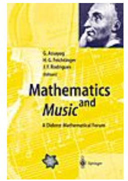
Un survol sur quelques modèles algébriques en théorie, analyse et composition assistées par ordinateur

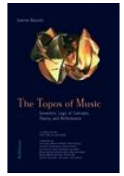
Moreno Andreatta
Equipe Représentations Musicales
IRCAM/CNRS UMR 9912

Mathématiques/Musique…une histoire récente!

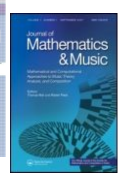
- 1999 : 4e Forum Diderot (Paris, Vienne, Lisbonne), *Mathematics and Music* (G. Assayag, H.G. Feichtinger, J.F. Rodrigues, Springer, 2001)
- 2000-2001 : Séminaire *MaMuPhi, Penser la musique avec les mathématiques ?* (Assayag, Mazzola, Nicolas éds., Coll. « M/S », Ircam/Delatour, 2006)
- 2000-2003 : International Seminar on *MaMuTh* (*Perspectives in Mathematical and Computational Music Theory* (Mazzola, Noll, Luis-Puebla eds, epOs, 2004)
- 2003 : *The Topos of Music* (G. Mazzola et al.)
- 2003: *Music and Mathematics. From Pythagoras to Fractals* (J. Fauvel et al.)
- 2001 - 2011 : Séminaire *MaMuX* de l'Ircam
- 2004 - 2011 : Séminaire *mamuphi* (Ens/Ircam)
- 2006 : *Mathematical Theory of Music* (F. Jedrzejewski), Coll. « M/S »
- 2007 : *La vérité du beau dans la musique* (G Mazzola), Coll. « Musique/Sciences »
- 2007 : Journal of Mathematics and Music (Taylor & Francis) et MCM 2007
- 2007: *Music. A Mathematical Offering* (Dave Benson), CUP
- 2008: *Music Theory and Mathematics* (Jack Douthett et al.), URP
- 2009 : *Computational Music Science Series* (Springer)
- 2009 : MCM 2009 (Yale) et Proceedings chez Springer
- 2010 : Mathematics Subject Classification : 00A65 Mathematics and music
- 2011 : Conférence de la SMCM (Ircam, 15-17 juin 2011)

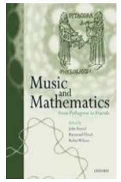


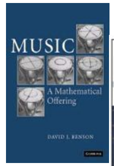


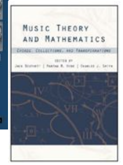


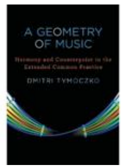






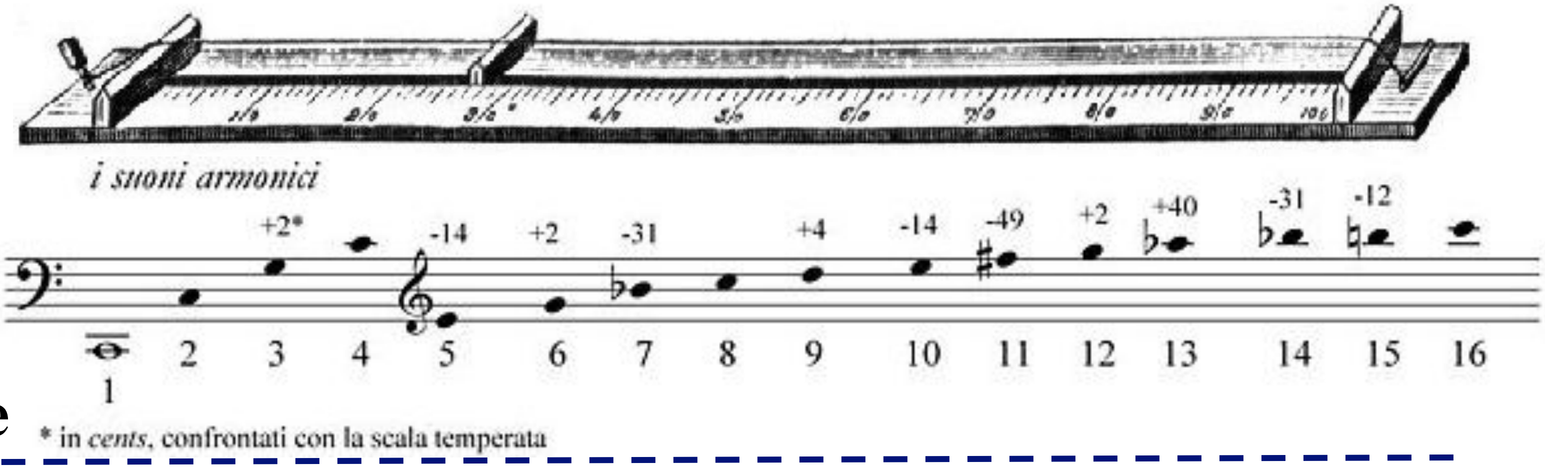




Société Mathématique de France

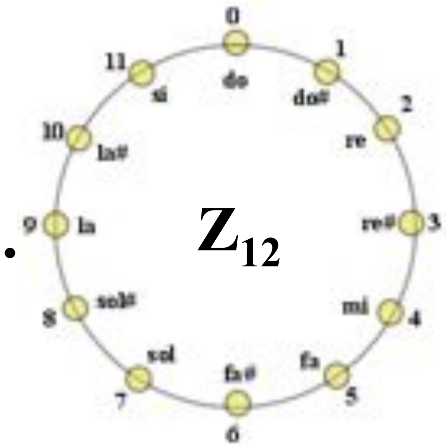
De Pythagore… à la théorie des groupes

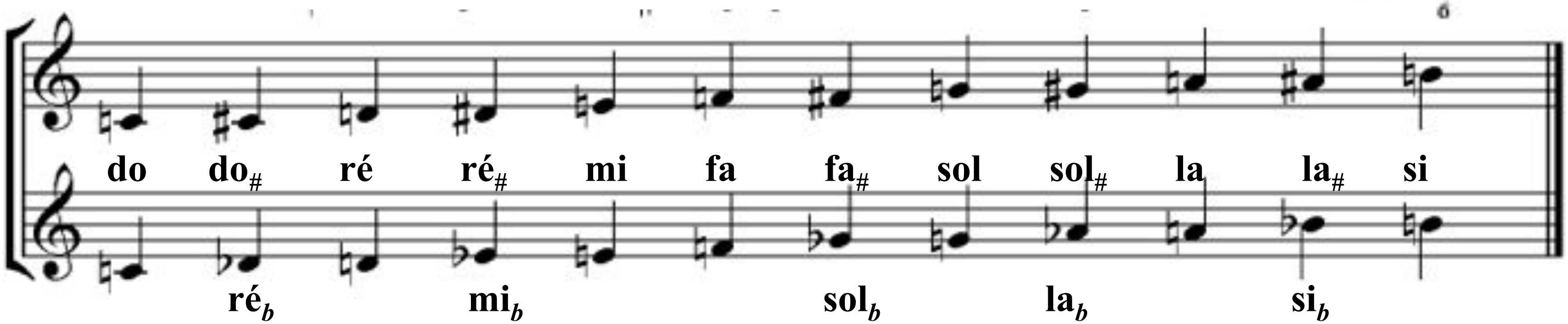


Physique

Mathématiques

…  …

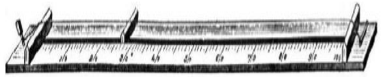




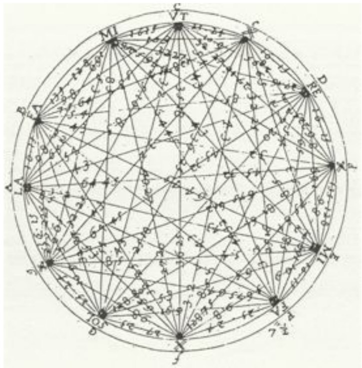
Musique et mathématiques : deux destinées parallèles

MUSIQUE	MATHS
500 av. J. C. Relation hauteur/longueur corde. La musique est source d'inspiration pour la théorie des nombres et la géométrie.	Nombres naturels et rationnels
300 a.J. Invention (théorique) de la gamme chromatique tempérée égale par Atistoxène de Tarente) et **prémonition de la théorie des groupes**. Isomorphismes entre les logarithmes (intervalles musicaux) et les exponentiels (longueur d'une corde)	*Aucune relation.*
1000 Invention de la représentation bidimensionnelle des hauteurs	*Aucune correspondance*
1500 *Aucune reprise des concepts précédents*	Nombres négatifs. Construction des rationnels
1600 *Aucune relation*	Nombres réels et les logarithmes
Marin Mersenne (1588-1648) : combinatoire musicale	Calcul des probabilités
1700 La fugue comme un automate abstrait. Manipulation inconsciente du groupe de Klein	Nombres complexes (Euler, Gauss), les quaternions (Hamilton), continuité (Cauchy), structure de groupe (Galois, Abel)
Leonhard Euler : *Speculum Musicum* (1773)	Théorie des graphes
1900 Libération de la prison de la tonalité (Loquin, Hauer, Schoenberg)	Nombres infinis et transfinis (Cantor). Axiomatique de Peano. Théorie de la mesure (Lebesgue, Borel)
1920 Formalisation radicale des macrostructures à travers le système sériel (Schoenberg)	*Aucun développement de la théorie des nombres.*
Ernst Krenek (1900-1991) : les axiomes dans le système dodécaphonique	*David Hilbert, Les fondements de la géométrie (1899)*

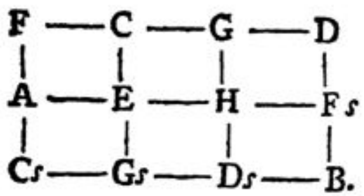
Iannis Xenakis, *Musique. Architecture*, Tournai, Casterman, 1971, (New, revised edition: Tournai, Casterman, 1976, 238 p.)



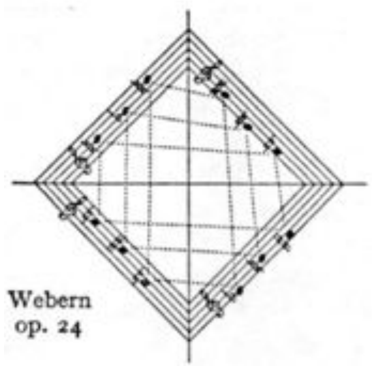
Pythagore et le monocorde, VI^{e}-V^{e} siècle av. J. C.

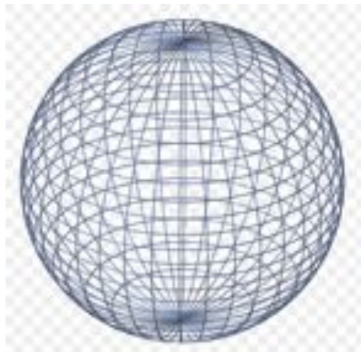


Mersenne, *Harmonicorum Libri* XII, 1648



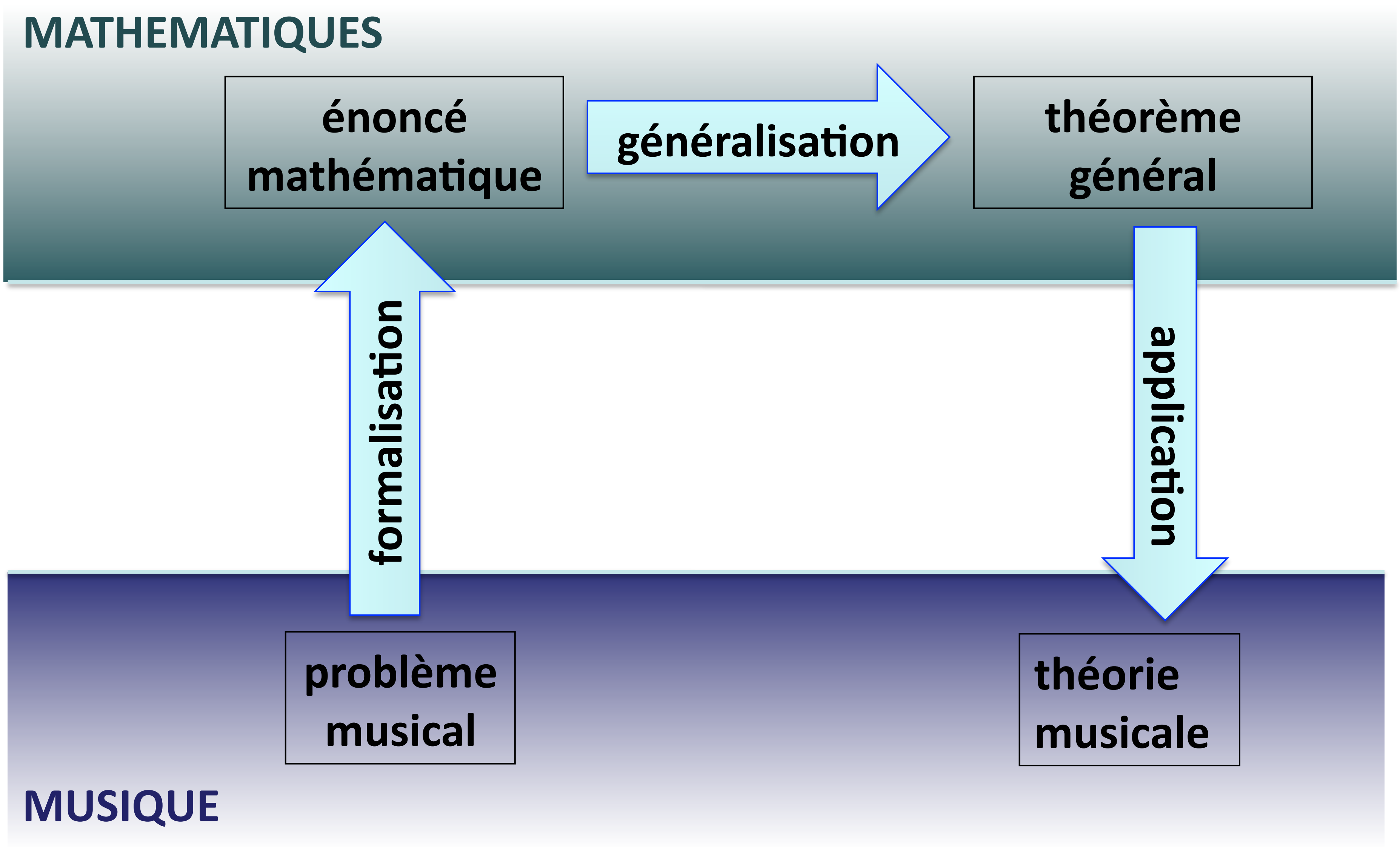
Euler : *Speculum musicum*, 1773





Double mouvement d'une dynamique mathémusicale

[*Cf.* M. Andreatta : *Mathematica est exercitium musicae*, HDR, octobre 2010]



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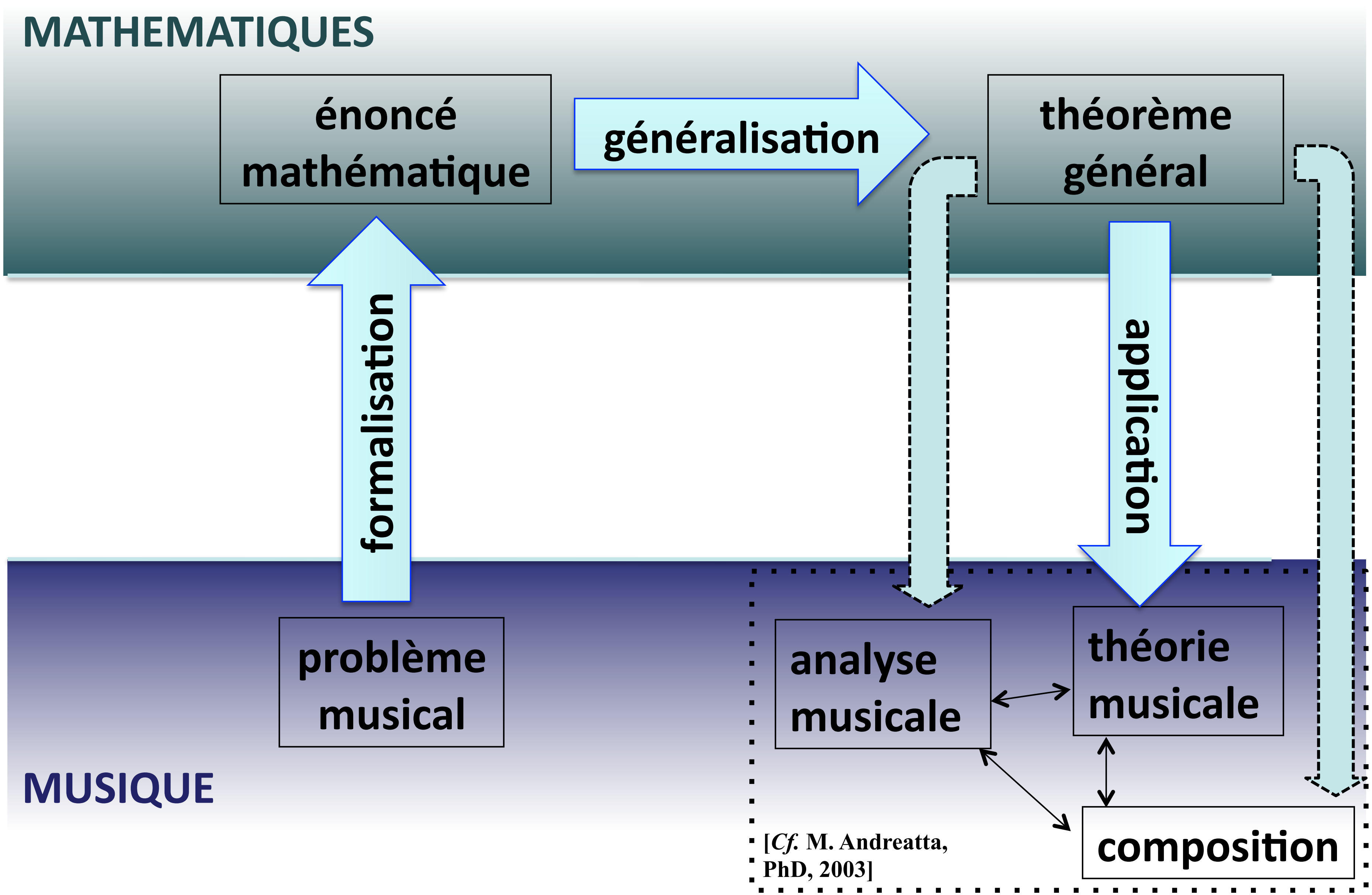

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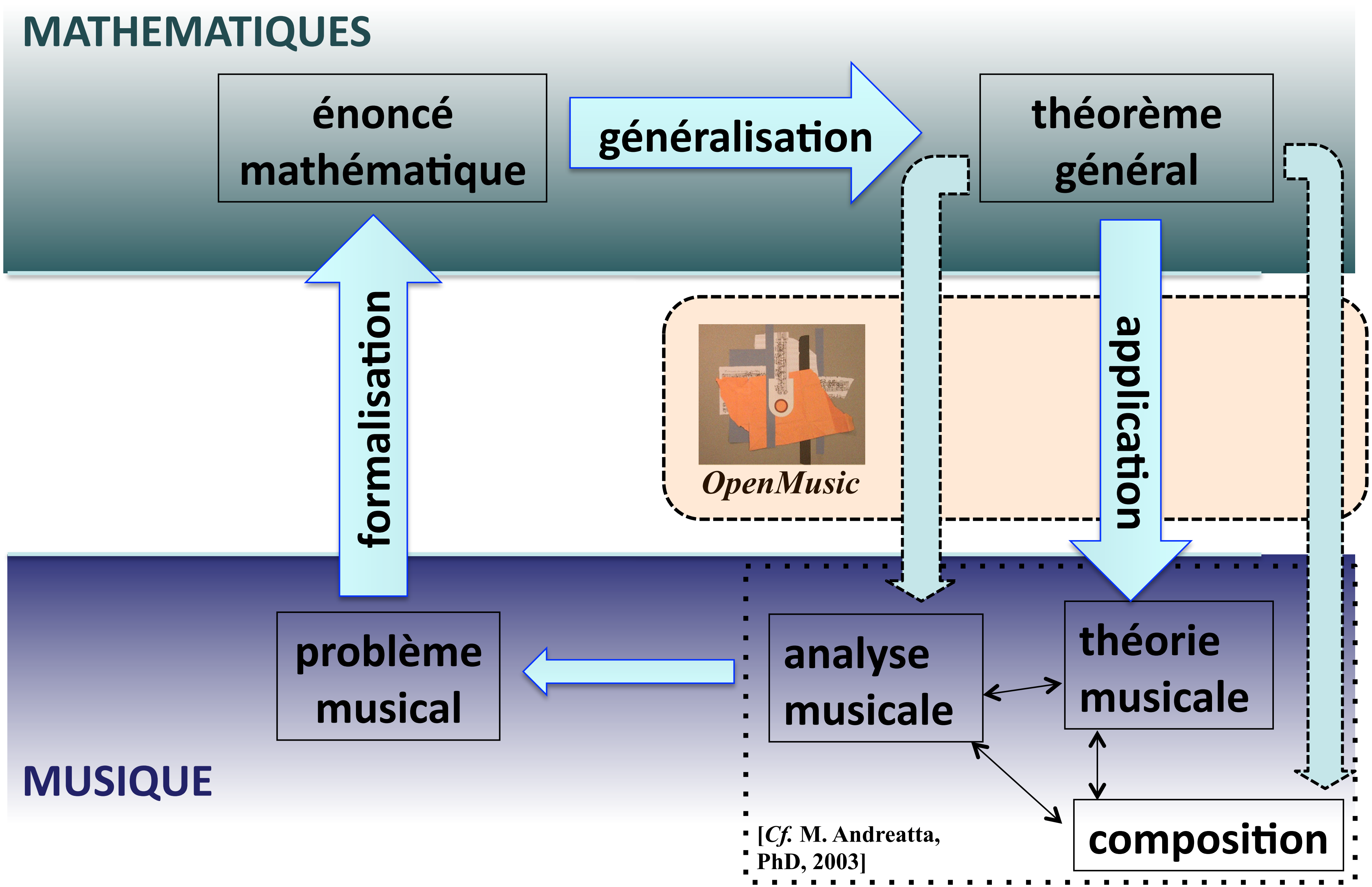

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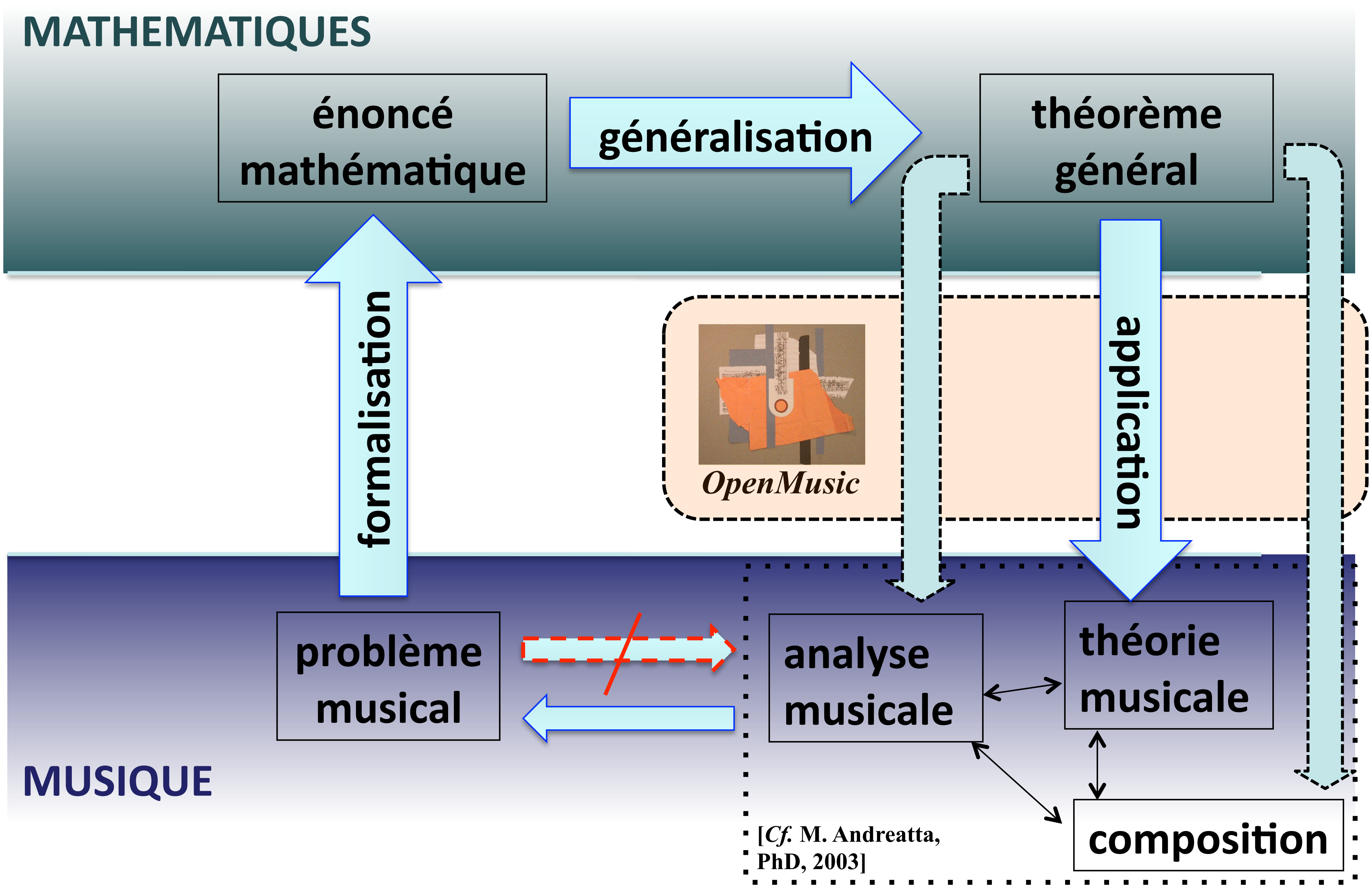
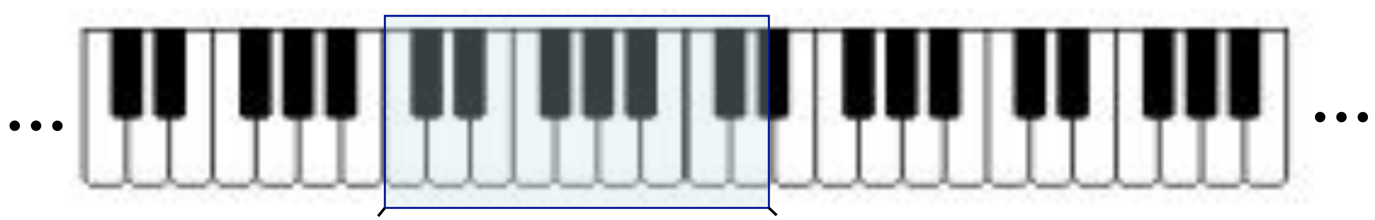
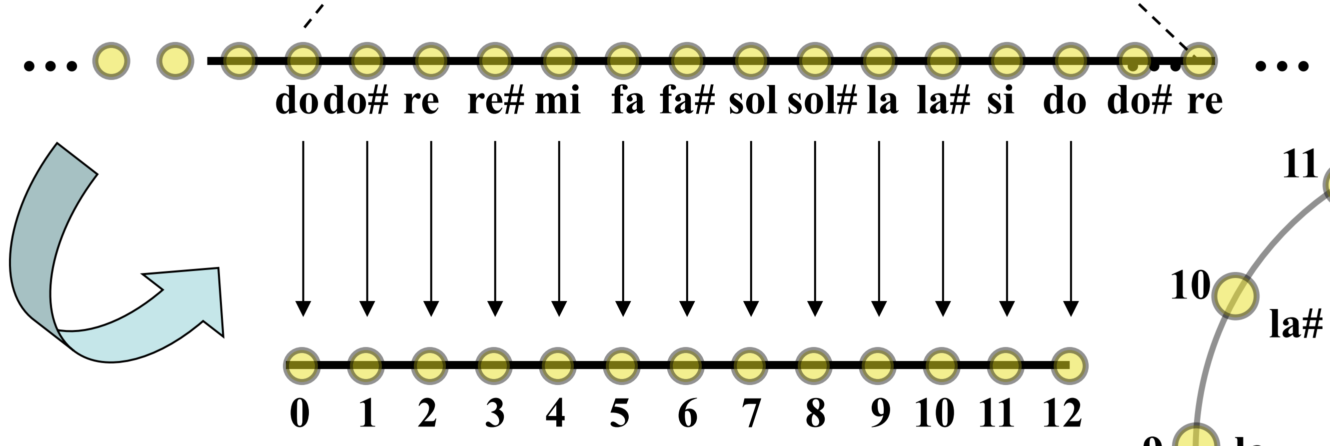
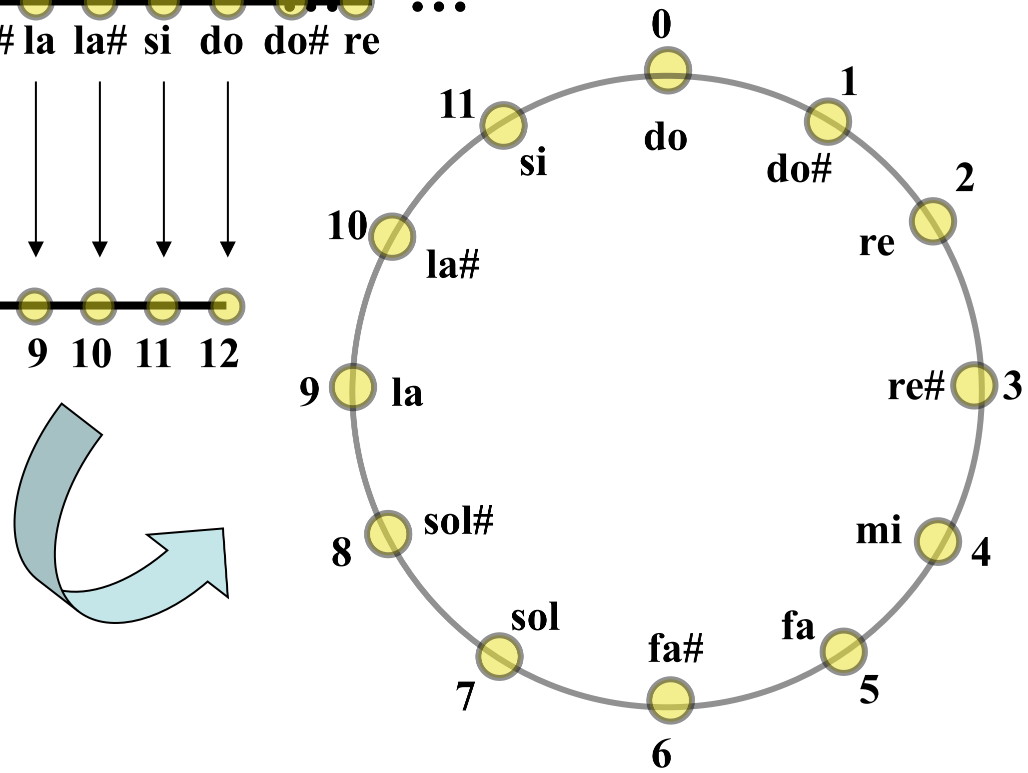


Réduction à l'octave et congruence modulo 12



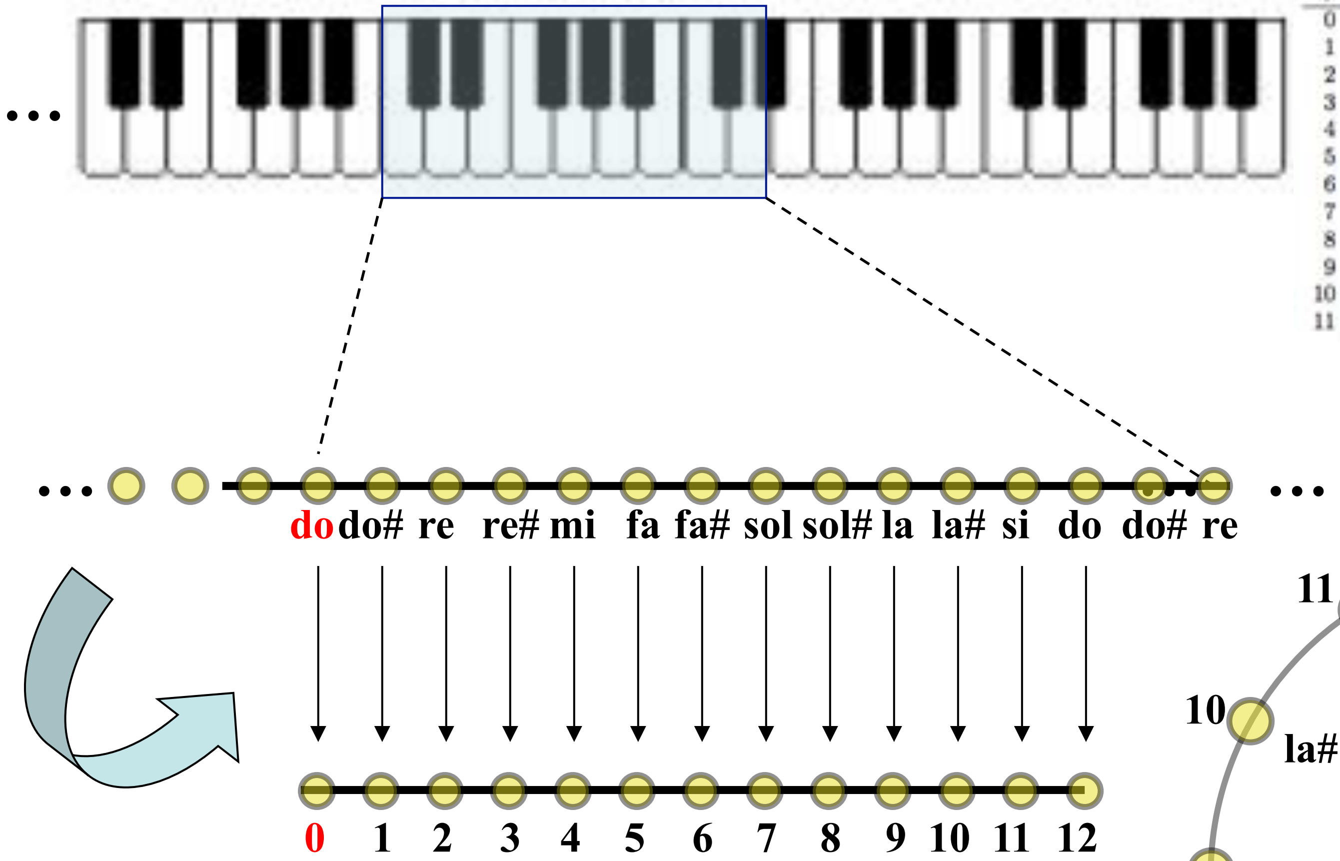
$$a \sim b \text{ mod } n \Leftrightarrow b\text{-}a = k \cdot n$$
$$a \sim b \text{ mod } 12 \Leftrightarrow b\text{-}a = k \cdot 12$$



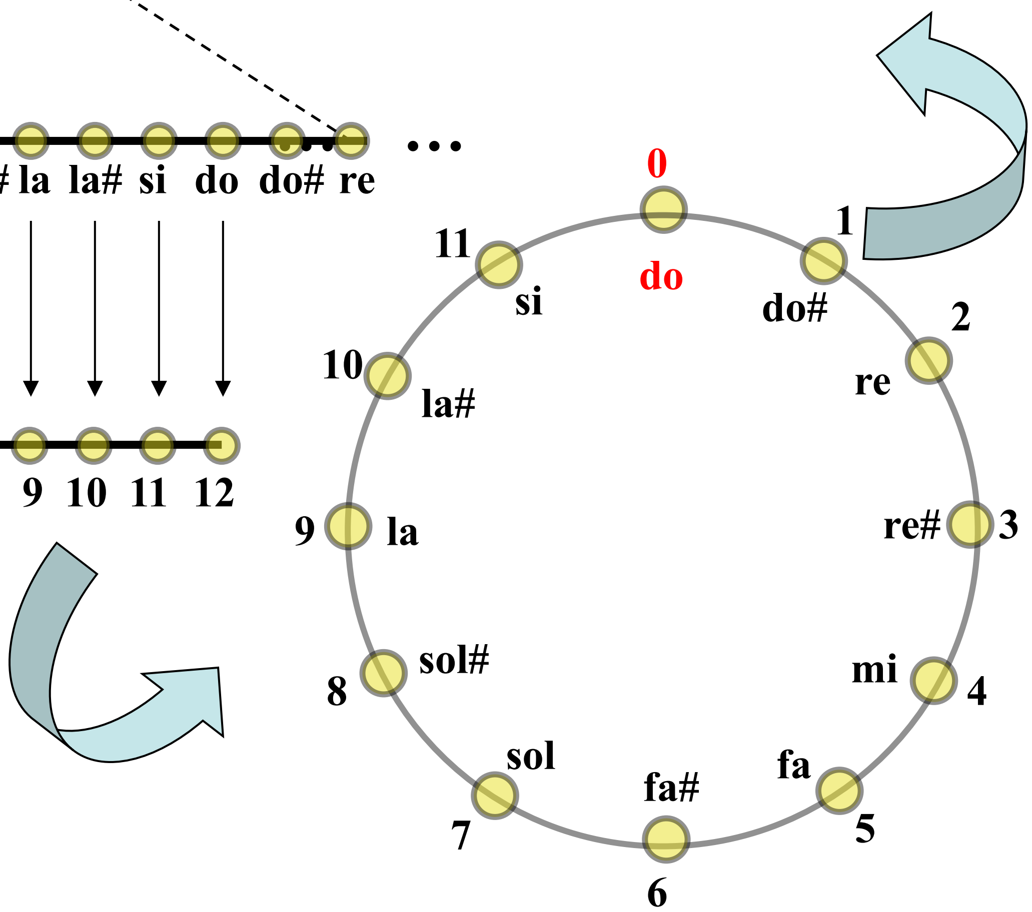


Relation d'*équivalence* :
- **Réflexive**
- **Symétrique**
- **Transitive**

Congruence modulo 12 et structure de groupe (cyclique)

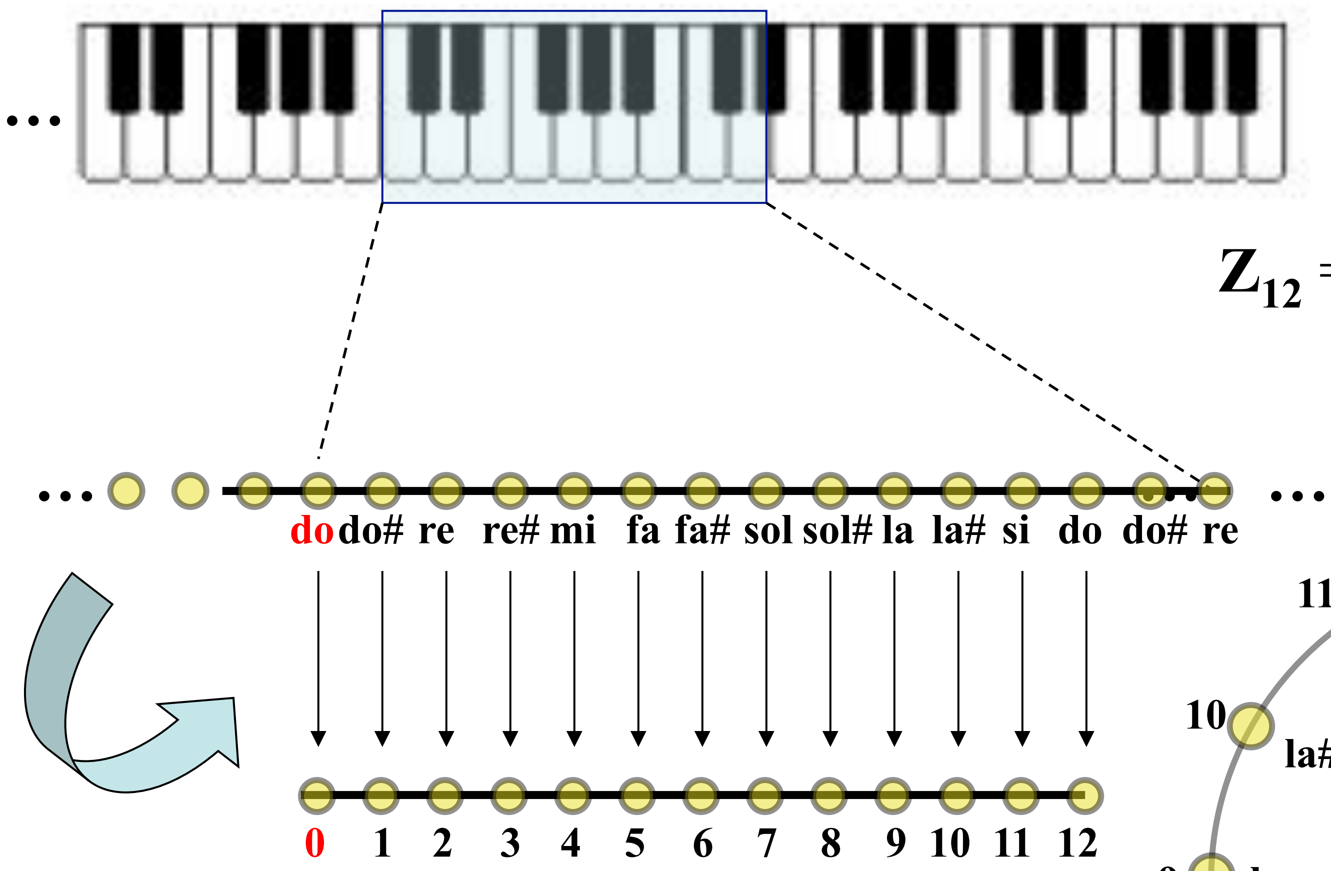


+	0	1	2	3	4	5	6	7	8	9	10	11
0	0	1	2	3	4	5	6	7	8	9	10	11
1	1	2	3	4	5	6	7	8	9	10	11	0
2	2	3	4	5	6	7	8	9	10	11	0	1
3	3	4	5	6	7	8	9	10	11	0	1	2
4	4	5	6	7	8	9	10	11	0	1	2	3
5	5	6	7	8	9	10	11	0	1	2	3	4
6	6	7	8	9	10	11	0	1	2	3	4	5
7	7	8	9	10	11	0	1	2	3	4	5	6
8	8	9	10	11	0	1	2	3	4	5	6	7
9	9	10	11	0	1	2	3	4	5	6	7	8
10	10	11	0	1	2	3	4	5	6	7	8	9
11	11	0	1	2	3	4	5	6	7	8	9	10



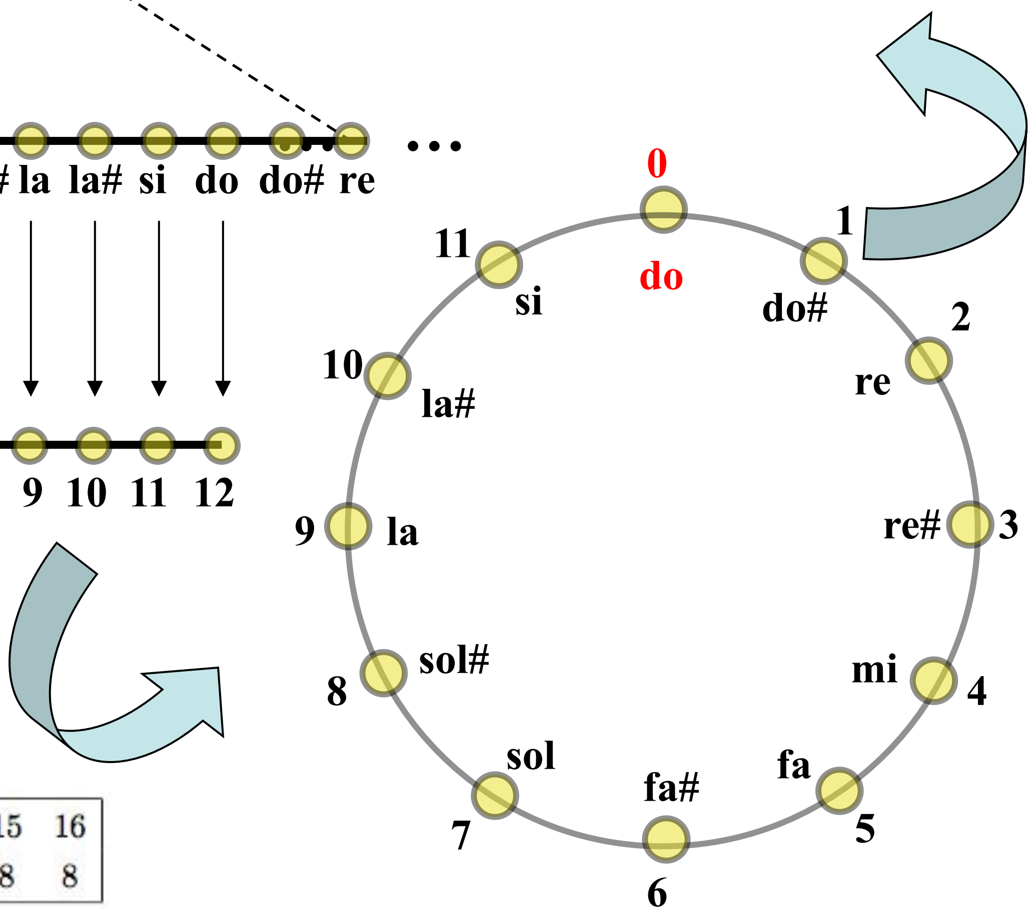
Structure de *groupe*
- **Clôture**
- **Existence de l'élément neutre**
- **Existence de l'inverse**
- **Associativité**

Ecriture par générateurs et relations



$$\mathbf{Z_{12}} = < a \mid a^{12} = 0 >$$

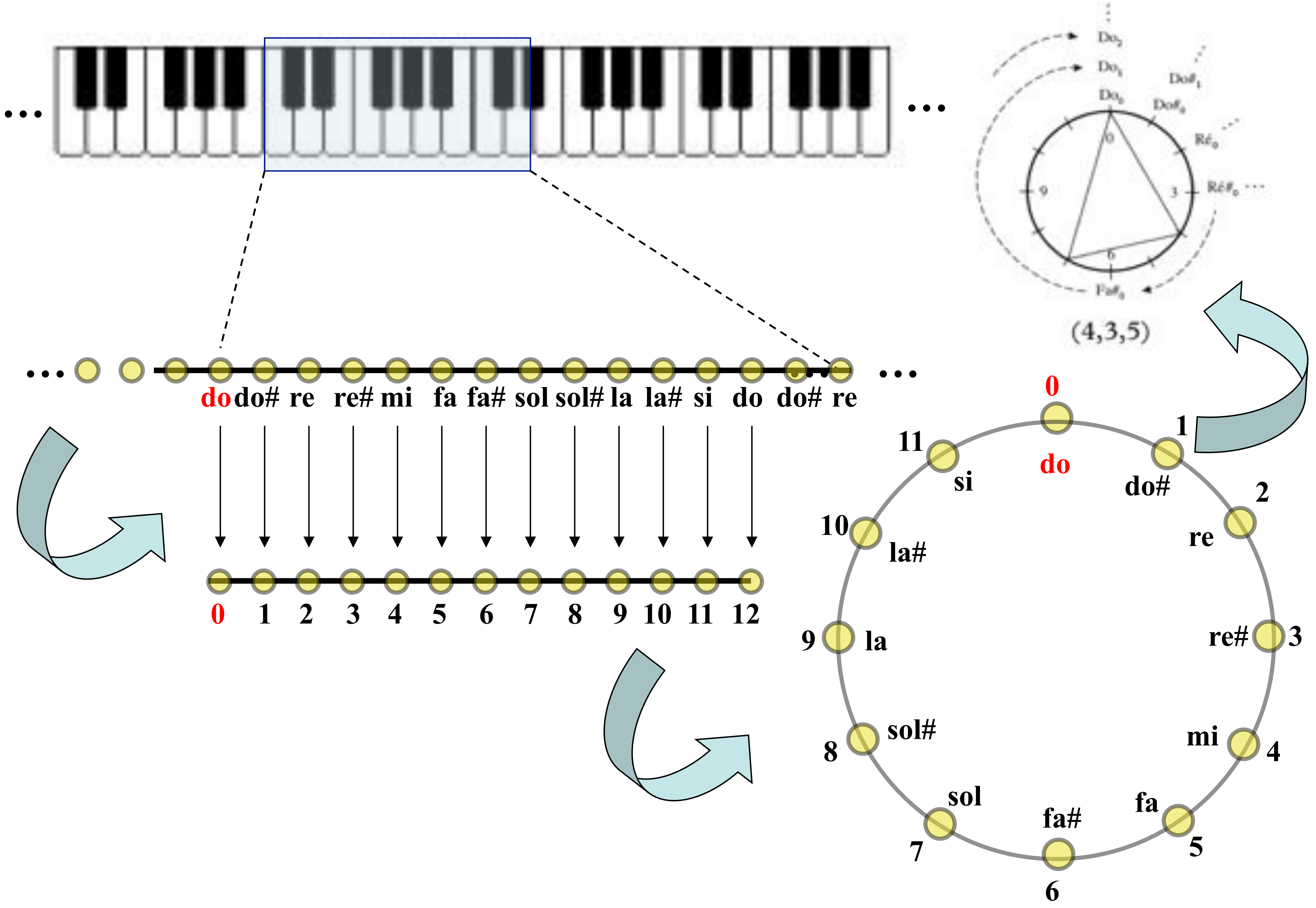
$$\mathbf{Z_{12}} = < \mathrm{T_k} \mid (\mathrm{T_k})^{12} = \mathrm{T_0} >$$



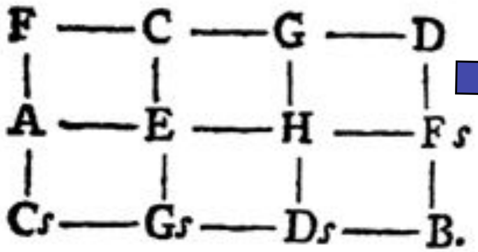
$$\begin{array}{rccl} \varphi & : & \mathbb{N}^* & \longrightarrow & \mathbb{N}^* \\ & & n & \longmapsto & \mathrm{card}(\{m \in \mathbb{N}^* \mid m \leq n \text{ et } m \text{ premier avec } n\}) \end{array}$$

n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
$\phi(n)$	0	1	2	2	4	2	6	4	6	4	10	4	12	6	8	8

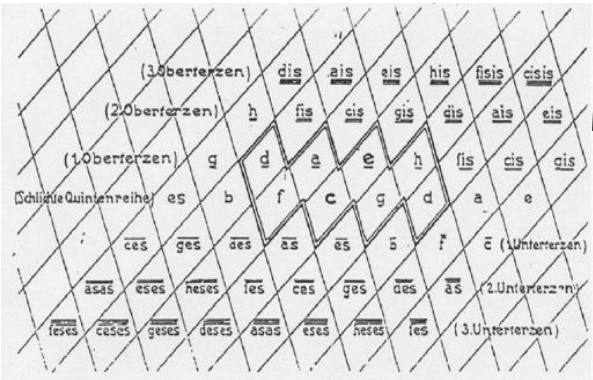
Premiers invariants algébriques : la structure intervallique



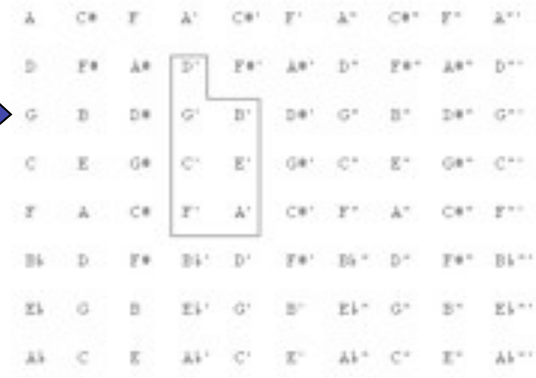
Du *Speculum musicum d'Euler* au *Tonnetz* de Riemann



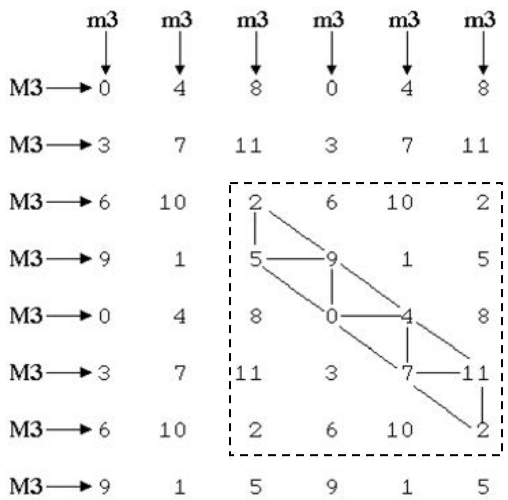
Euler : *Speculum musicum*, 1773



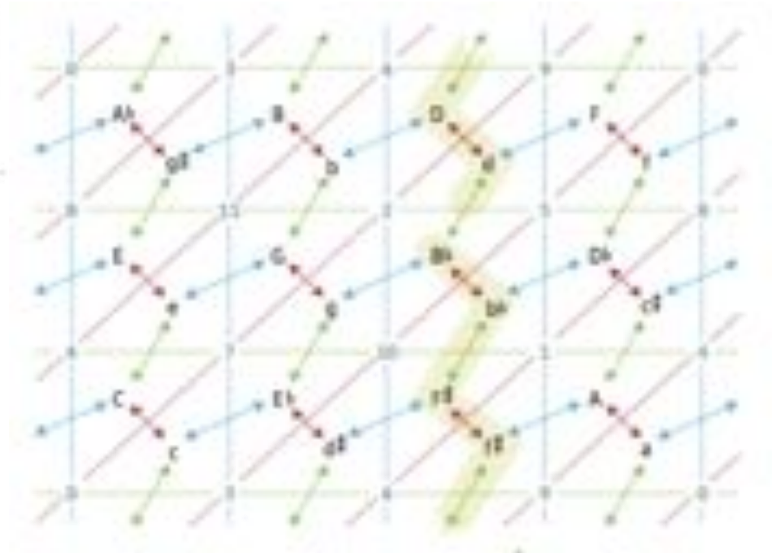
Hugo Riemann : « Ideen zu einer Lehre von den Tonvorstellung », 1914



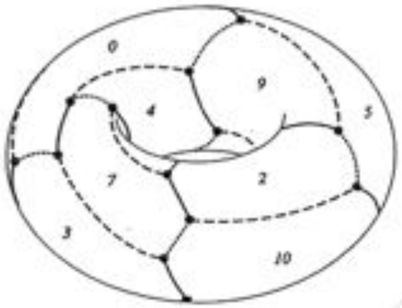
Longuet-Higgins (1962)



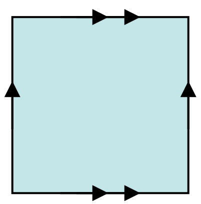
Balzano (1980)



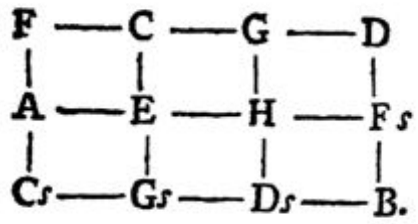
J. Hook, « Exploring Musical Space », *Science, 2006*



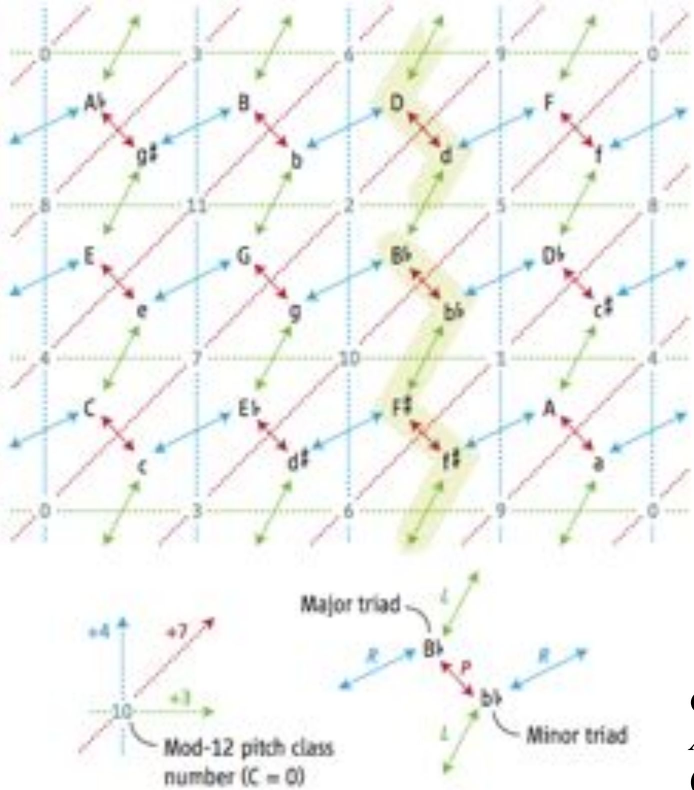
Douthett & Steinbach, *JMT*, 1998



Du réseau d’Euler au *Tonnetz*



Euler : *Speculum musicum*, 1773



**ρ = <L,R | $L^2 = (LR)^{12} = 1$
$LRL = L(LR)^{-1}$ >**

$$\rho \subseteq C_{Sym}\,(D_{12})$$

$$D_{12} \subseteq C_{Sym}\,(\rho)$$

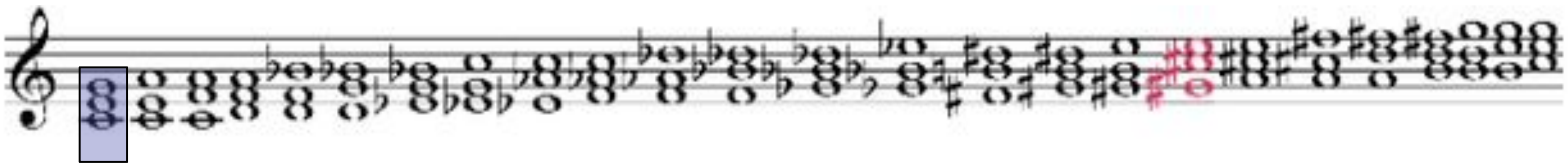
**D_{12} = <I, T | $I^2 = T^{12} = 1$
$ITI = I(IT)^{-1}$ >**

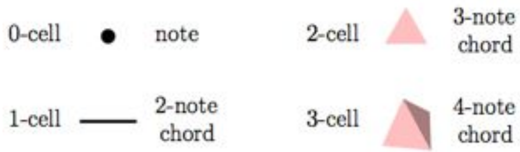
$$\begin{tikzcd} \circ \arrow[r, "f"] \arrow[d, "g"] & \circ \arrow[d, "g"] \\ \circ \arrow[r, "f"] & \circ \end{tikzcd}$$

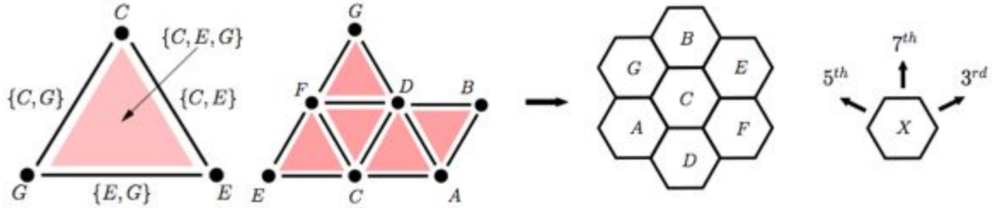
Tout diagramme commute
$\forall f \in D_{12}$
$\forall g \in \rho$

**Crans A., Fiore T., and Satyendra R. "Musical Actions of Dihedral Groups." *The American Mathematical Monthly*, Vol. 116 (2009), No. 6: 479 – 495
(Winner of the The Mathematical Association of America's Merten M. Hasse Prize 2011)**

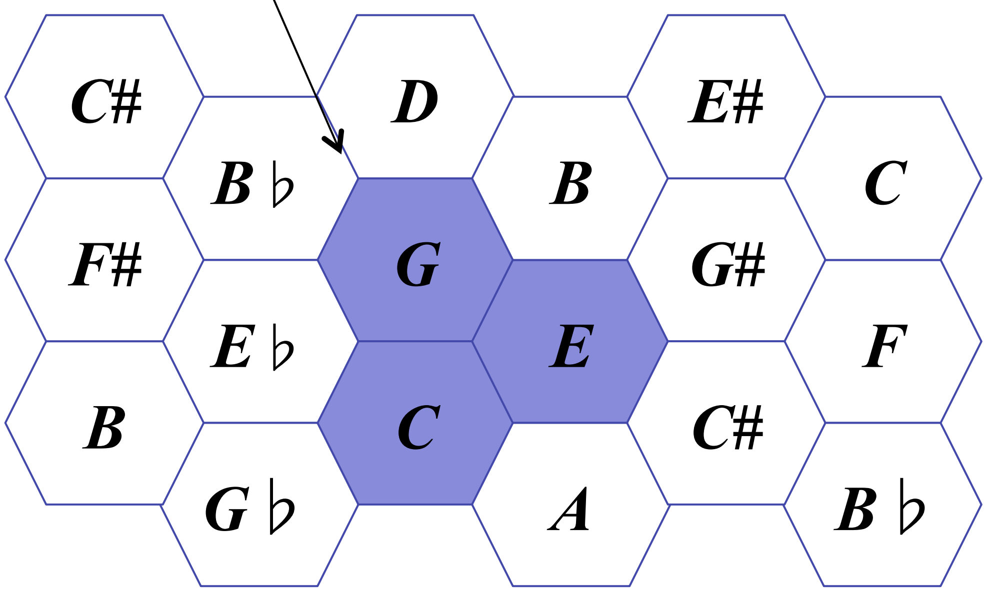
Du *Tonnetz* à la programmation spatiale

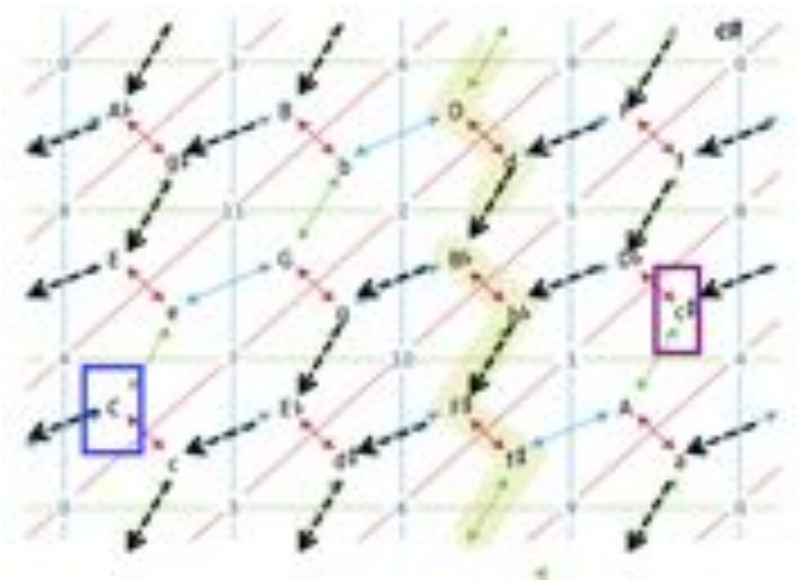




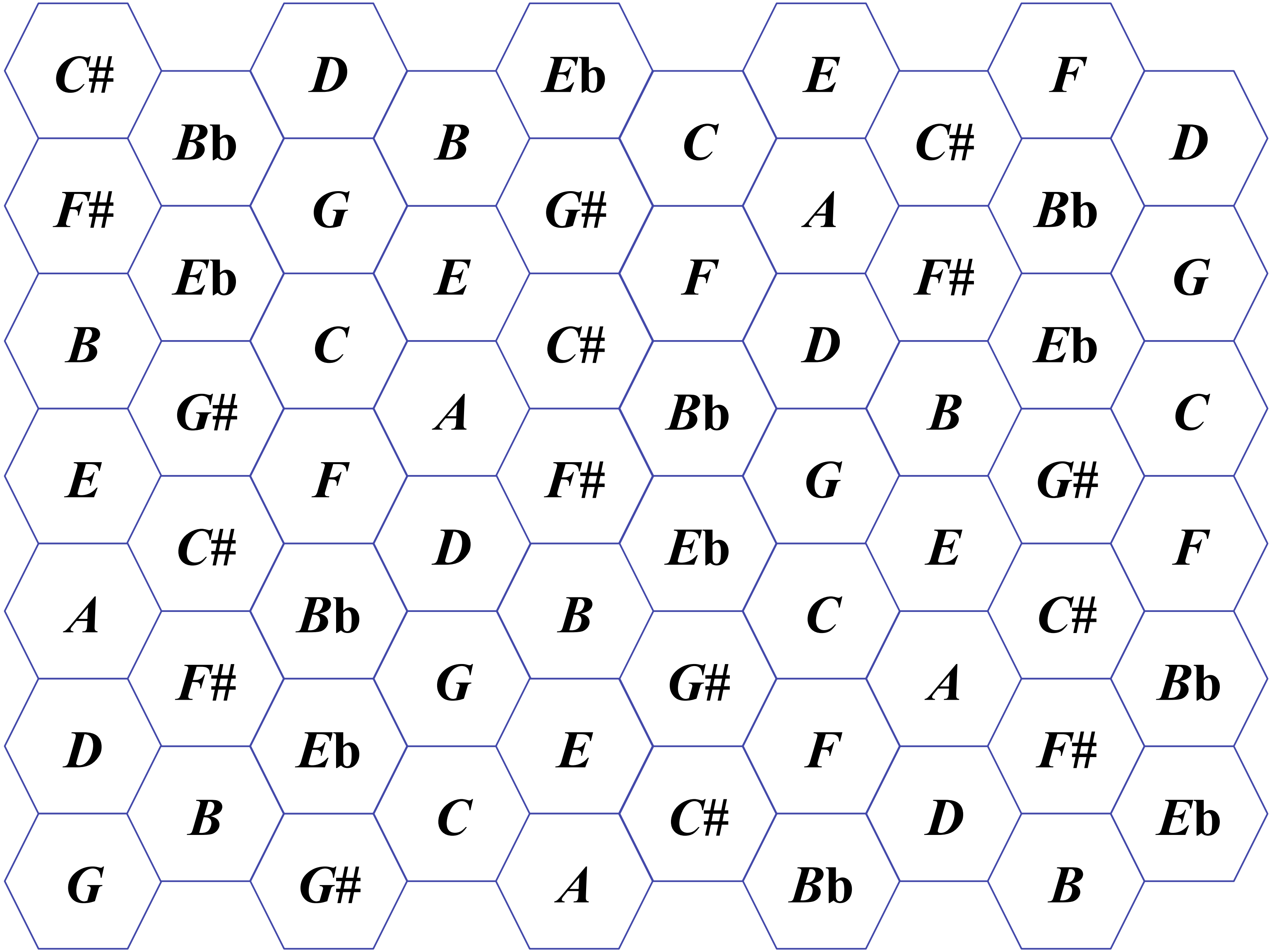


L. Bigo, J.-L. Giavitto, A. Spicher, "Building Topological Spaces for Musical Objects", MCM 2011
L. Bigo, thèse (en cours), Ircam / Université Paris 12





Extract of the 2nd movement of the Symphony No. 9 (L. van Beethoven)



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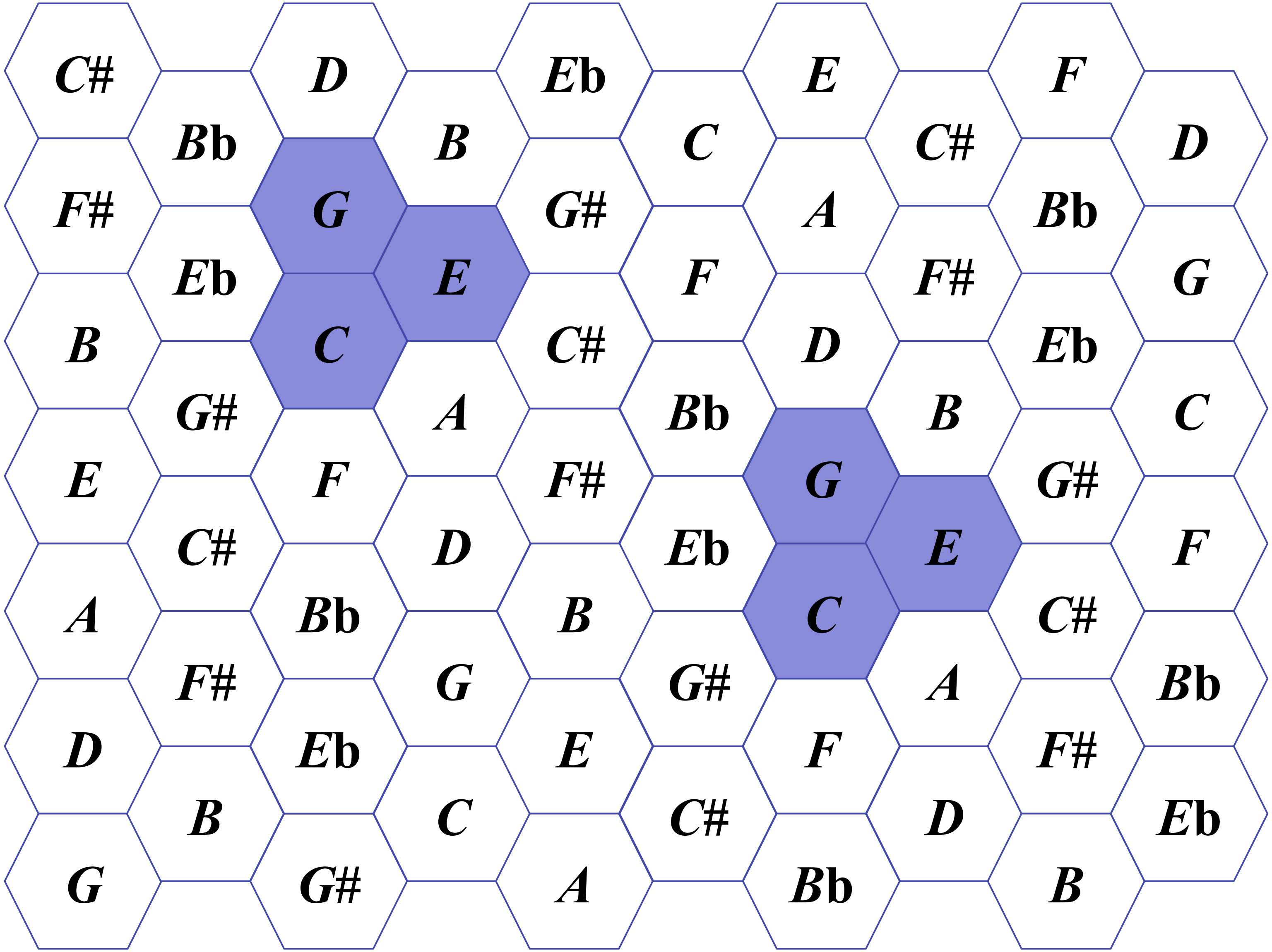

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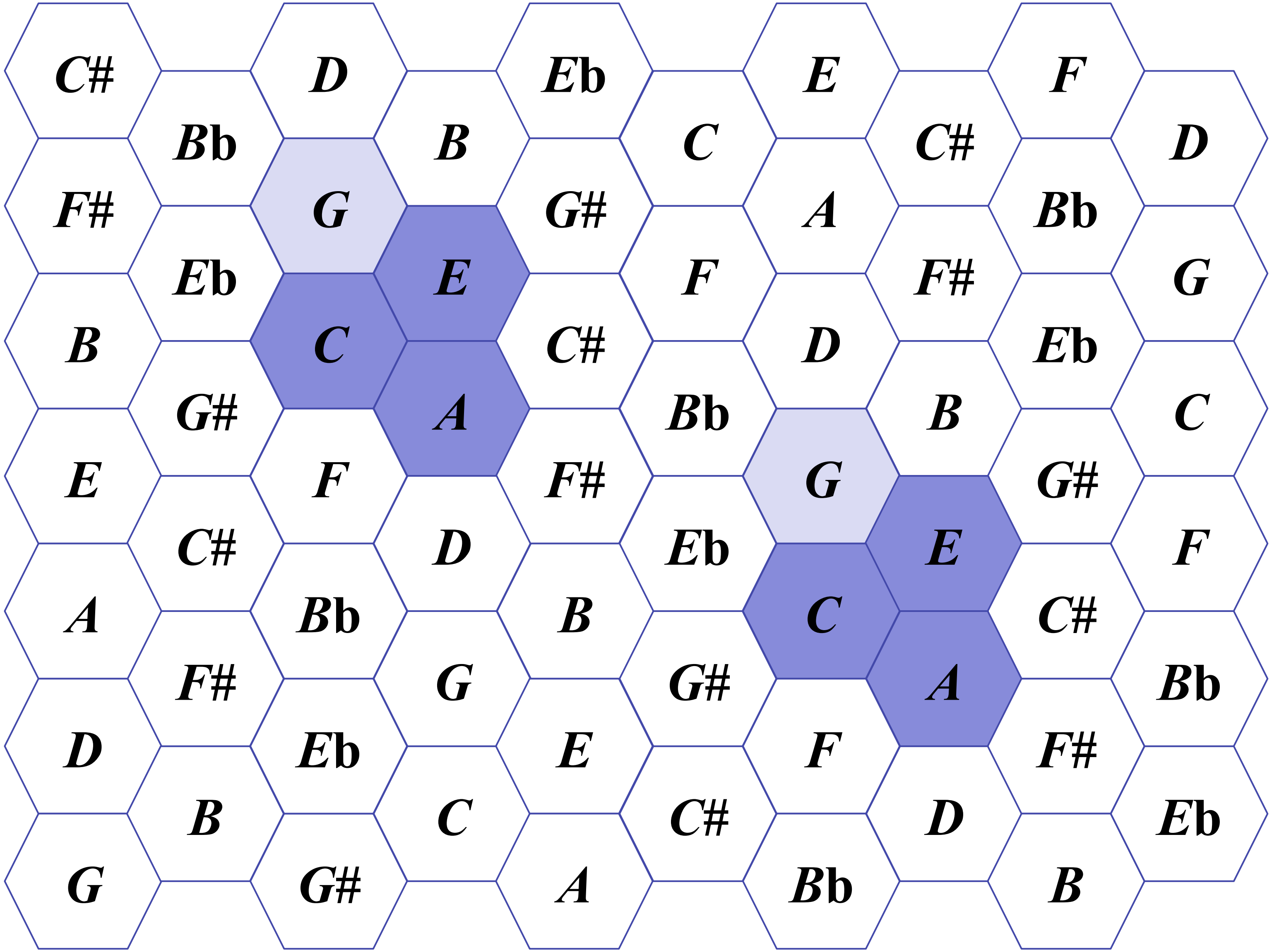

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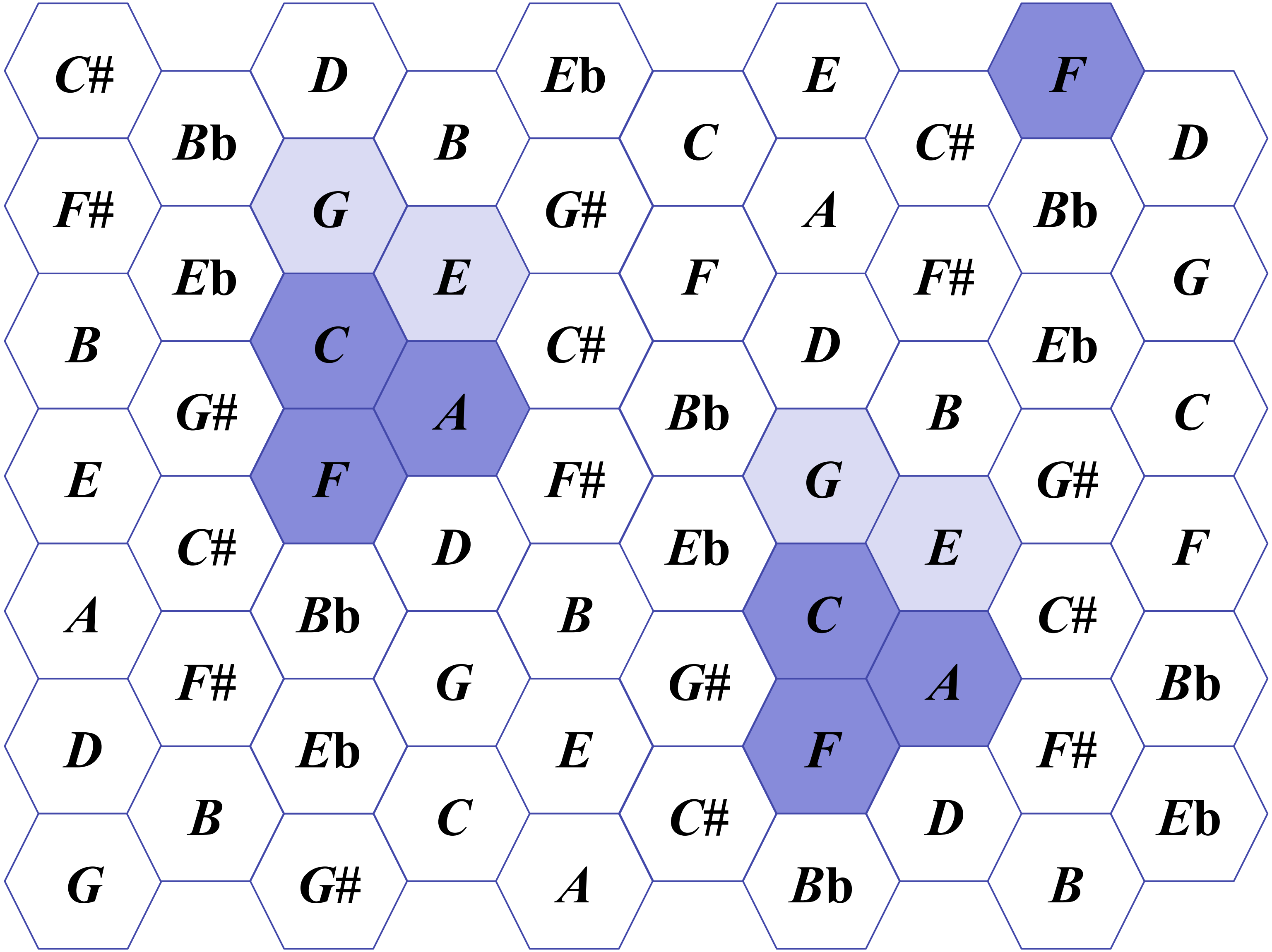

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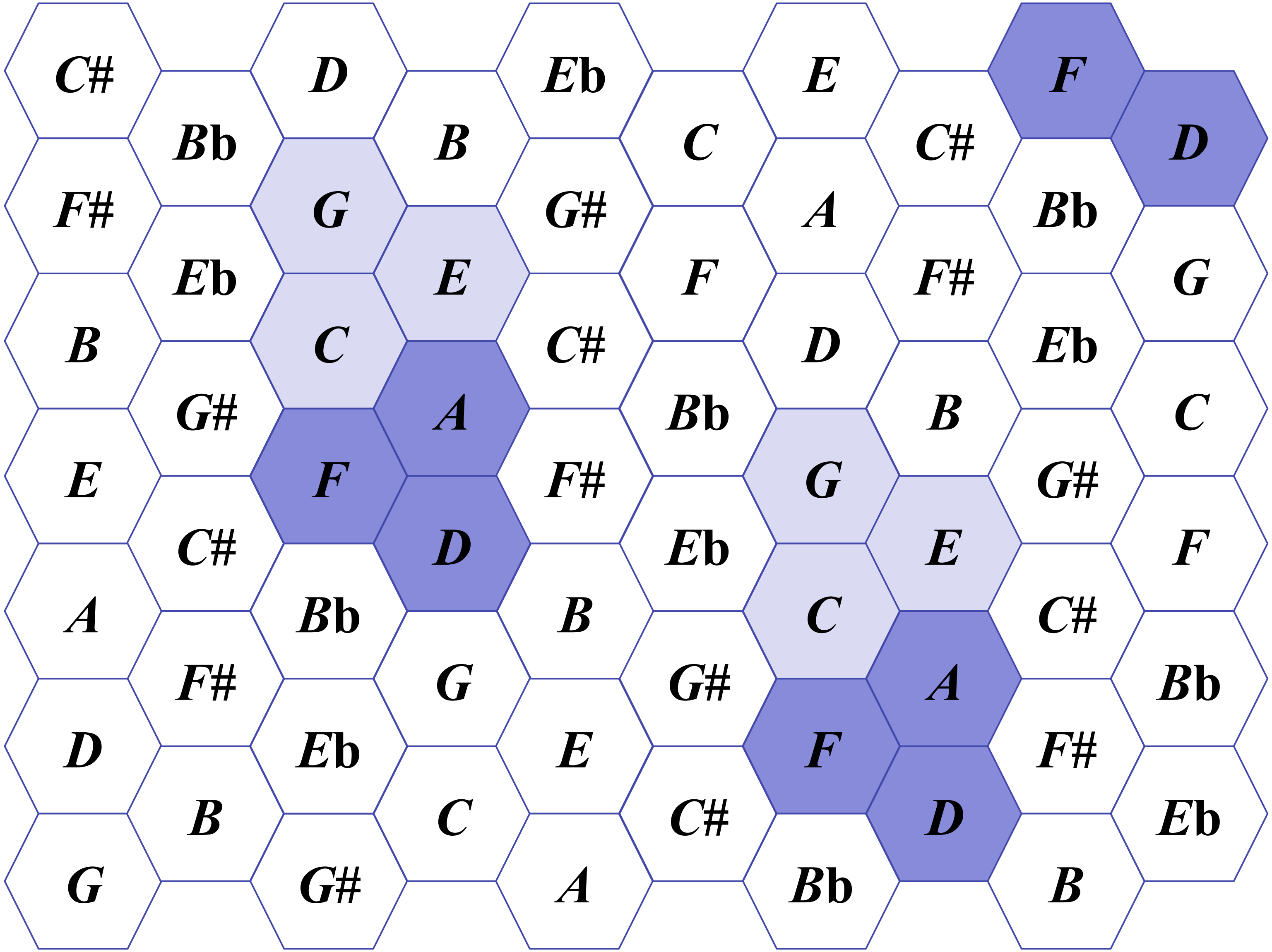

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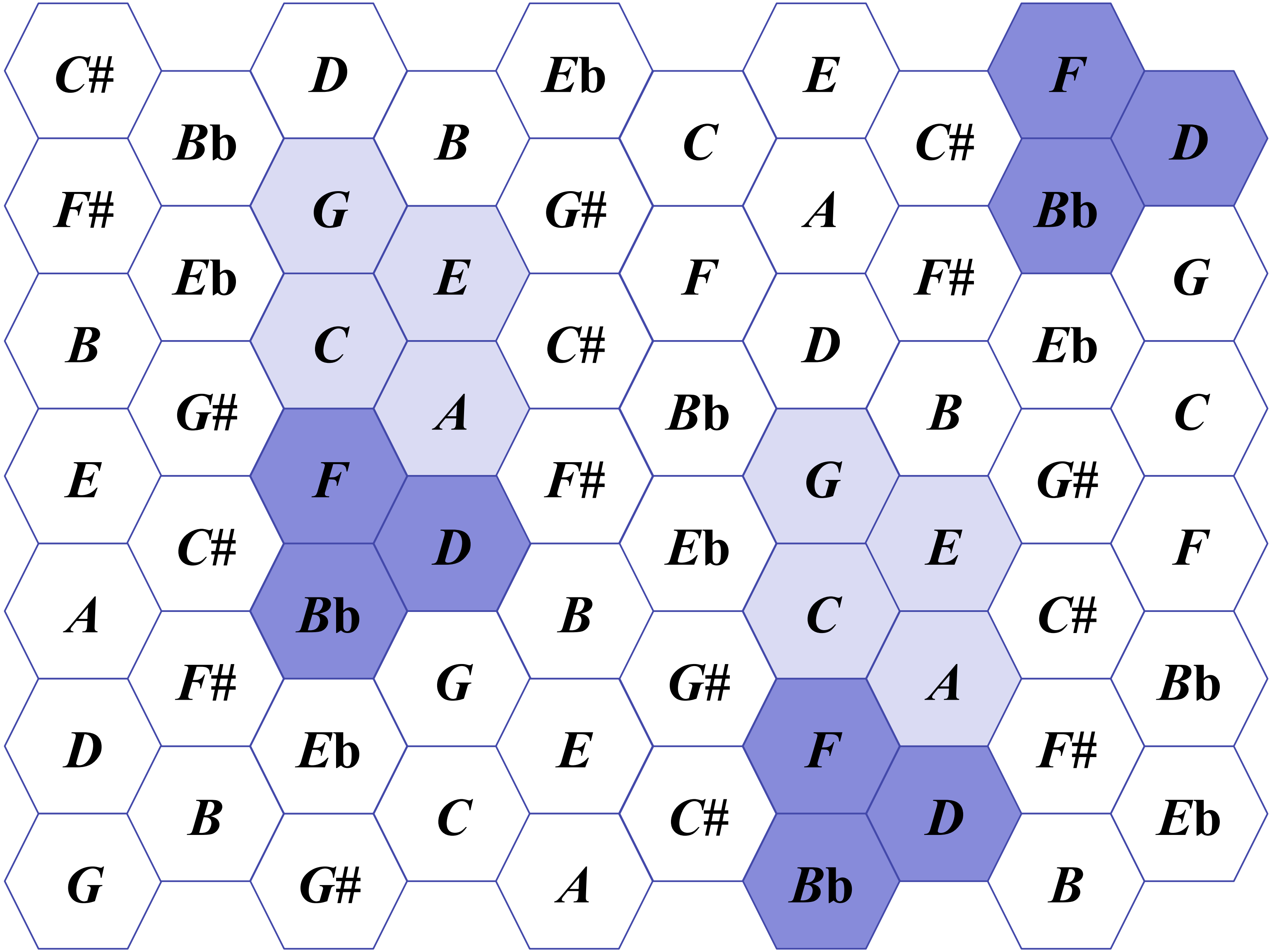

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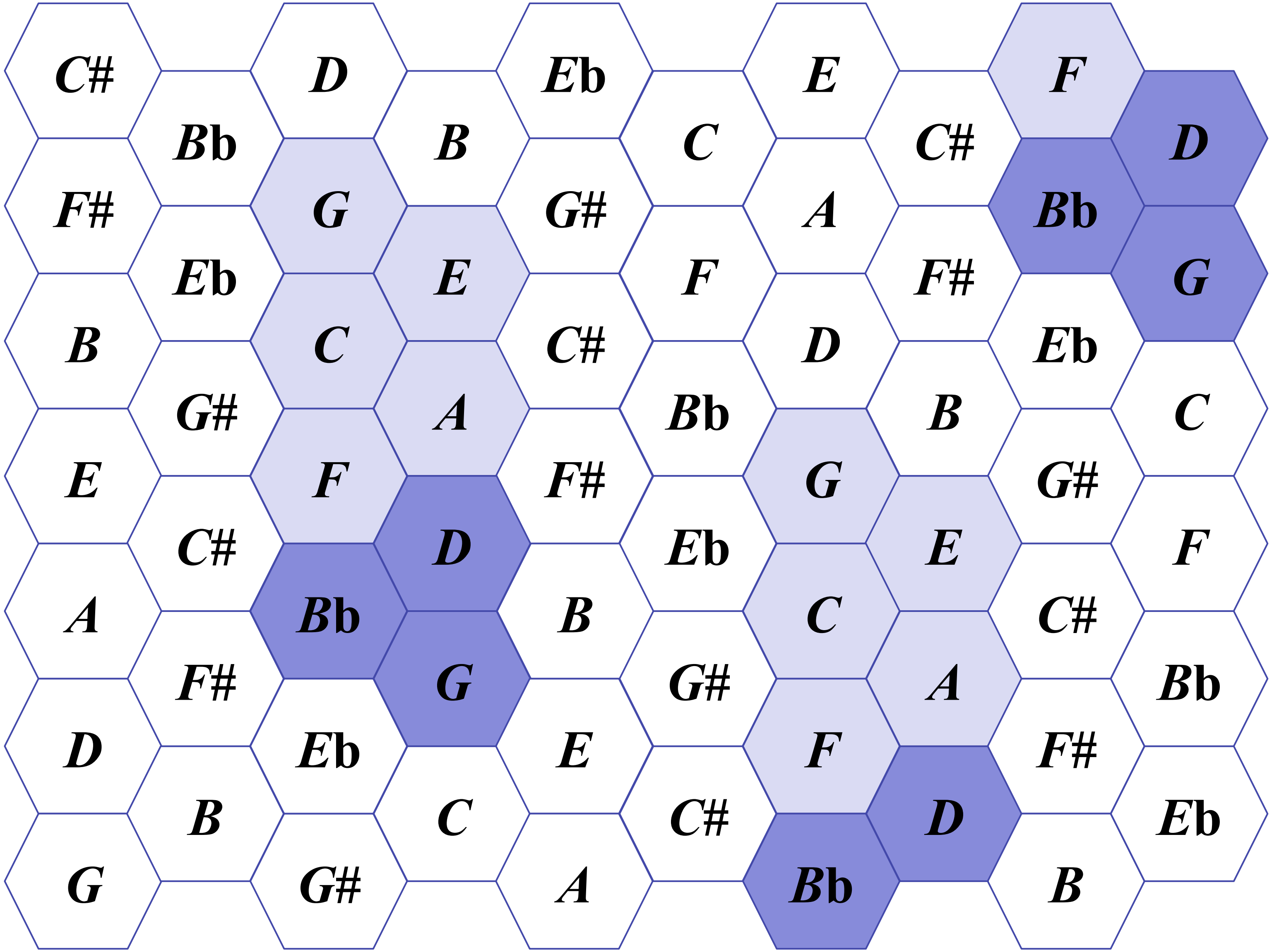

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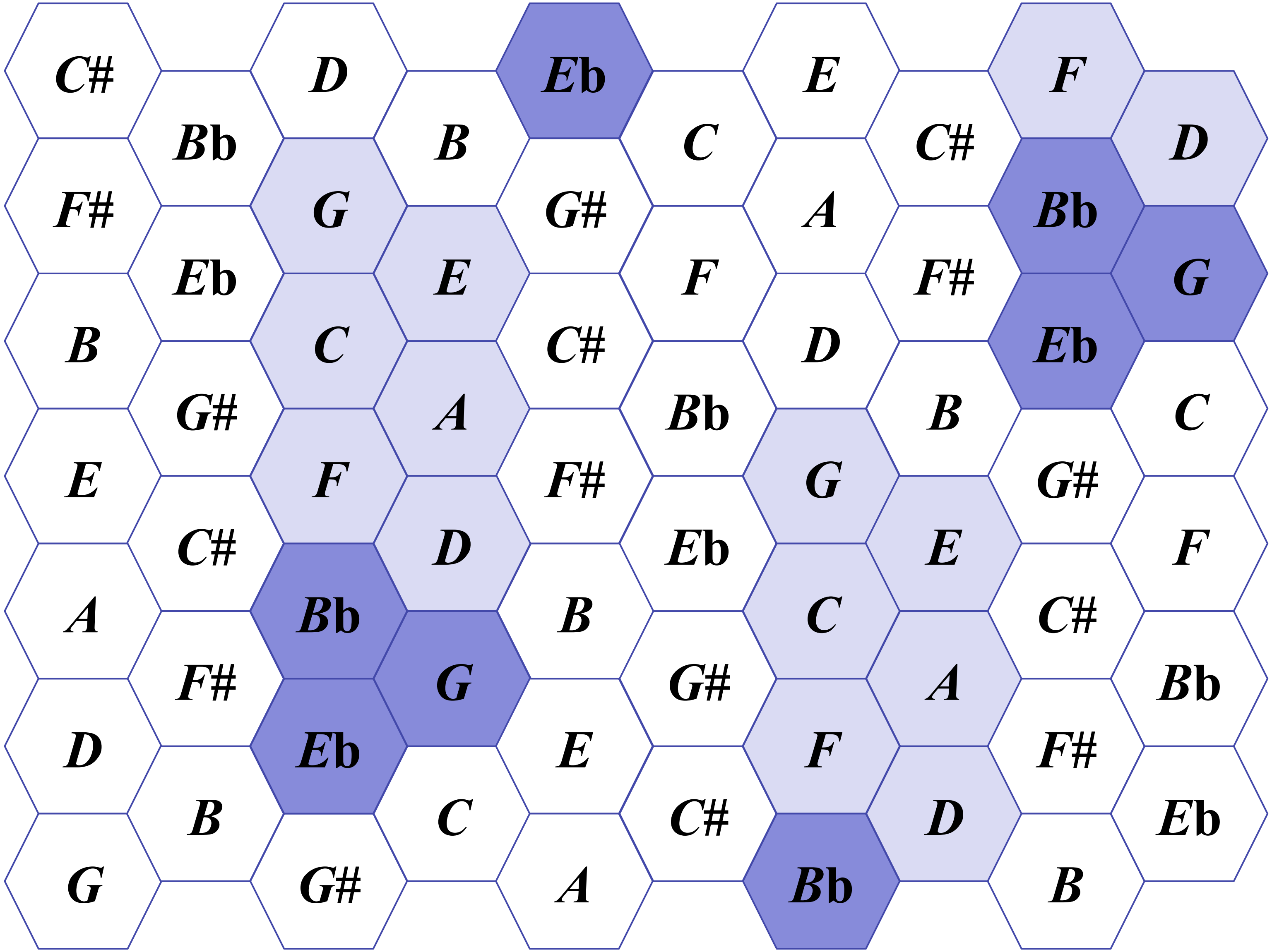

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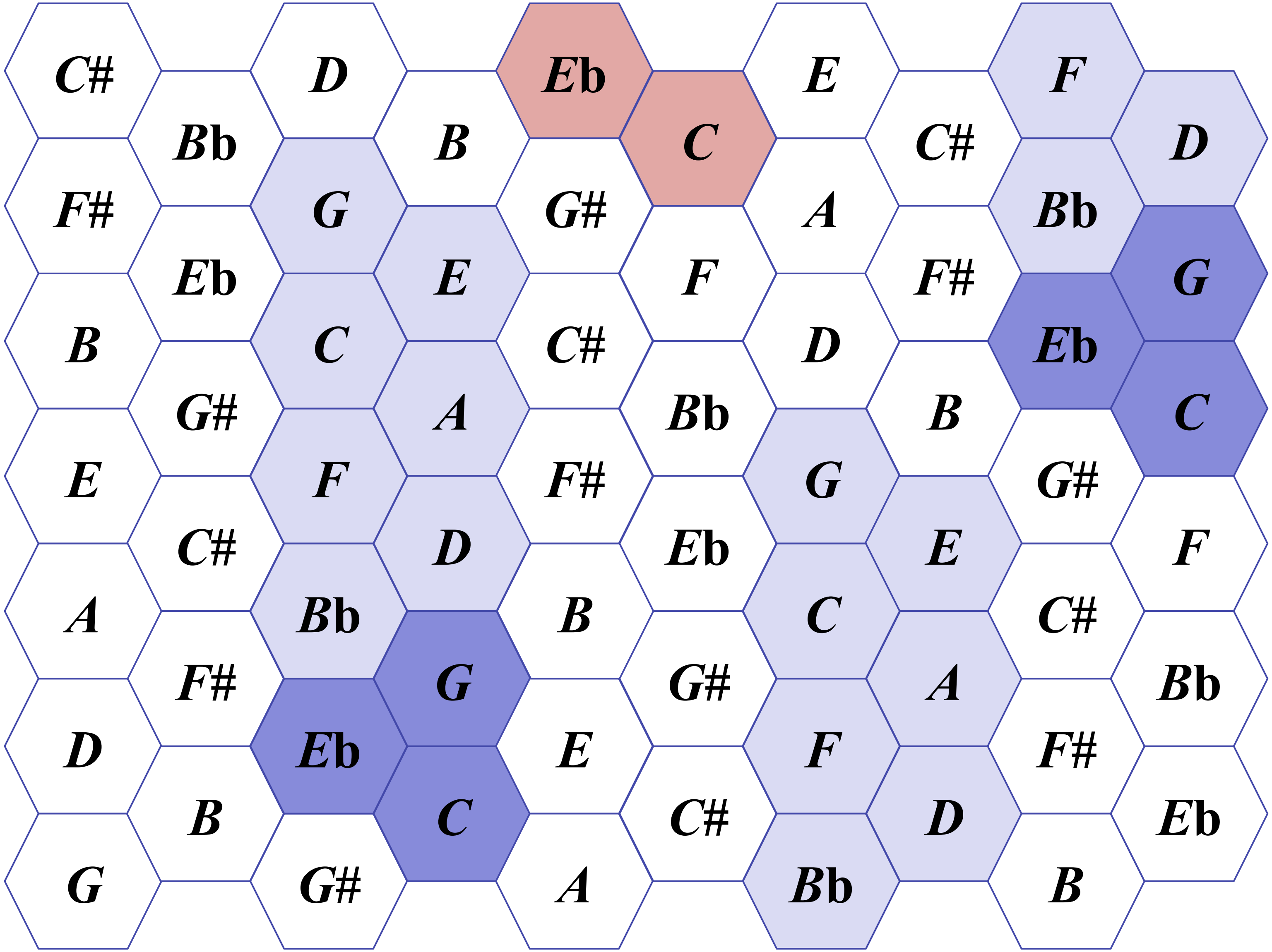

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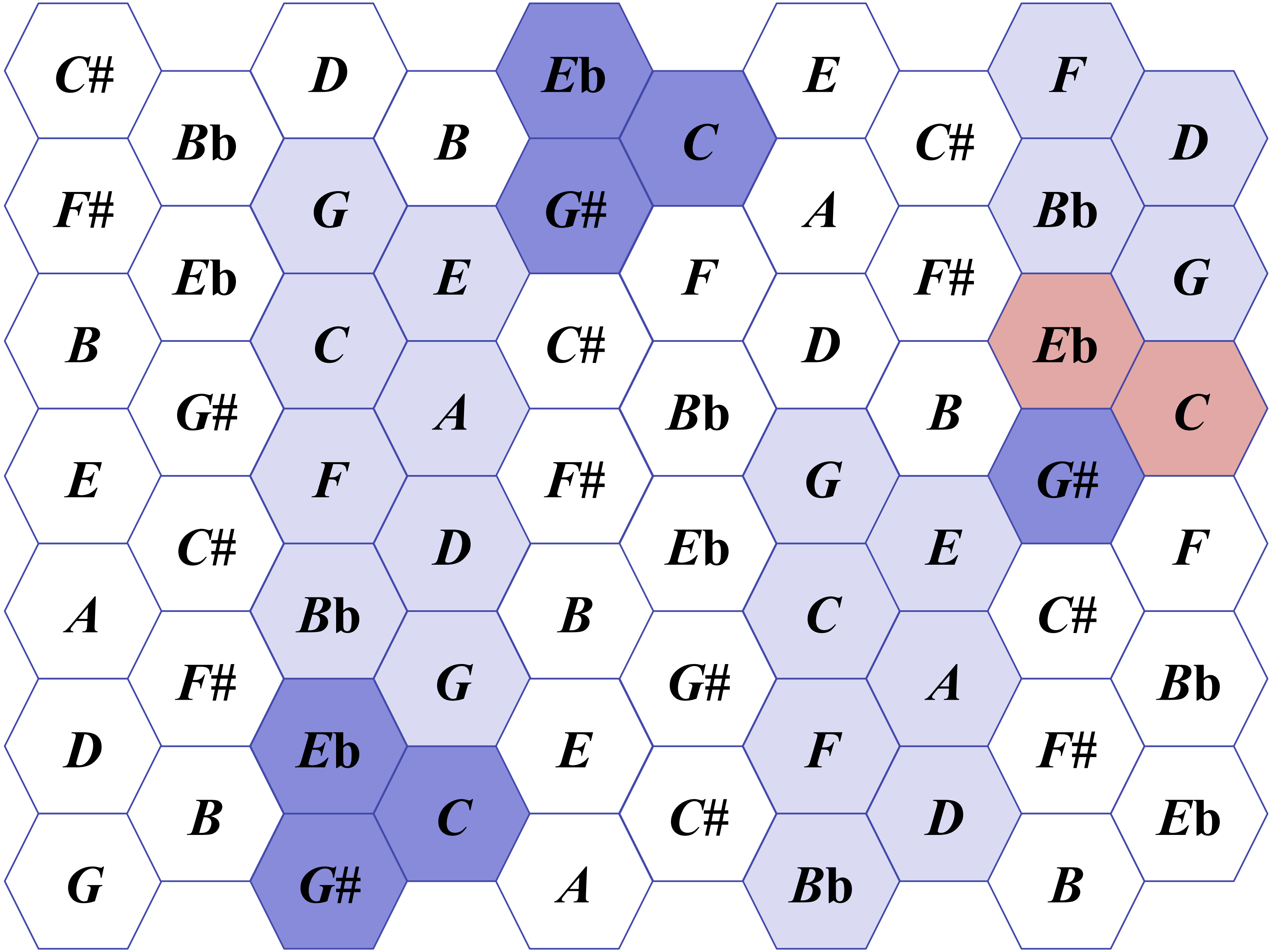
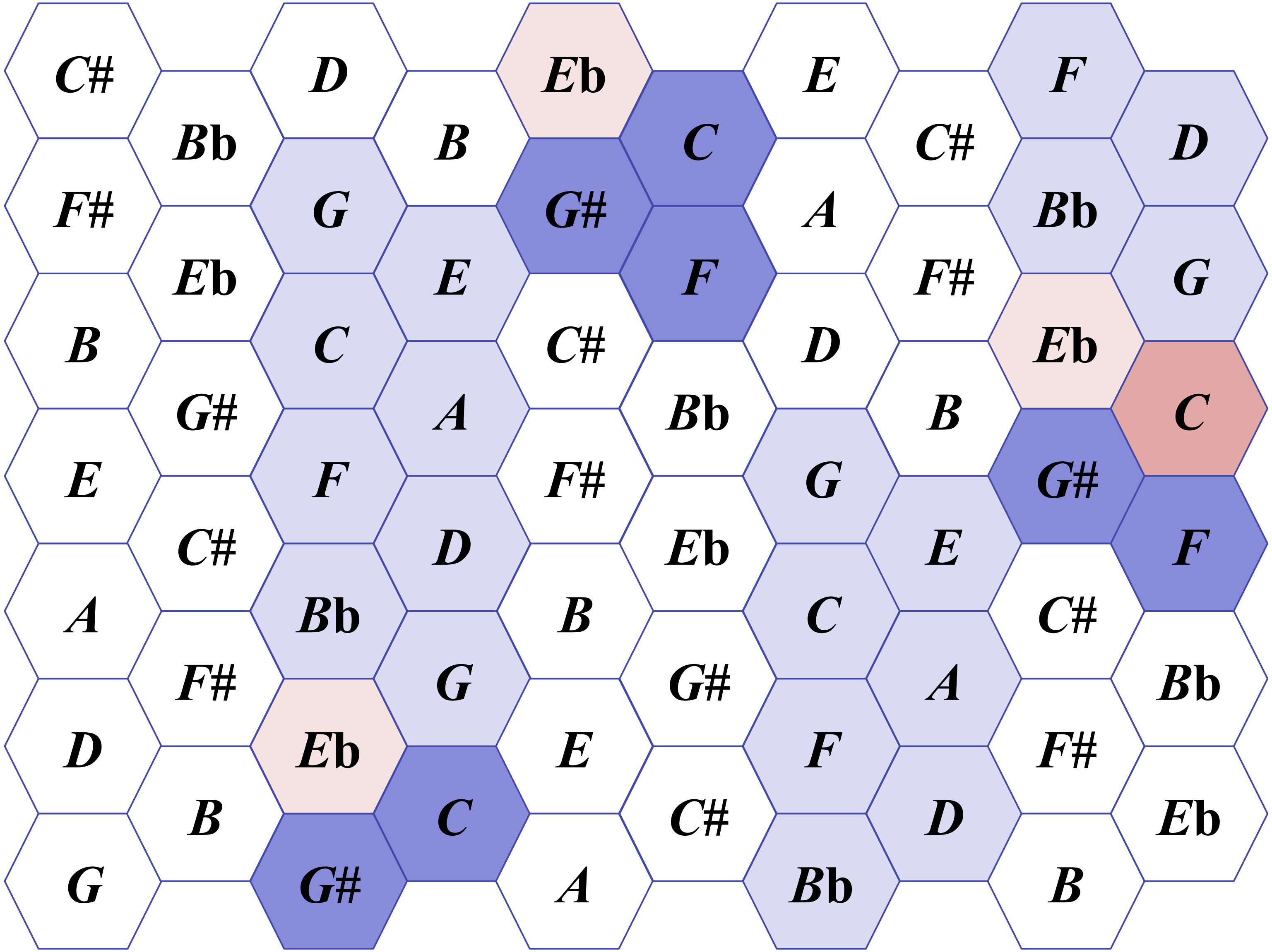
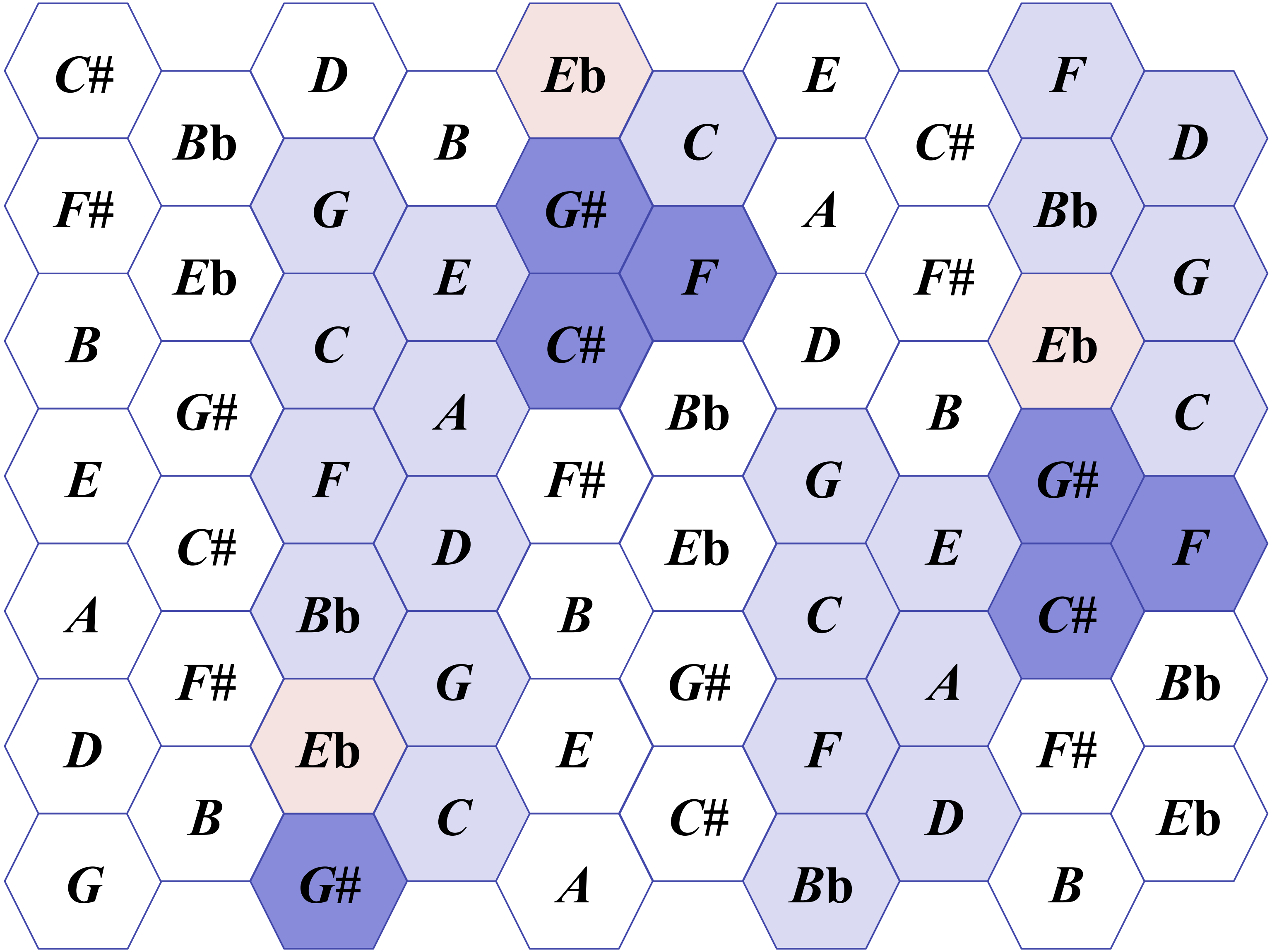


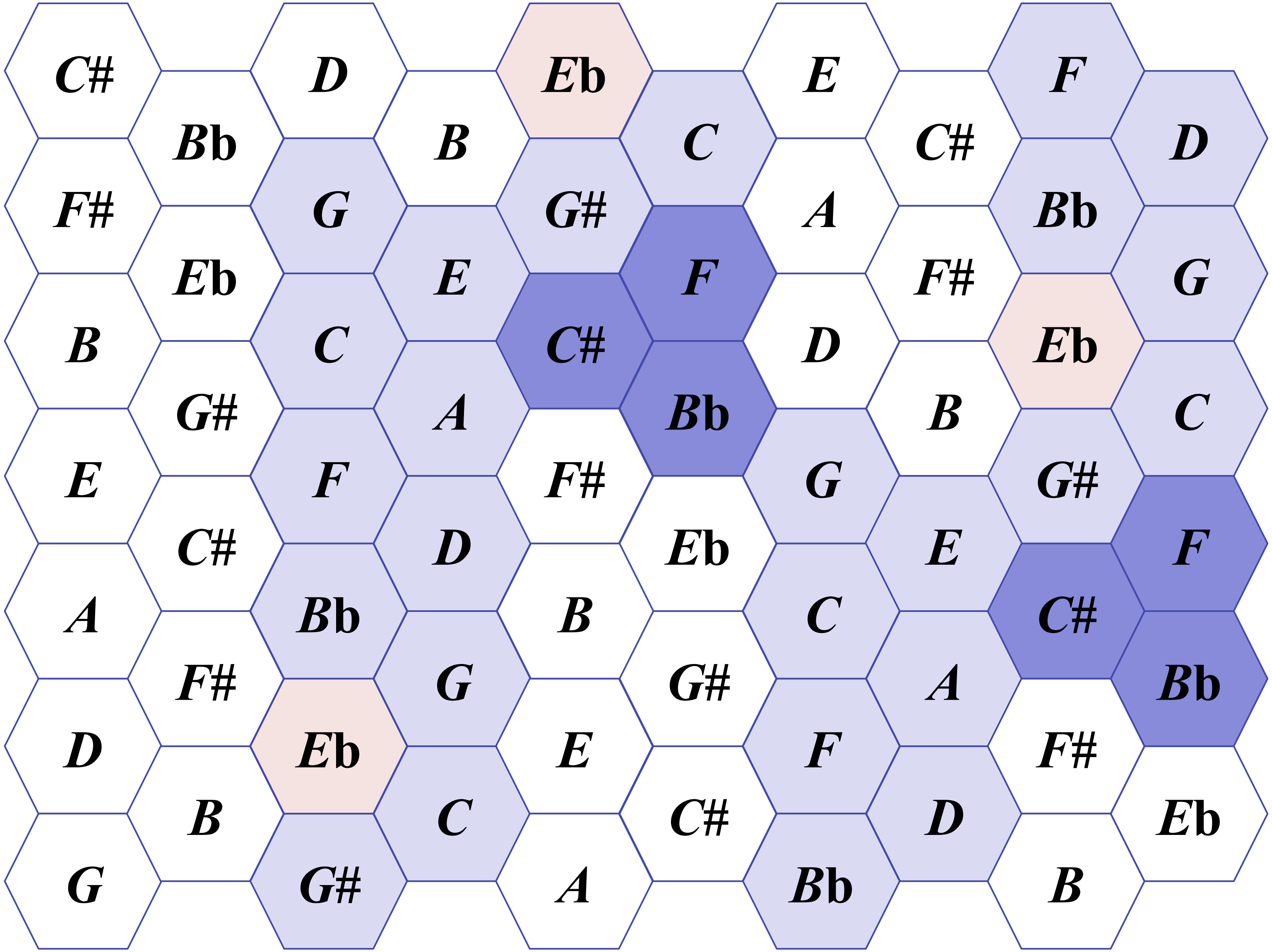
Extract of the 2nd movement of the Symphony No. 9 (L. van Beethoven)



Extract of the 2nd movement of the Symphony No. 9 (L. van Beethoven)



Extract of the 2nd movement of the Symphony No. 9 (L. van Beethoven)



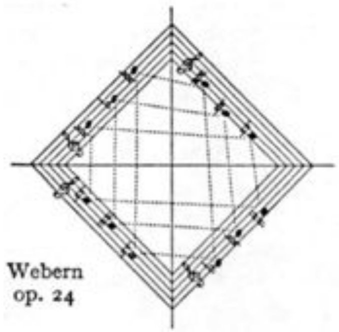
Vers l’émergence de la notion de groupe en musique

Ernst Krenek et l’approche axiomatique en musique

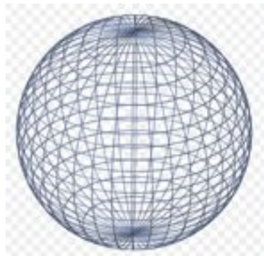
- *The Relativity of Scientific Systems*
- *The Significance of Axioms*
- *Axioms in music*
- *Musical Theory and Musical Practice*

Ernst Krenek : *Über Neue Musik*, 1937 (Engl. Transl. *Music here and now*, 1939)

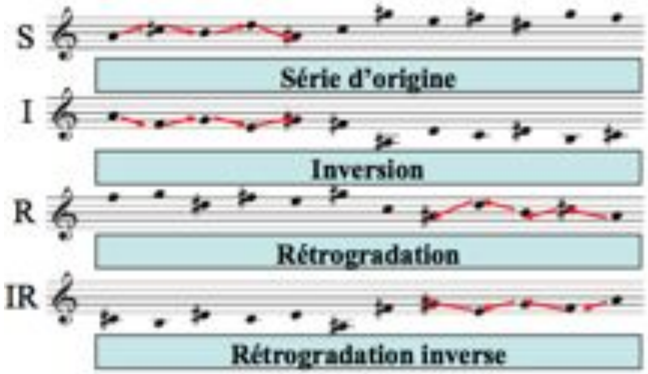
Physicists and mathematicians *are far in advance of musicians in realizing that their respective sciences do not serve to establish a concept of the universe conforming to an objectively existent nature*



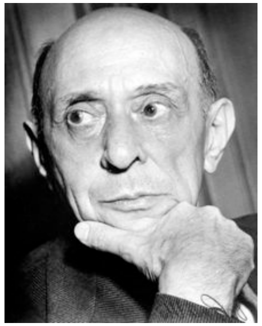
As the study of axioms eliminates the idea that ***axioms*** *are something absolute, conceiving them instead as* ***free propositions of the human mind****, just so would this* ***musical theory*** *free us from the concept of major/minor tonality […] as an irrevocable law of nature.*

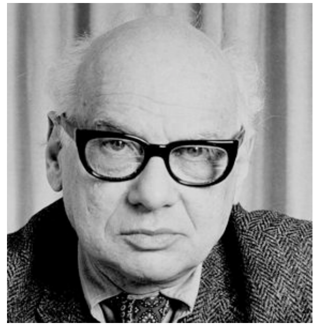


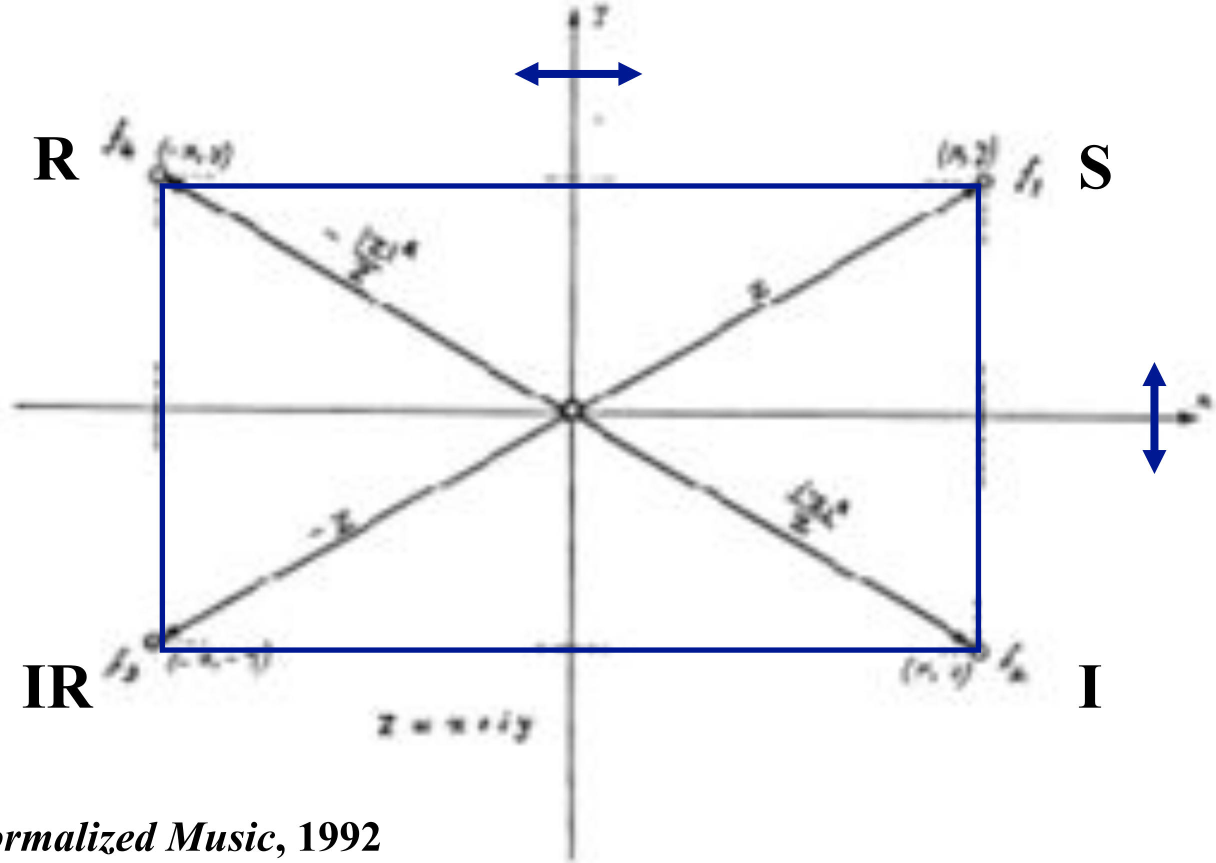
Opérations dodécaphoniques et structures algébriques



	S	I	R	RI
S	S	I	R	RI
I	I	S	RI	R
R	R	RI	S	I
RI	RI	R	I	S





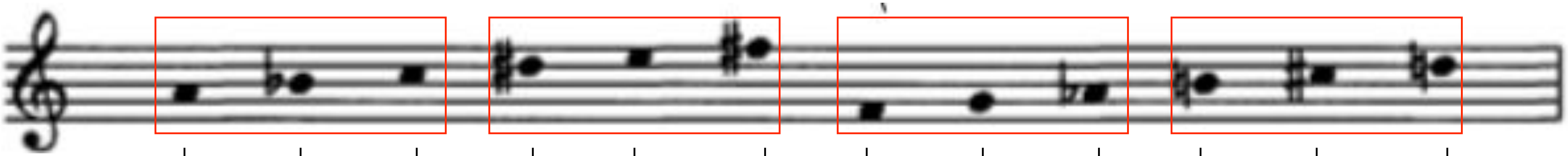


Iannis Xenakis, *Formalized Music*, 1992

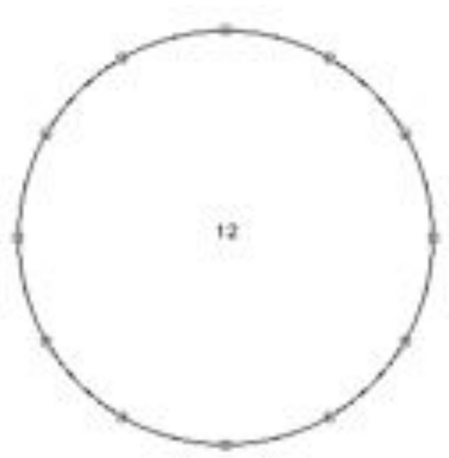
Felix Klein

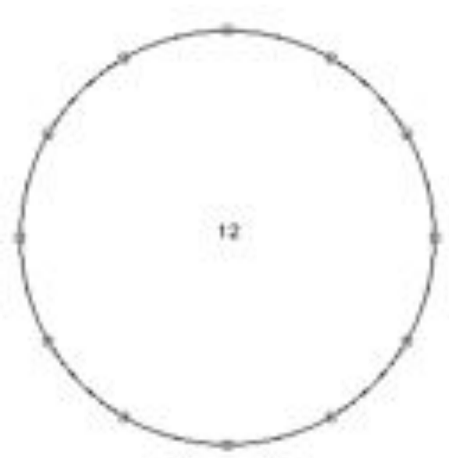
Exercice : retrouver les symétries dans une série (I)

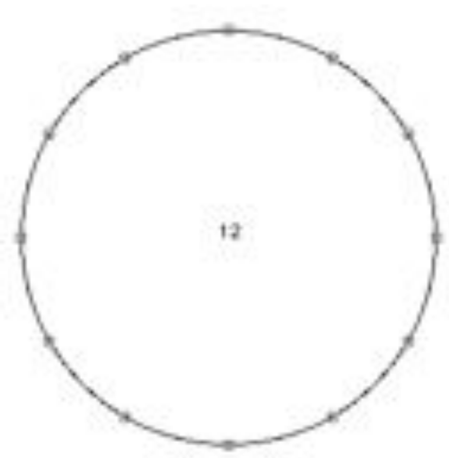
Schoenberg: Serenade Op.24, Mouvement 5

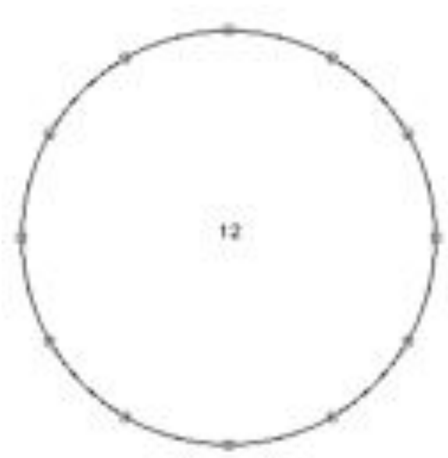


{… , … , …} {… , … , …} {… , … , …} {… , … , …}





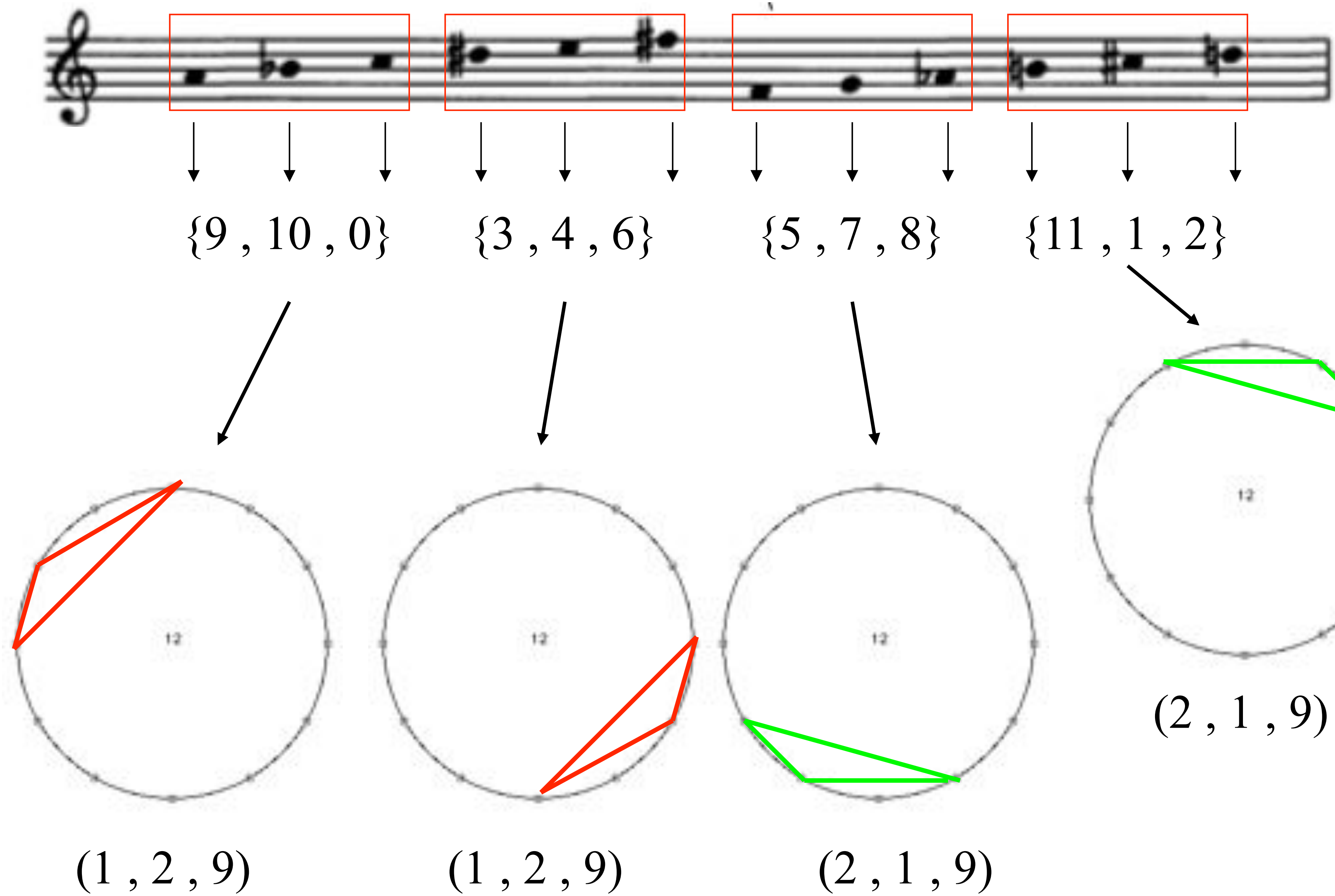




(… , … , …) (… , … , …) (… , … , …) (… , … , …)

Exercice : retrouver les symétries dans une série (I)

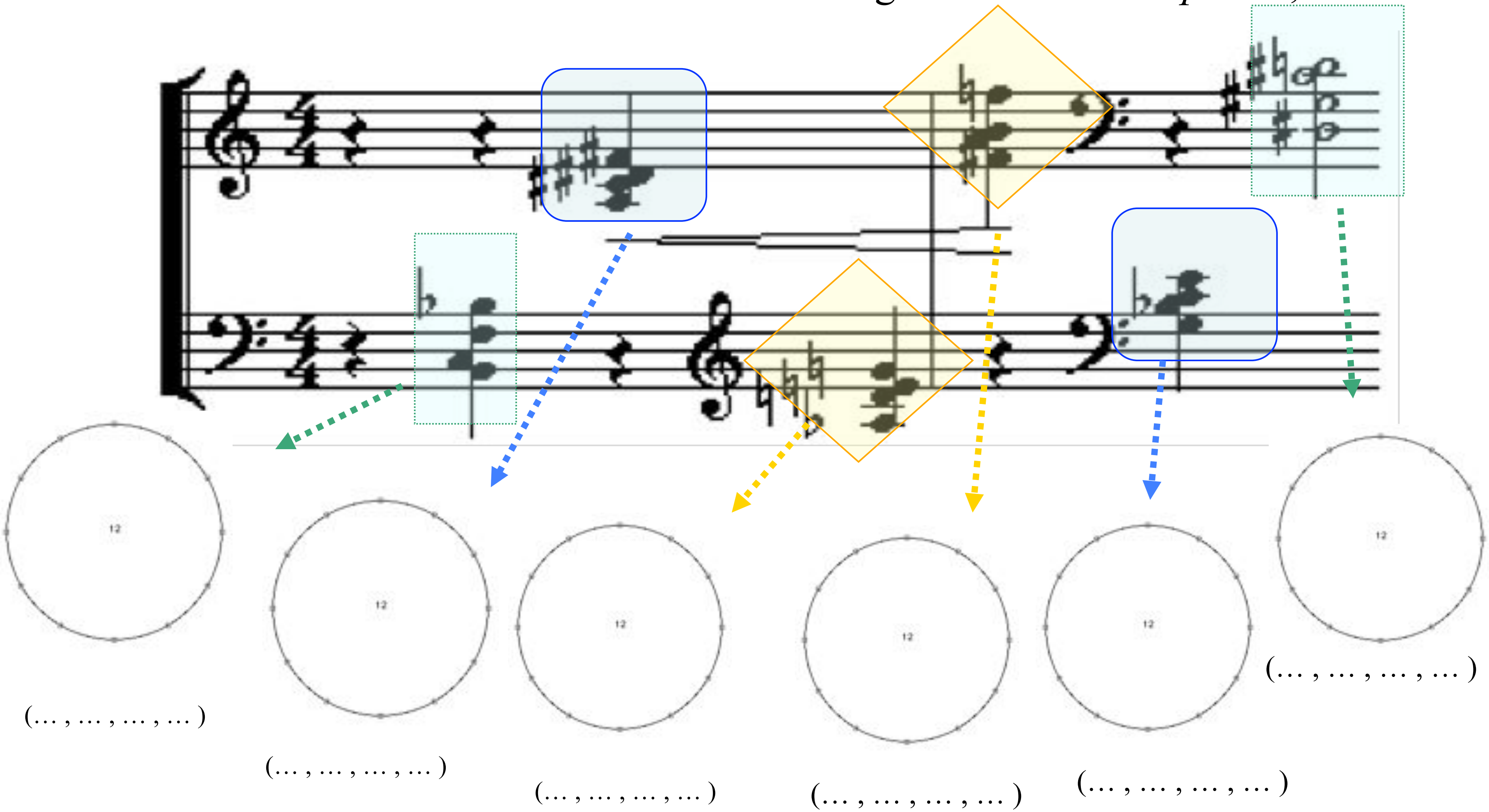
Schoenberg: Serenade Op.24, Mouvement 5



L'analyse formalisée ou les entités formelles en musique

André Riotte e Marcel Mesnage

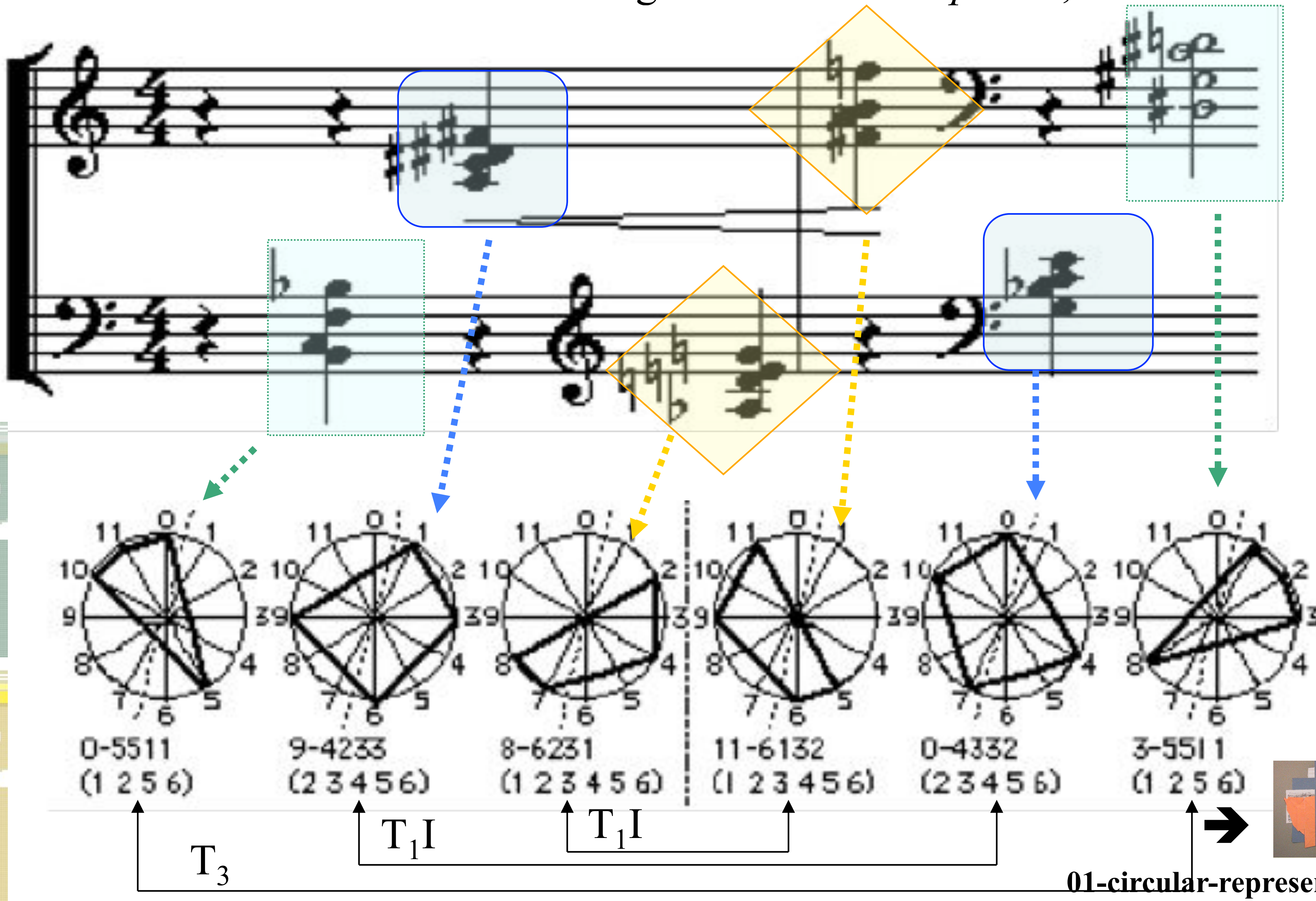
A. Schoenberg : *Klavierstück Op. 33a*, 1929

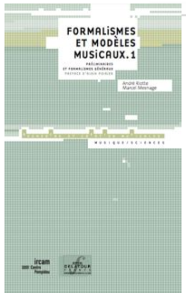


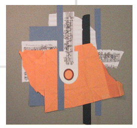
L'analyse formalisée ou les entités formelles en musique

André Riotte & Marcel Mesnage

A. Schoenberg : *Klavierstück Op. 33a*, 1929

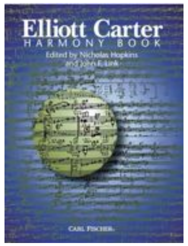






01-circular-representation

Elliott Carter : 90+ (1994)

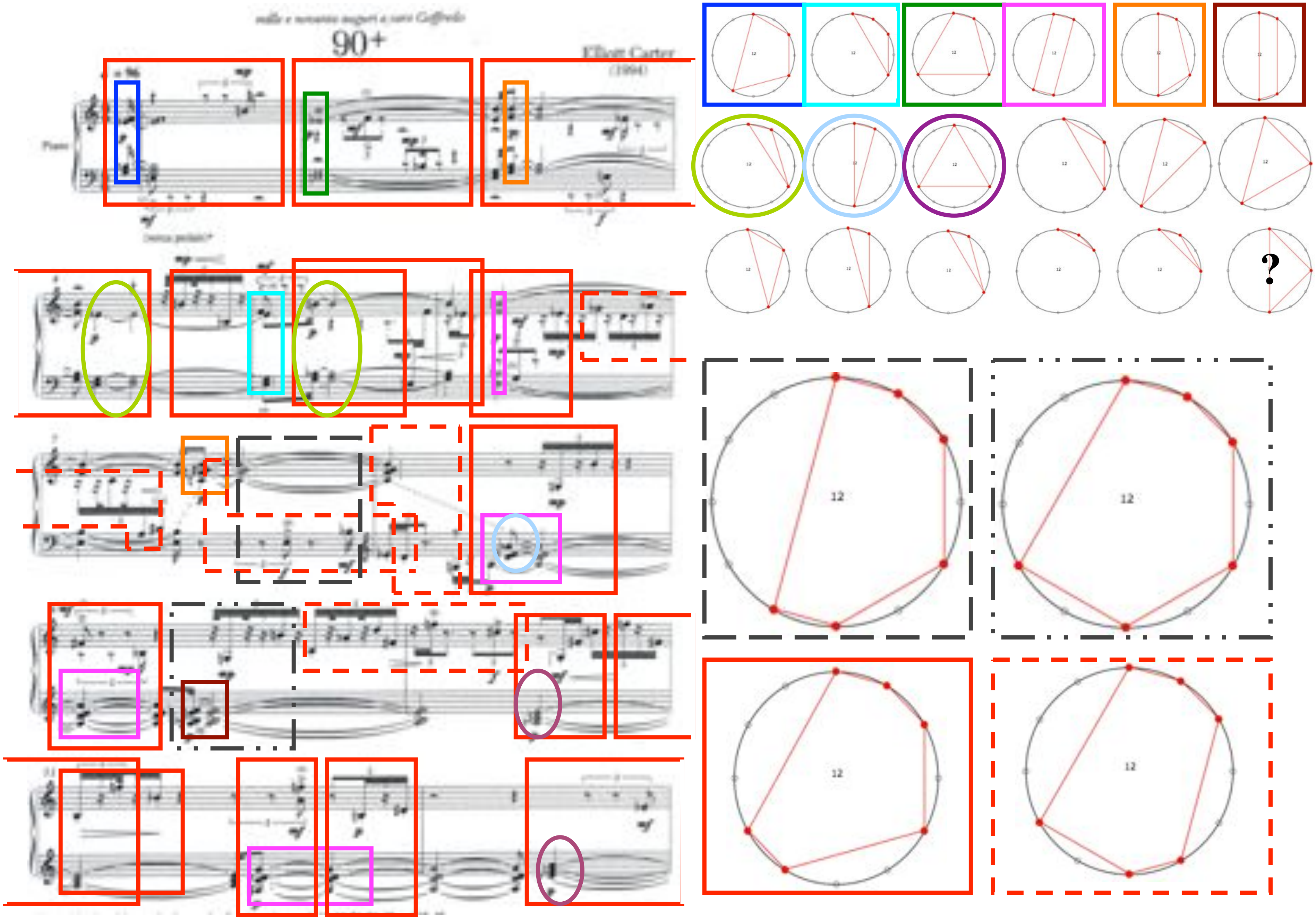


- **Combinatoire d’accords**
  - **Hexacordes**
  - **Tétracordes**
  - **Triades**
  - **Relation Z**

- **Séries tous–intervalles**
  - ***Link–chords***

(piano: John Snijders)

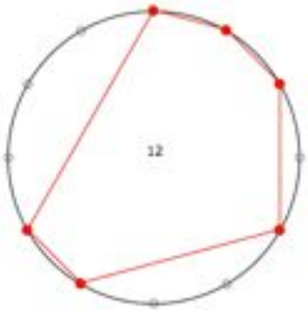
Elliott Carter : 90+ (1994) : combinatoire tetra/tricordale



Elliott Carter: 90+ (1994)

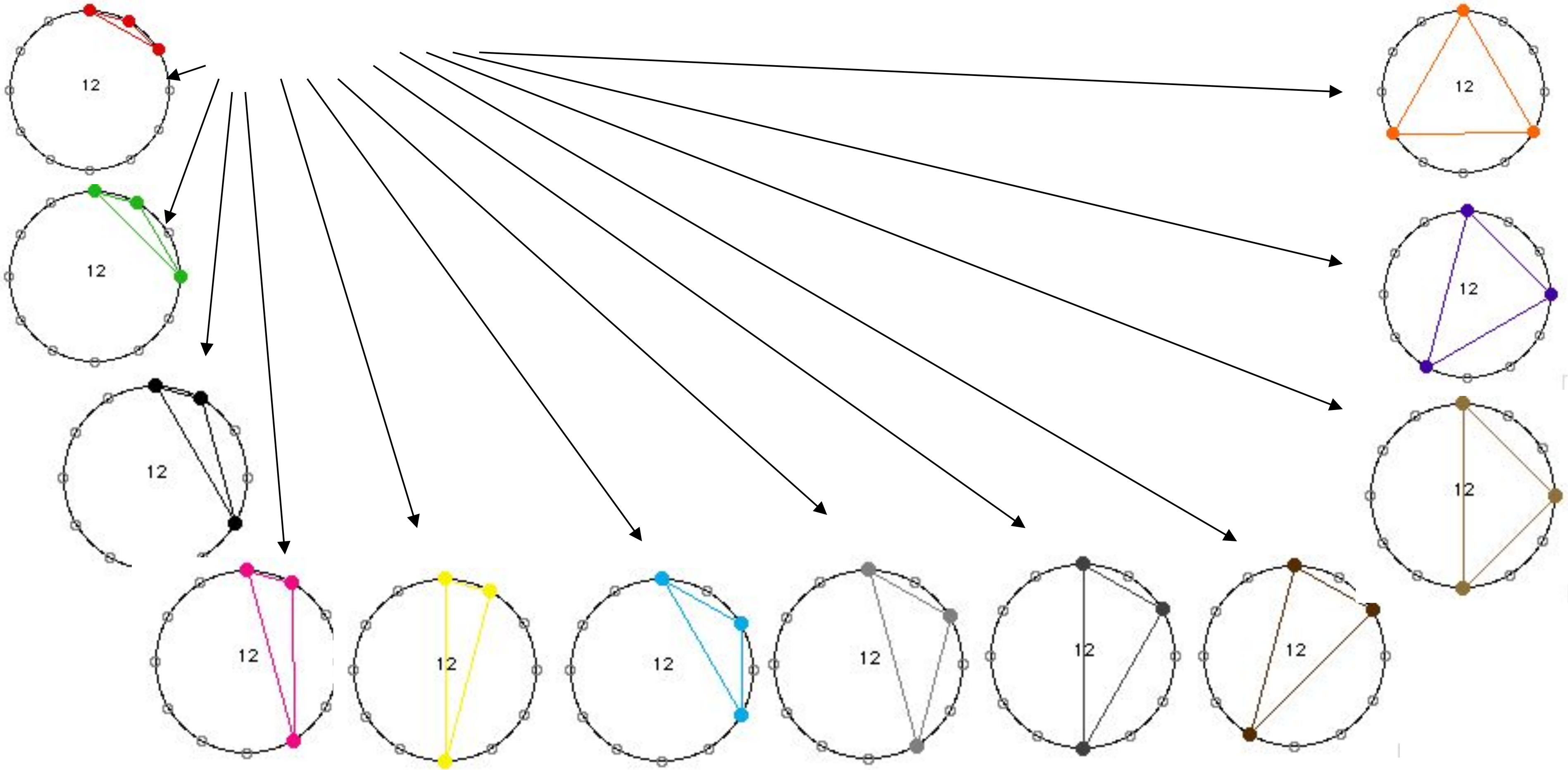
$G\backslash k$	1	2	3	4	5	6	7	8	9	10	11	12
C_{12}	1	6	19	43	66	80	66	43	19	6	1	1
D_{12}	1	6	12	29	38	50	38	29	12	6	1	1
$\mathrm{Aff}_1(Z_{12})$	1	5	9	21	25	34	25	21	9	5	1	1

35



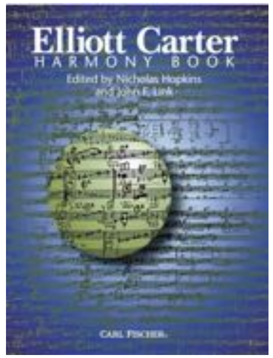
pcset	*vector*	*name*
(0 1 2 4 7 8)	[322332]	6-Z17

All-triad hexachord

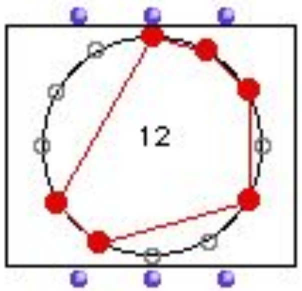


Elliott Carter: 90+ (1994)

« From about 1990, I have reduced my vocabulary of chords more and more to the six note chord n° 35 and the four note chords n° 18 and 23, which encompass all the intervals » (*Harmony Book*, 2002, p. ix)



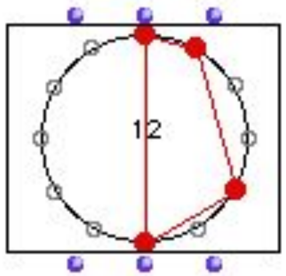
35



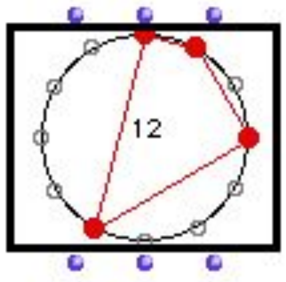
	pcset	*vector*	*name*
35	(0 1 2 4 7 8)	[322332]	6-Z17
18	(0 1 4 6)	[111111]	4-Z15
23	(0 1 3 7)	[111111]	4-Z29

All-triad hexachord

18



23



La *Set Theory* d'Allen Forte: catalogue des *pitch-class sets*

complementare

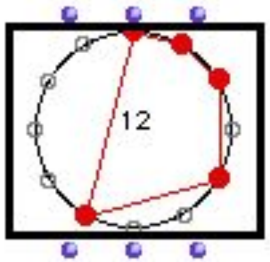
name	pcs	vector
5-30	0,1,4,6,8	121321
5-31	0,1,3,6,9	114112
5-32	0,1,4,6,9	113221
5-33(12)	0,2,4,6,8	040402
5-34(12)	0,2,4,6,9	032221
5-35(12)	0,2,4,7,9	032140
5-Z36	0,1,2,4,7	222121
5-Z37(12)	0,3,4,5,8	212320
5-Z38	0,1,2,5,8	212221
6-1(12)	0,1,2,3,4,5	543210
6-2	0,1,2,3,4,6	443211
		432321
6-Z4(12)	0,1,2,4,5,6	
6-5	0,1,2,3,6,7	422232
6-Z6(12)	0,1,2,5,6,7	421242
6-7(6)	0,1,2,6,7,8	420243
6-8(12)	0,2,3,4,5,7	343230
6-9	0,1,2,3,5,7	342231
6-Z10	0,1,3,4,5,7	333321
6-Z11	0,1,2,4,5,7	333231
6-Z12	0,1,2,4,6,7	332232
6-Z13(12)	0,1,3,4,6,7	324222

name	pcs	vector
7-30	0,1,2,4,6,8,9	343542
7-31	0,1,3,4,6,7,9	336333
7-32	0,1,3,4,6,8,9	335442
7-33	0,1,2,4,6,8,10	262623
7-34	0,1,3,4,6,8,10	254442
7-35	0,1,3,5,6,8,10	254361
7-Z36	0,1,2,3,5,6,8	444342
7-Z37	0,1,3,4,5,7,8	434541
7-Z38	0,1,2,4,5,7,8	434442
6-Z37(12)	0,1,2,3,4,8	
6-Z38(12)	0,1,2,3,7,8	
6-Z39	0,2,3,4,5,8	
6-Z40	0,1,2,3,5,8	
6-Z41	0,1,2,3,6,8	
6-Z42(12)	0,1,2,3,6,9	

5-Z36	0,1,2,4,7	222121	7-Z36	0,1,2,3,5,6,8	444342

Relation Z

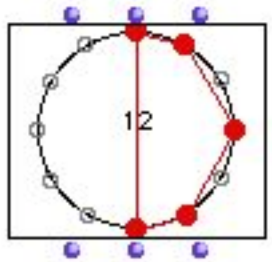
Vecteur d'intervalles et relation Z



5-30	0,1,4,6,8	121321
5-31	0,1,3,6,9	114112
5-32	0,1,4,6,9	113221
5-33(12)	0,2,4,6,8	040402
5-34(12)	0,2,4,6,9	032221
5-35(12)	0,2,4,7,9	032140
5-Z36	0,1,2,4,7	222121
5-Z37(12)	0,3,4,5,8	212320
5-Z38	0,1,2,5,8	212221
6-1(12)	0,1,2,3,4,5	543210
6-2	0,1,2,3,4,6	443211
5-Z36	**0,1,2,4,7**	**222121**
6-Z4(12)	0,1,2,4,5,6	432321
6-5	0,1,2,3,6,7	422232
6-Z6(12)	0,1,2,5,6,7	421242
6-7(6)	0,1,2,6,7,8	420243
6-8(12)	0,2,3,4,5,7	343230
6-9	0,1,2,3,5,7	342231
6-Z10	0,1,3,4,5,7	333321
6-Z11	0,1,2,4,5,7	333231
6-Z12	0,1,2,4,6,7	332232
6-Z13(12)	0,1,3,4,6,7	324222

7-30	0,1,2,4,6,8,9	343542
7-31	0,1,3,4,6,7,9	336333
7-32	0,1,3,4,6,8,9	335442
7-33	0,1,2,4,6,8,10	262623
7-34	0,1,3,4,6,8,10	254442
7-35	0,1,3,5,6,8,10	254361
7-Z36	0,1,2,3,5,6,8	444342
7-Z37	0,1,3,4,5,7,8	434541
7-Z38	0,1,2,4,5,7,8	434442

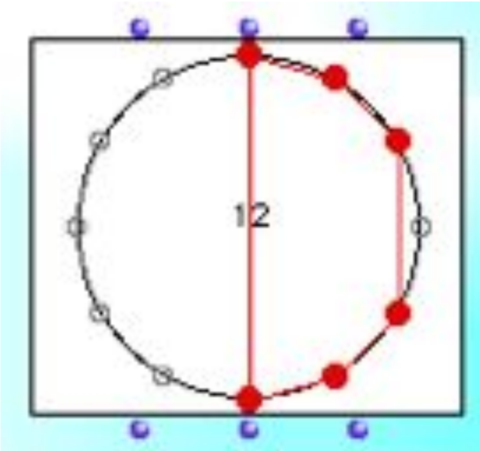
6-Z36	0,1,2,3,4,7
6-Z37(12)	0,1,2,3,4,8
6-Z38(12)	0,1,2,3,7,8
6-Z39	0,2,3,4,5,8
6-Z40	0,1,2,3,5,8
6-Z41	0,1,2,3,6,8
6-Z42(12)	0,1,2,3,6,9



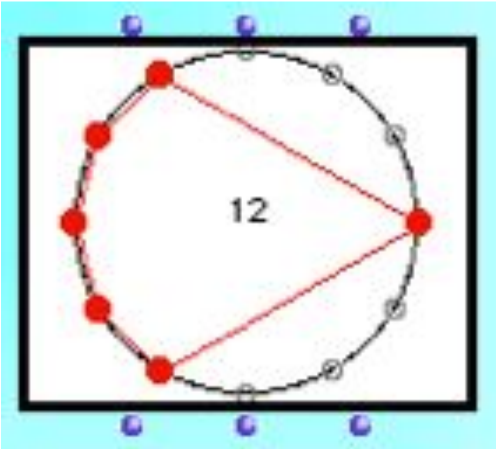
5-Z12

Théorème de l'hexacorde (ou théorème de Babbitt)

(Wilcox, Ralph Fox (?), Chemillier, Lewin, Mazzola, Schaub, …, Amiot [2006])



A



A'

$$\mathrm{IV}(A) = [4, 3, 2, 3, 2, 1] = [4, 3, 2, 3, 2, 1] = \mathrm{IV}(A')$$

Un hexacorde et son complementaire ont le même vecteur d'intervalles

Relation Z en musique et théorie de l'homométrie

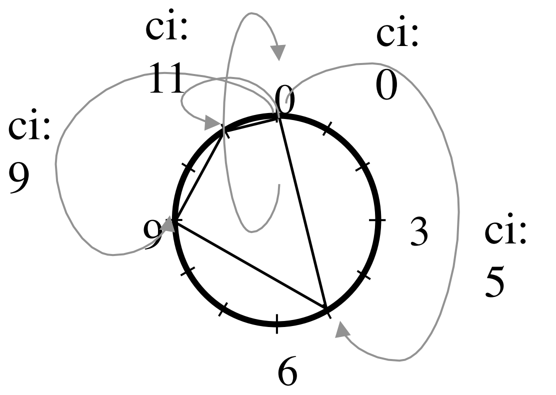
- Le contenu intervallique est équivalent à un produit de convolution de fonctions caractéristiques (Lewin, 1958)

$$\mathcal{F}_A : t \mapsto \sum_{k \in A} e^{-2i\pi kt/c}$$

$$IC_A(k) = (1_A \star 1_{-A})(k)$$

$$1_A * \tilde{1}_B(k) = \sum_i 1_A(i) \times 1_B(i-k) = \sum_{\substack{i \in A \\ i-k \in B}} 1$$

$$\mathcal{F}(1_A * \tilde{1}_B) = \mathcal{F}(1_A) \times \mathcal{F}(\tilde{1}_B)$$



$A = \{0, 5, 9, 11\} \Rightarrow IC_A(k) = 1 \ \forall k$

$$\mathcal{F}(IC_A) = \mathcal{F}_A \times \mathcal{F}_{-A} = |\mathcal{F}_A|^2$$ ➔ **Relation Z**

$$\forall k \ \mathcal{F}(IC_{\mathbb{Z}_c \setminus A})(k) = \mathcal{F}(IC_A)(k)$$ ➔ **Théorème de l'hexacorde**

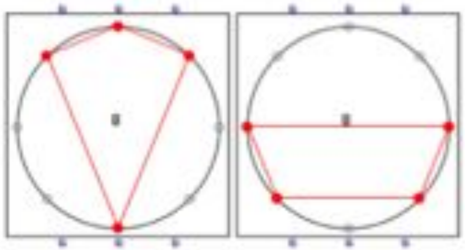
A  *A'* ➔ $IC_A = IC_{A'}$

FIGURE 1.4 – Plus petite Z-relation non triviale

- P. Beauguitte, *Transformée de Fourier discrète et structures musicales*, Master ATIAM, 2011

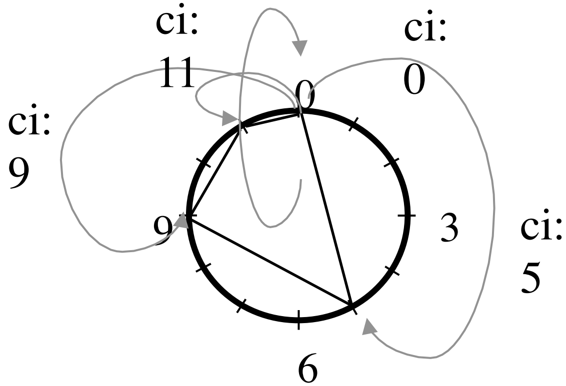
Z-relation, homométrie et reconstruction de la phase

- Deux ensembles sont en Z-relations s'ils ont le même module de la DFT

$$IC_A(k) = (1_A \star 1_{-A})(k)$$

$$\mathcal{F}_A : t \mapsto \sum_{k \in A} e^{-2i\pi kt/c}$$

$$\mathcal{F}(IC_A) = \mathcal{F}_A \times \mathcal{F}_{-A} = |\mathcal{F}_A|^2$$



A = {0, 5, 9, 11}

$IC_A\,(k) = 1\ \forall\ k = 1\ldots 11$

{0, 1, 4, 6}$_{12}$

iv= [4, 1, 1, 1, 1, 1, 2, 1, 1, 1, 1, 1]

{0, 1, 3, 7}$_{12}$

• Mandereau J., D. Ghisi, E. Amiot, M. Andreatta, C. Agon, (2011a), « Z-relation and homometry in musical distributions », *Journal of Mathematics and Music*, vol. 5, n° 2, p. 83-98.
• Mandereau J, D. Ghisi, E. Amiot, M. Andreatta, C. Agon (2011b), « Discrete phase retrieval in musical structures », *Journal of Mathematics and Music*, vol. 5, n° 2, p. 99-116.

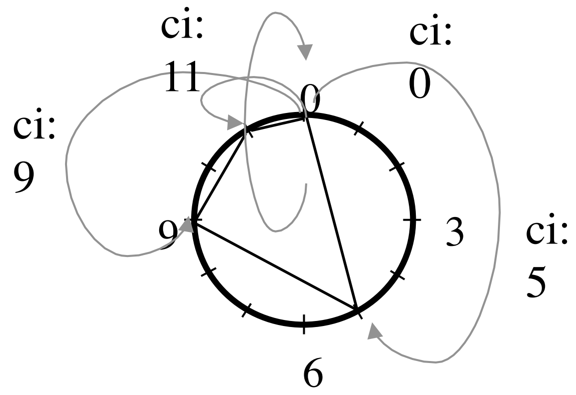
Z-relation, homométrie et reconstruction de la phase

- Deux ensembles sont en Z-relations s'ils ont le même module de la DFT

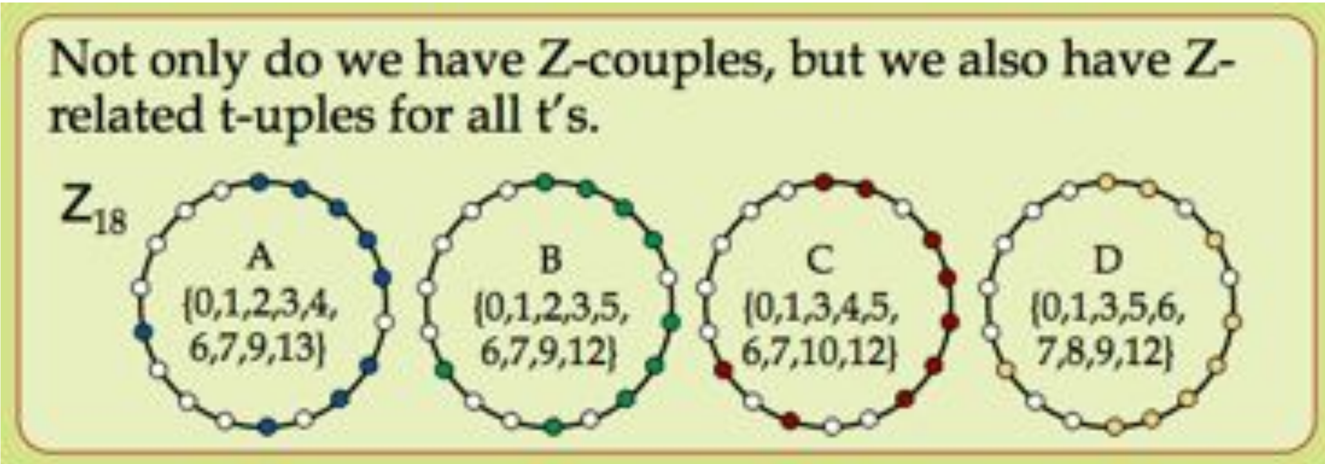
$$IC_A(k) = (1_A \star 1_{-A})(k)$$

$$\mathcal{F}_A : t \mapsto \sum_{k \in A} e^{-2i\pi kt/c}$$

$$\mathcal{F}(IC_A) = \mathcal{F}_A \times \mathcal{F}_{-A} = |\mathcal{F}_A|^2$$



A = {0, 5, 9, 11}
$IC_A(k) = 1 \; \forall \; k = 1 \ldots 11$



• Daniele Ghisi, "From Z-relation to homometry: an introduction and some musical examples", Sémnaire MaMuX, 10 décembre 2010 [http://repmus.ircam.fr/mamux/saisons/saison10-2010-2011/2010-12-10]

Z-relation, homométrie et reconstruction de la phase

- Contenu intervallaire et Fonction de Patterson

$$IC_A(k) = (1_A \star 1_{-A})(k)$$

Expression polynomiale :

$$D(x) = 1 + x + x^2 + x^5$$

Fonction de Patterson (ou d'autocorrelation):

$$D(x)D(x^{-1}) =$$

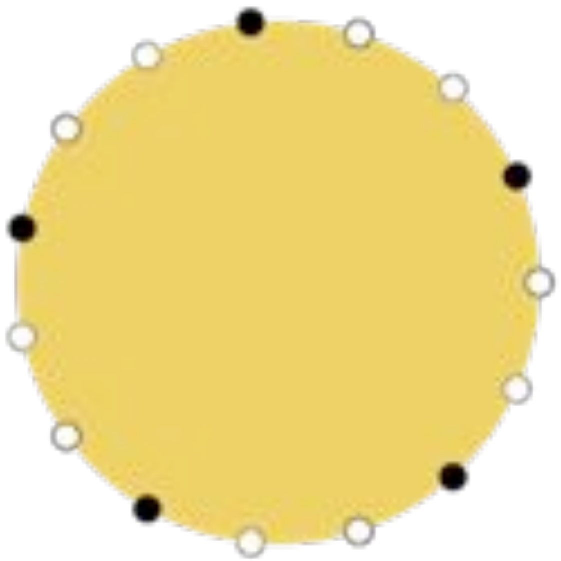
$$= (1 + x + x^2 + x^5)(1 + x^{11} + x^{10} + x^7)$$

$$= 1 + x + x^2 + x^5 + x^7 + x^8 + x^9 + x^{10} + 2x^{11} + 3x^{12} + x^{13} + x^{15} + x^{16}$$

$$= 4 + 2x + x^2 + x^3 + x^4 + x^5 + x^7 + x^8 + x^9 + x^{10} + 2x^{11}$$

• Daniele Ghisi, "From Z-relation to homometry: an introduction and some musical examples", Sémnaire MaMuX, 10 décembre 2010 [http://repmus.ircam.fr/mamux/saisons/saison10-2010-2011/2010-12-10]

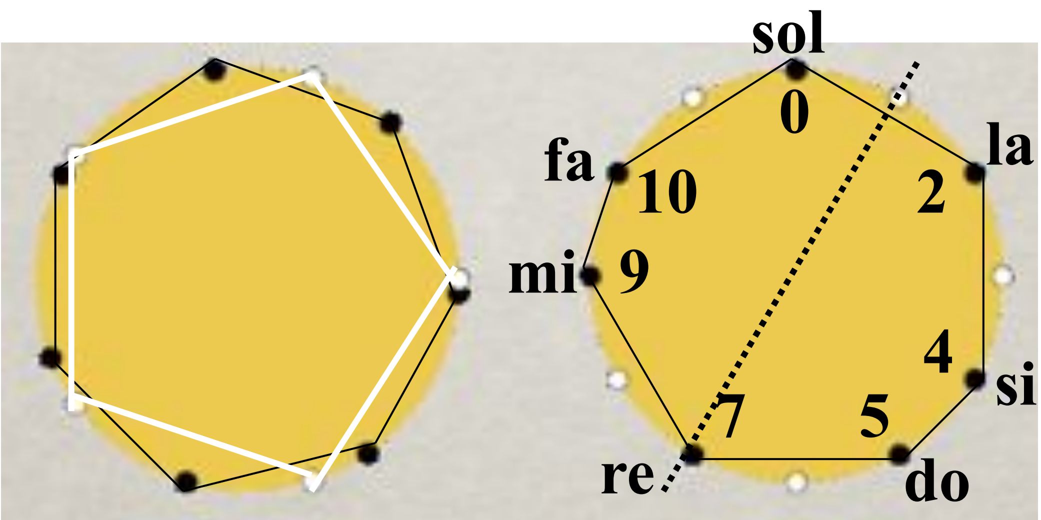
Maximally-Even Sets (ME–sets)



$$\mathcal{F}_A : t \mapsto \sum_{k \in A} e^{-2i\pi kt/c}$$

$|F_A(5)| = 1+1+1+1+1 = 5$

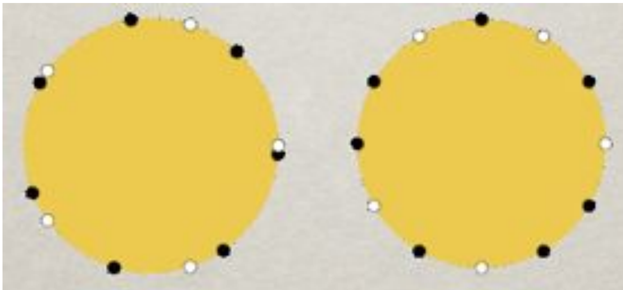
En général, $| F_A(t) | \leq \#A$

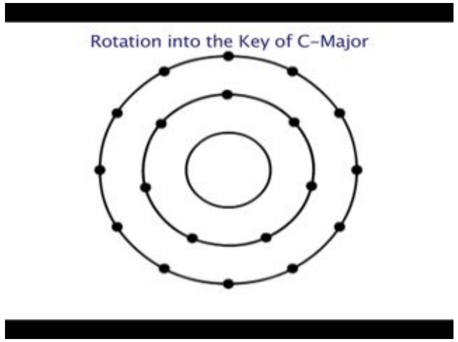


Gamme diatonique:
{0, 2, 4, 5, 7, 9, 10}

Gamme pentatonique:
{1, 3, 6, 8, 11}

Nouvelle définition de ME sets

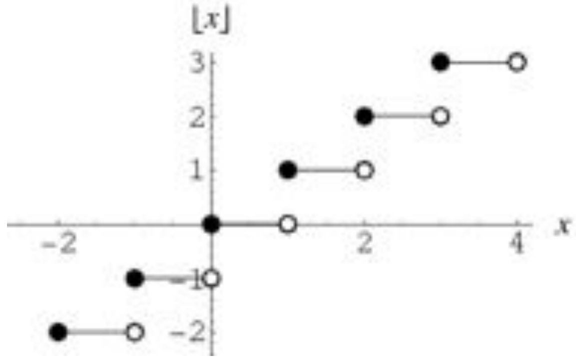




Definition (Clough-Myerson-Douthett) A set A with cardinality d in a given equal tempered space $\mathbf{Z}_c$ is maximally even if $A = \{a_k\}$

$$a_k = J^{\alpha}_{c,d}(k) = \left\lfloor \frac{kc+\alpha}{d} \right\rfloor$$

where $\alpha \in \mathbf{R}$
$\lfloor x \rfloor$ is the integer part of x



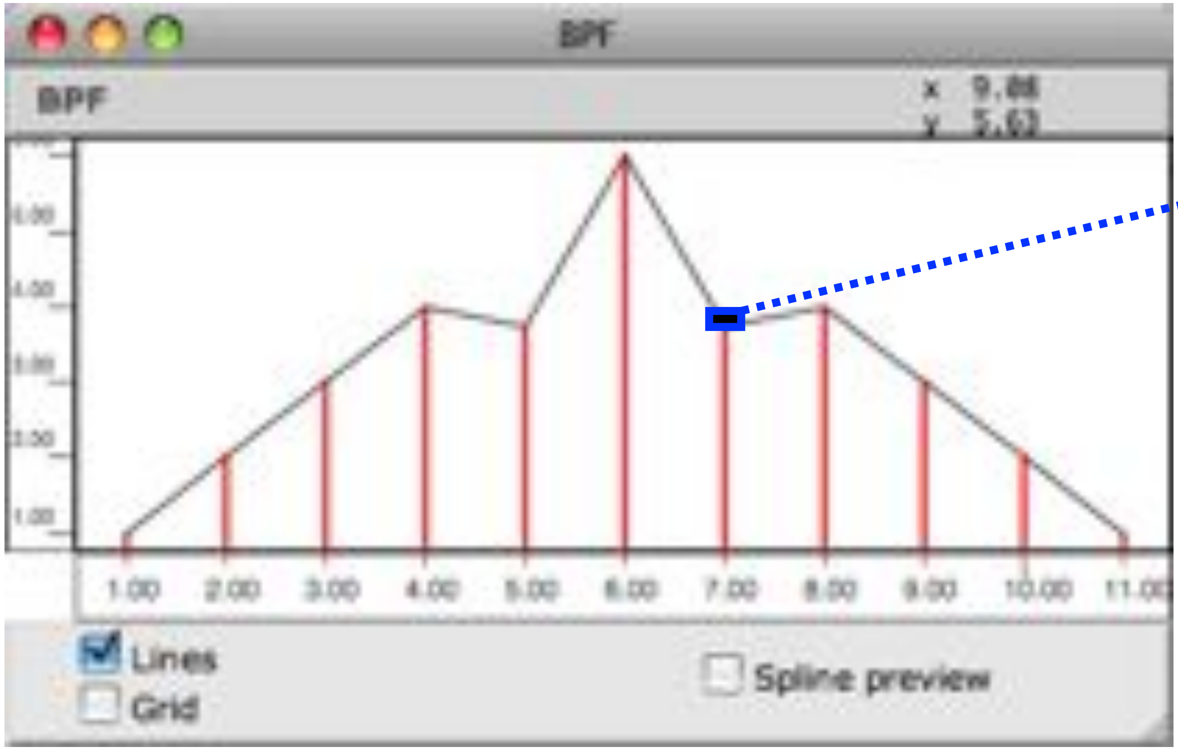
$$J^{5}_{12,7} = \left\{ \left\lfloor \frac{12k+5}{7} \right\rfloor \right\}_{k=0}^{6} = \{0,2,4,5,7,9,11\}$$

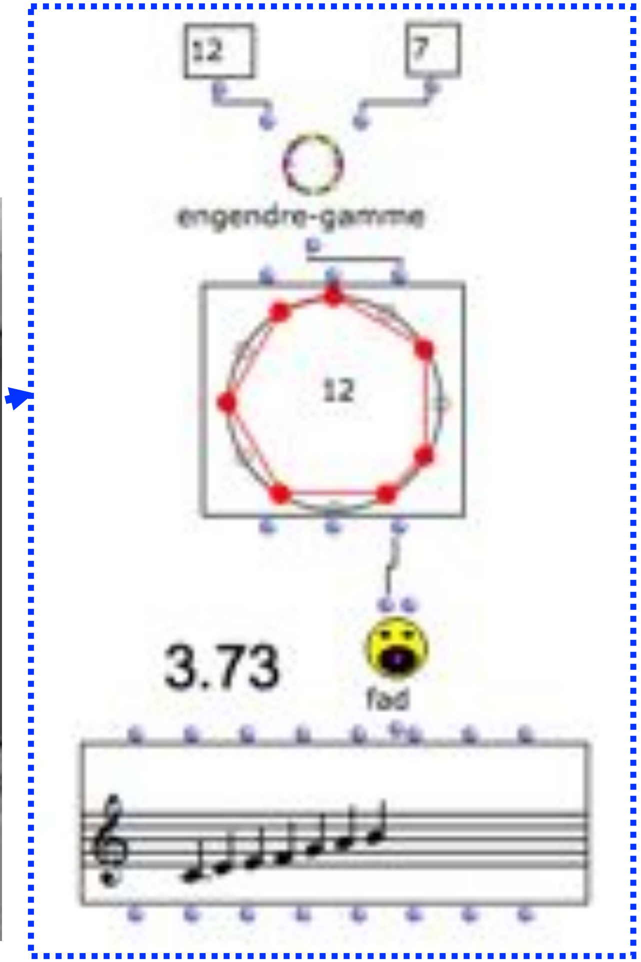
Definition (Amiot, 2005) A set A with cardinality d given equal tempered space $\mathbf{Z}_c$ is maximally even if $|F_A(d)| \geq |F_B(d)|$ for all subsets B of cardinality d in $\mathbf{Z}_c$.

where $F_{set}(t) : \sum_{k \in set} e^{2i\pi kt/12}$

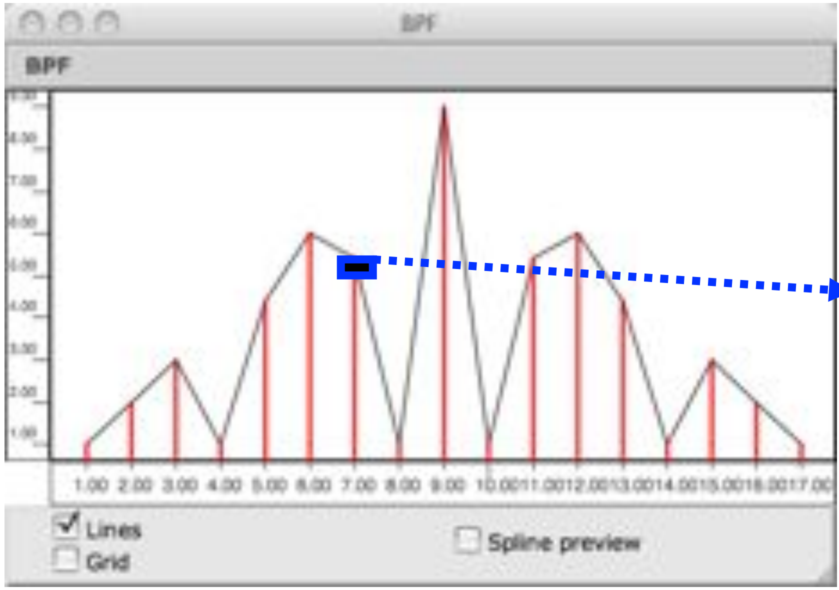
Nouvelle classification des structures musicales à l'aide de la DFT

$$\mathcal{F}_A : t \mapsto \sum_{k \in A} e^{-2i\pi kt/c}$$

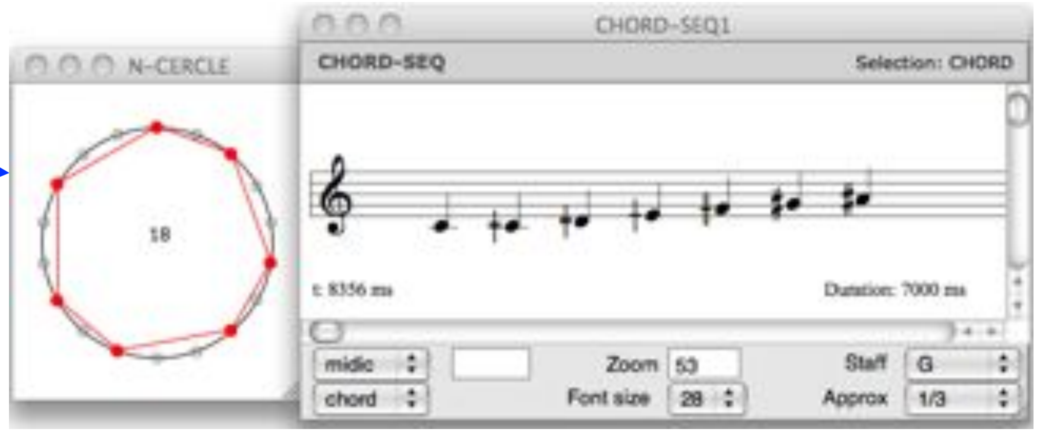


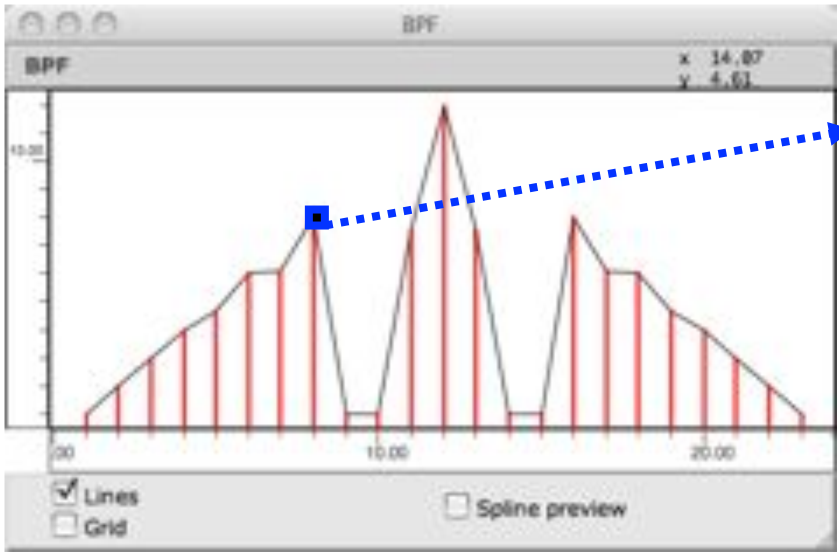


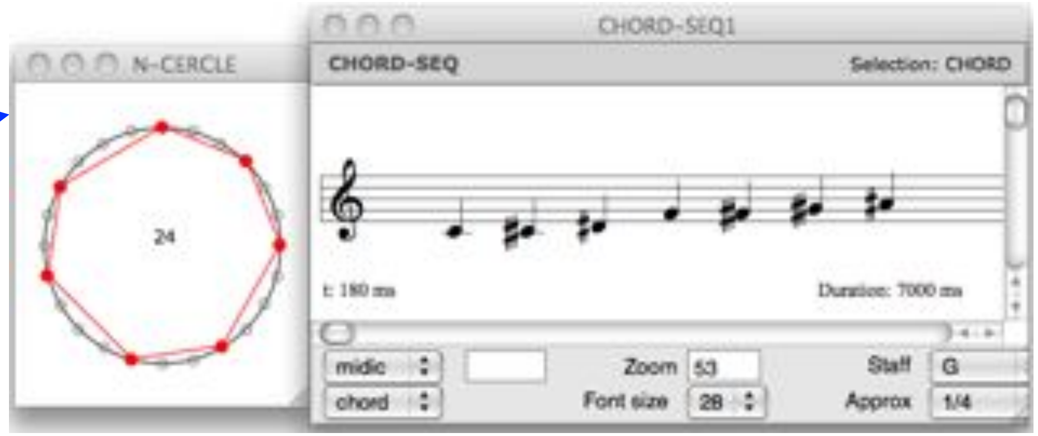
Classification des structures microtonales à l'aide de la DFT



Système en tiers de ton







Système en quarts de ton

The one-dimensional antiferromagnetic spin-1/2 Ising Model

Jack Douthett &Richard Krantz, “Energy extremes and spin configurations for the one-dimensional antiferromagnetic Ising model with arbitrary-range interaction”, *J. Math. Phys.* 37 (7), July 1996