of styles with checkboxes: Tresillo, Son, Shiko, Soukous, Rumba, Bossa Nova, Gahu, Samba, Fume-fume, Bemba, Steve Reich, Basic 1, Basic 2, Basic 3, and Basic 4. Below the diagram are controls for 'Snare Drum', 'Kick Drum', and 'Hi Hat', each with 'Subdivisions' and 'Rotation' settings. An 'Export' button is at the bottom right.

THE RHYTHM CIRCLE

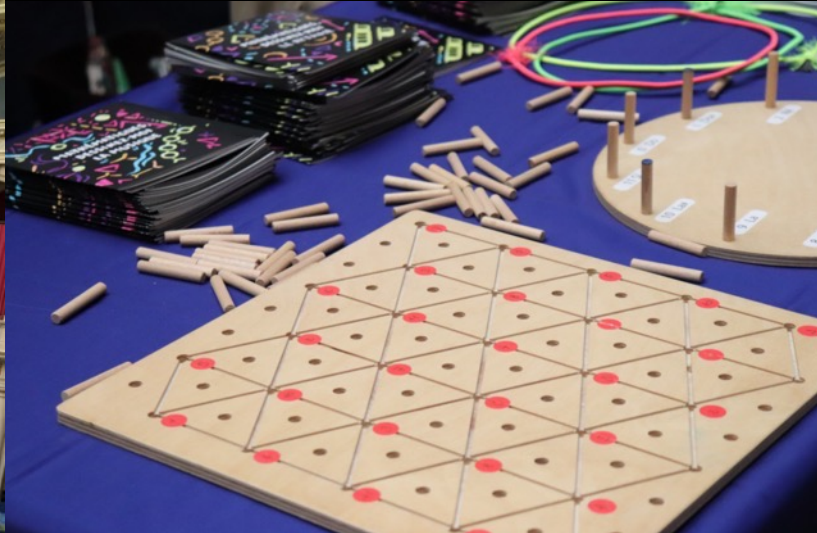
(P. Lascabettes & C. Guichaoua)

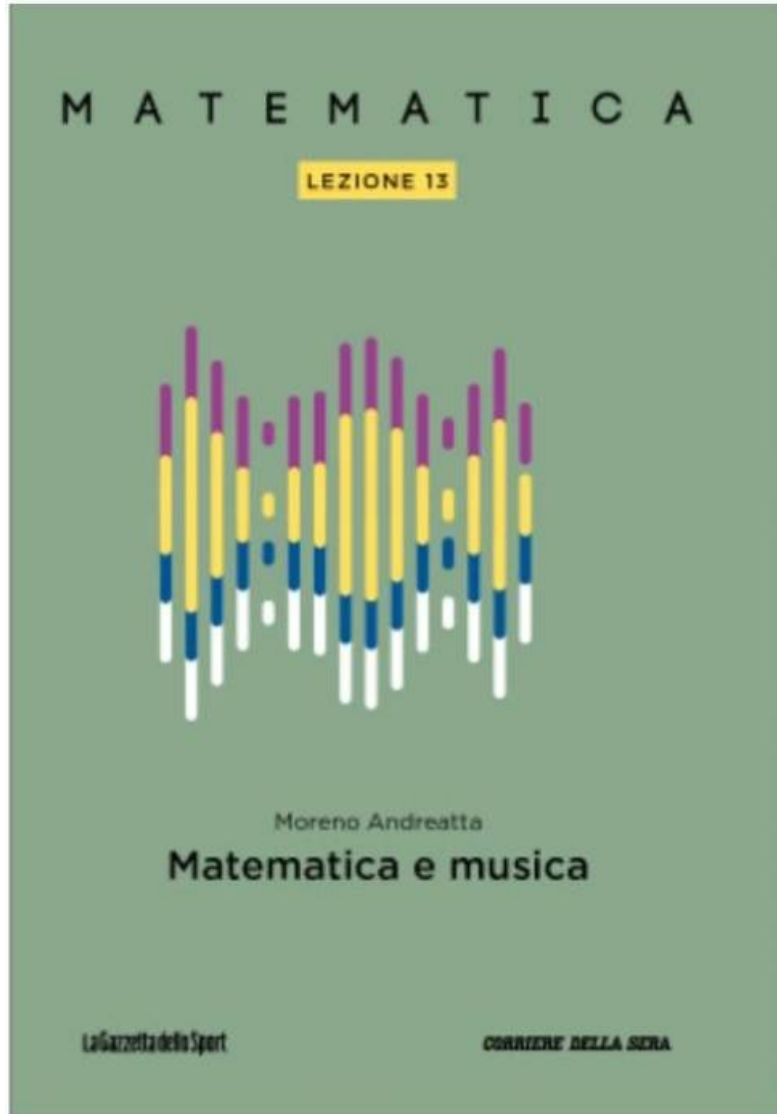
➔ <https://rhythm-circle.com>

Pedagogical school activities



Large public events





Sommario

- Introduzione
- **MATEMATICA E MUSICA**
 - Qualche parola per cominciare
 - Quando la musica precede la matematica...
 - Formalizzazione algebrica delle strutture musicali
 - Il Tonnetz e le sue generalizzazioni
 - Da Minkowski a Fuglede: canoni ritmici a mosaico e congetture aperte
- **I MAESTRI DELLA STORIA DELLA MATEMATICA**
 - Babbage e Lovelace
- **GIOCHI MATEMATICI**
 - Matemusica
 - Soluzioni
- **ESERCIZI**
 - Soluzioni

Thank you for your attention!

