



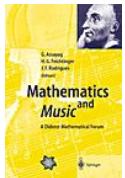
Un survol sur quelques modèles algébriques en théorie, analyse et composition assistées par ordinateur

Moreno Andreatta
Equipe Représentations Musicales
IRCAM/CNRS UMR 9912



Mathématiques/Musique...une histoire récente!

- 1999 : 4^e Forum Diderot (Paris, Vienne, Lisbonne), *Mathematics and Music* (G. Assayag, H.G. Feichtinger, J.F. Rodrigues, Springer, 2001)



- 2000-2001 : Séminaire *MaMuPhi*, *Penser la musique avec les mathématiques ?* (Assayag, Mazzola, Nicolas éds., Coll. « M/S », Ircam/Delatour, 2006)



- 2000-2003 : International Seminar on *MaMuTh* (*Perspectives in Mathematical and Computational Music Theory*) (Mazzola, Noll, Luis-Puebla eds, epOs, 2004)



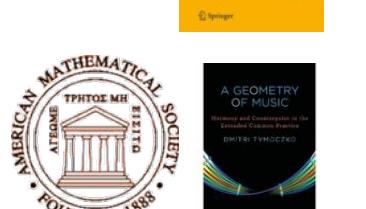
- 2003 : *The Topos of Music* (G. Mazzola et al.)



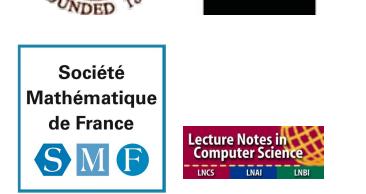
- 2003: *Music and Mathematics. From Pythagoras to Fractals* (J. Fauvel et al.)



- 2001 - 2011 : Séminaire *MaMuX* de l'Ircam



- 2004 - 2011 : Séminaire *mamuphi* (Ens/Ircam)



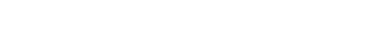
- 2006 : *Mathematical Theory of Music* (F. Jedrzejewski), Coll. « M/S »



- 2007 : *La vérité du beau dans la musique* (G Mazzola), Coll. « Musique/Sciences »



- 2007 : *Journal of Mathematics and Music* (Taylor & Francis) et *MCM 2007*



- 2007: *Music. A Mathematical Offering* (Dave Benson), CUP



- 2008: *Music Theory and Mathematics* (Jack Douthett et al.), URP

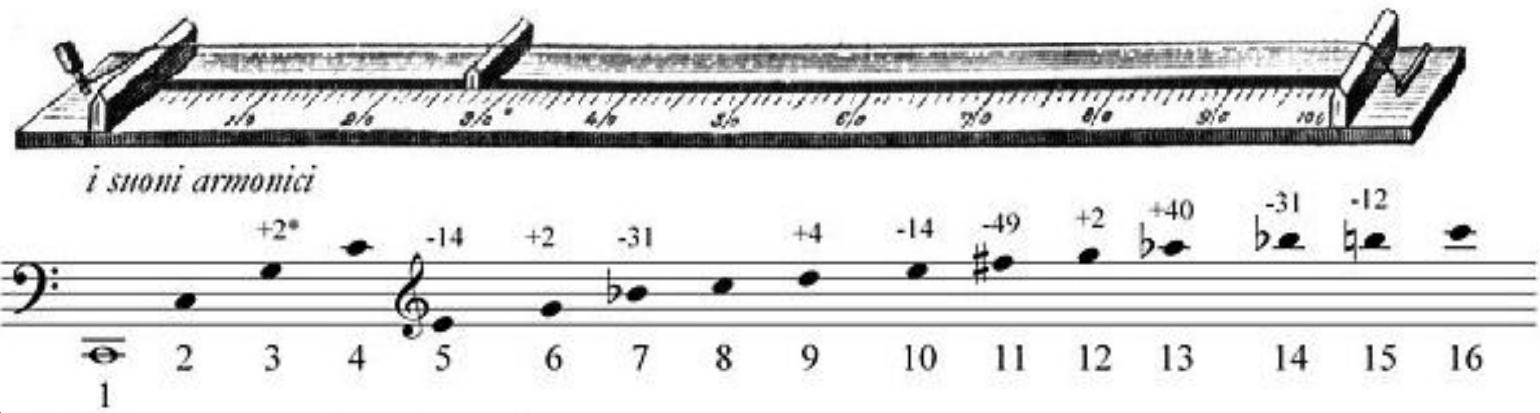
- 2009 : *Computational Music Science Series* (Springer)

- 2009 : *MCM 2009* (Yale) et Proceedings chez Springer

- 2010 : Mathematics Subject Classification : 00A65 Mathematics and music

- 2011 : Conférence de la SMCM (Ircam, 15-17 juin 2011)

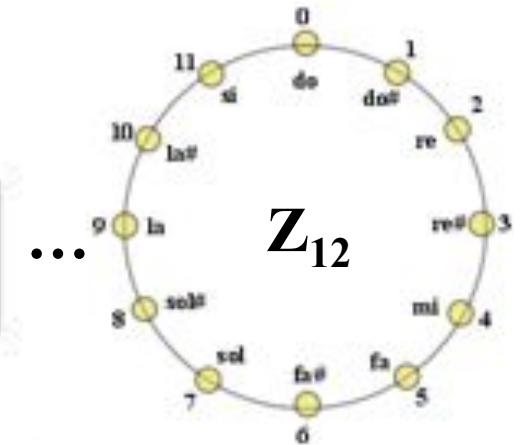
De Pythagore... à la théorie des groupes



Physique

* in cents, confrontati con la scala temperata

Mathématiques

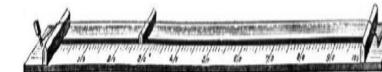


A musical staff with two staves. The top staff uses a treble clef and the bottom staff uses a bass clef. Notes are placed on the staff and labeled with their note names in French and English. The notes are: do, do#, ré, ré#, mi, fa, fa#, sol, sol#, la, la#, si, réb, mi_b, sol_b, la_b, si_b.

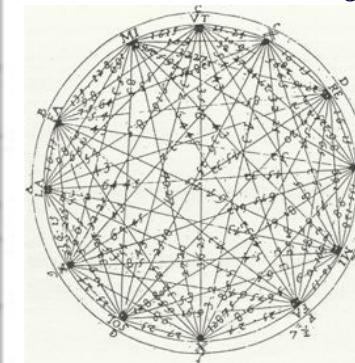
Musique et mathématiques : deux destinées parallèles

MUSIQUE	MATHS
500 av. J. C. Relation hauteur/longueur corde. La musique est source d'inspiration pour la théorie des nombres et la géométrie.	Nombres naturels et rationnels
300 a.J. Invention (théorique) de la gamme chromatique tempérée égale par Aristoxène de Tarente) et prémonition de la théorie des groupes. Isomorphismes entre les logarithmes (intervalles musicaux) et les exponentiels (longueur d'une corde)	Aucune relation.
1000 Invention de la représentation bidimensionnelle des hauteurs	Aucune correspondance
1500 Aucune reprise des concepts précédents	Nombres négatifs. Construction des rationnels
1600 Aucune relation	Nombres réels et les logarithmes
Marin Mersenne (1588-1648) : combinatoire musicale	Calcul des probabilités
1700 La fugue comme un automate abstrait. Manipulations inconscientes du groupe de Klein	Nombres complexes (Euler, Gauss), les quaternions (Hamilton), continuité (Cauchy), structure de groupe (Galois, Abel)
Leonhard Euler : <i>Speculum Musicum</i> (1773)	Théorie des graphes
1900 Libération de la prison de la tonalité (Loquin, Hauer, Schoenberg)	Nombres infinis et transfinis (Cantor). Axiomatique de Peano. Théorie de la mesure (Lebesgue, Borel)
1920 Formalisation radicale des macrostructures à travers le système sériel (Schoenberg)	Aucun développement de la théorie des nombres.
Ernst Krenek (1900-1991) : les axiomes dans le système dodécaphonique	David Hilbert, <i>Les fondements de la géométrie</i> (1899)

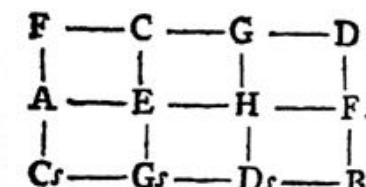
Iannis Xenakis, *Musique. Architecture*, Tournai, Casterman, 1971, (New, revised edition: Tournai, Casterman, 1976, 238 p.)



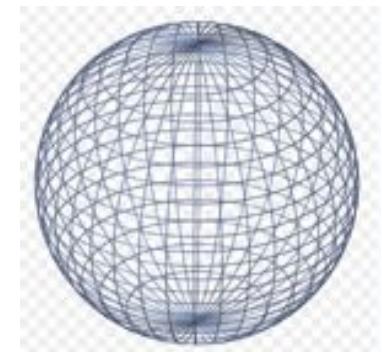
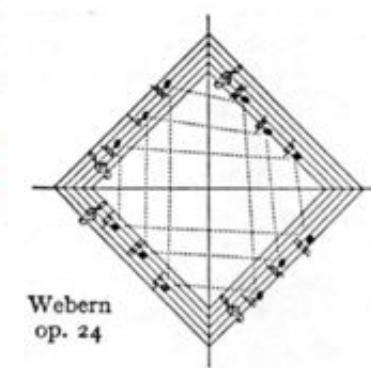
Pythagore et le monocorde, VI^e-V^e siècle av. J. C.



Mersenne,
Harmonicorum Libri XII, 1648



Euler : *Speculum musicum*, 1773



Double mouvement d'une dynamique mathémusicale

[Cf. M. Andreatta : *Mathematica est exercitium musicae*, HDR, octobre 2010]

MATHEMATIQUES

énoncé
mathématique

généralisation

théorème
général

formalisation

application

problème
musical

théorie
musicale

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composition

[Cf. M. Andreatta,
PhD, 2003]

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énoncé
mathématique

généralisation

théorème
général

formalisation



OpenMusic

application

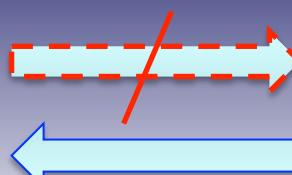
problème
musical

analyse
musicale

théorie
musicale

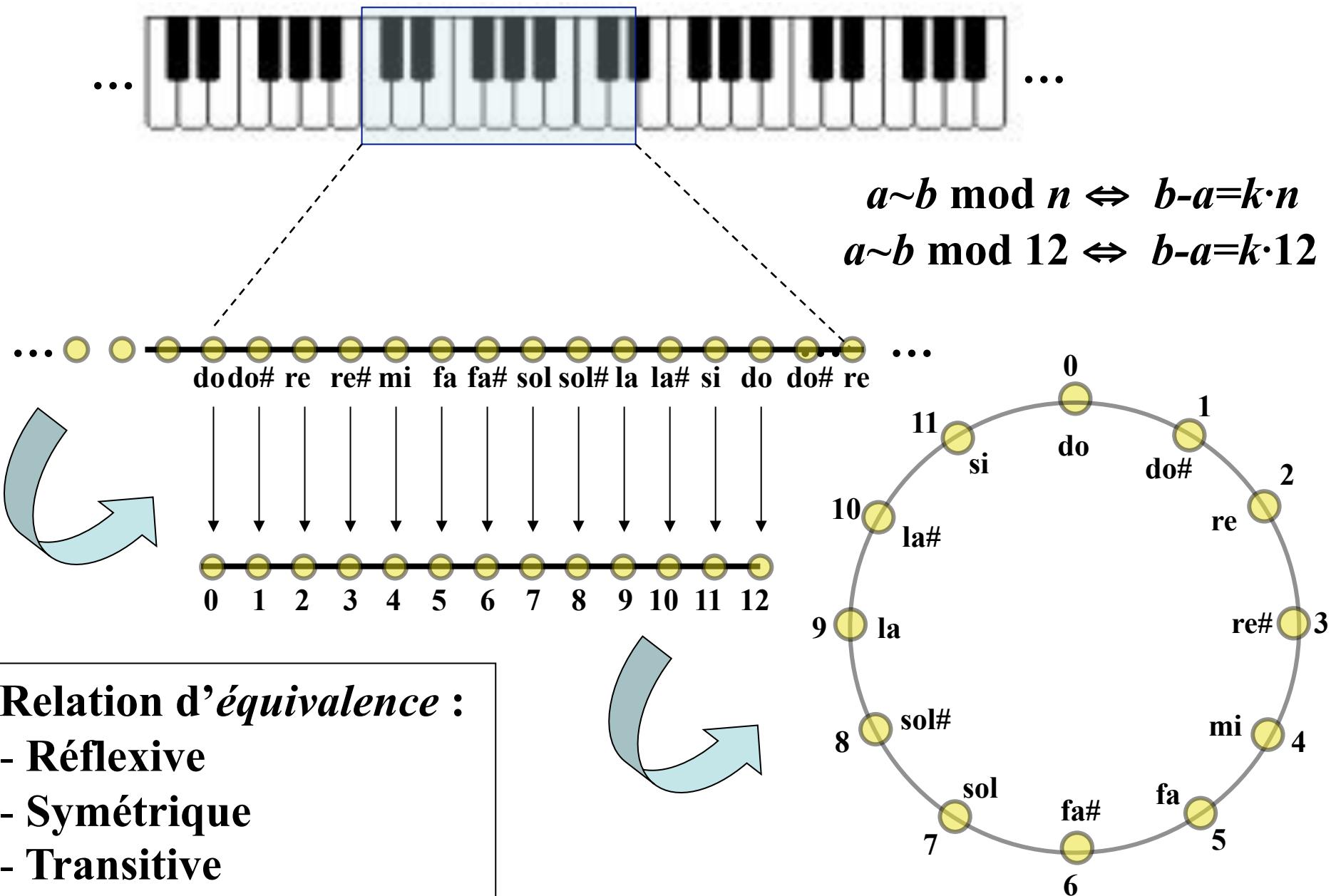
MUSIQUE

composition



[Cf. M. Andreatta,
PhD, 2003]

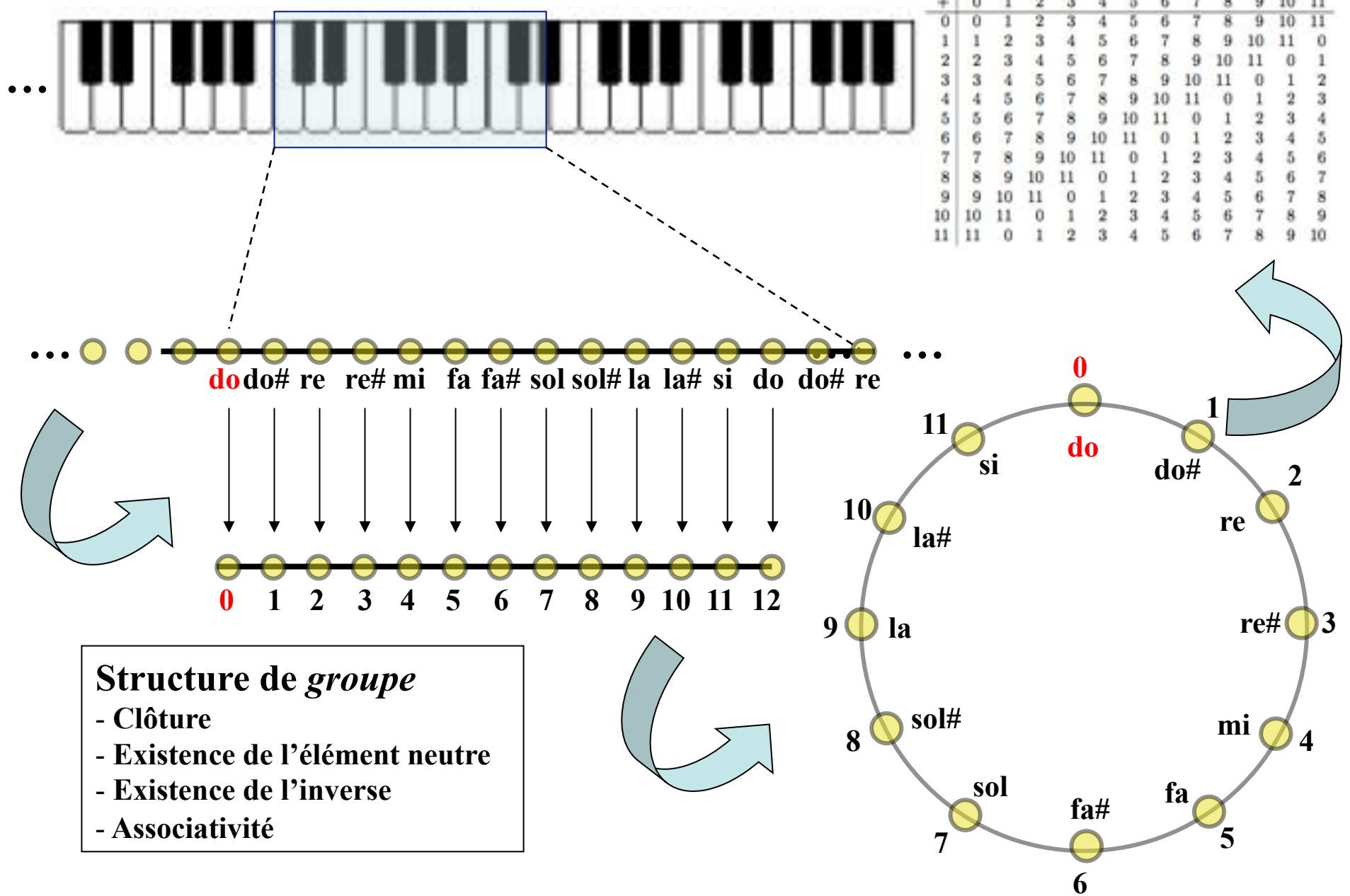
Réduction à l'octave et congruence modulo 12



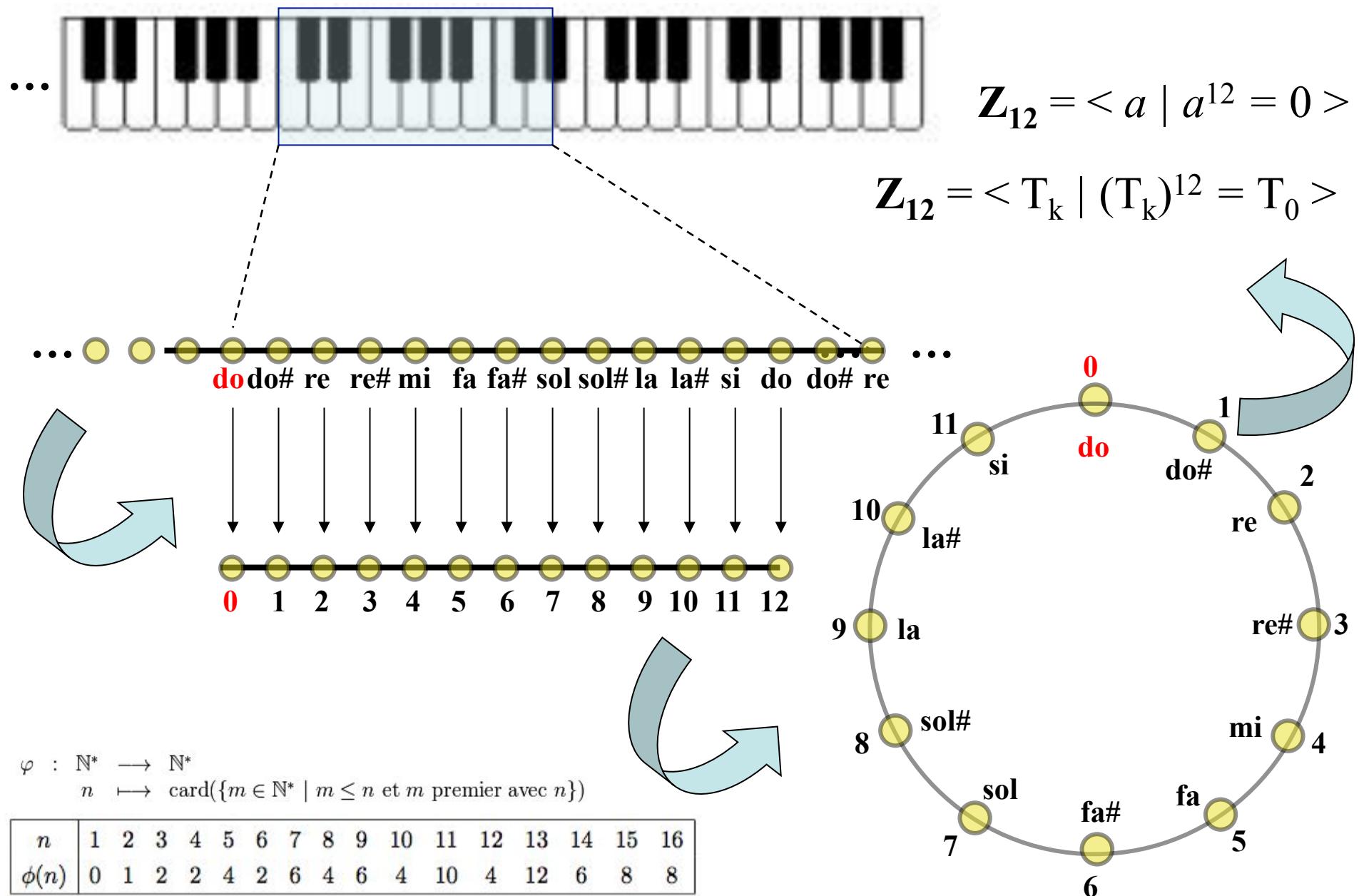
Relation d'équivalence :

- Réflexive
- Symétrique
- Transitive

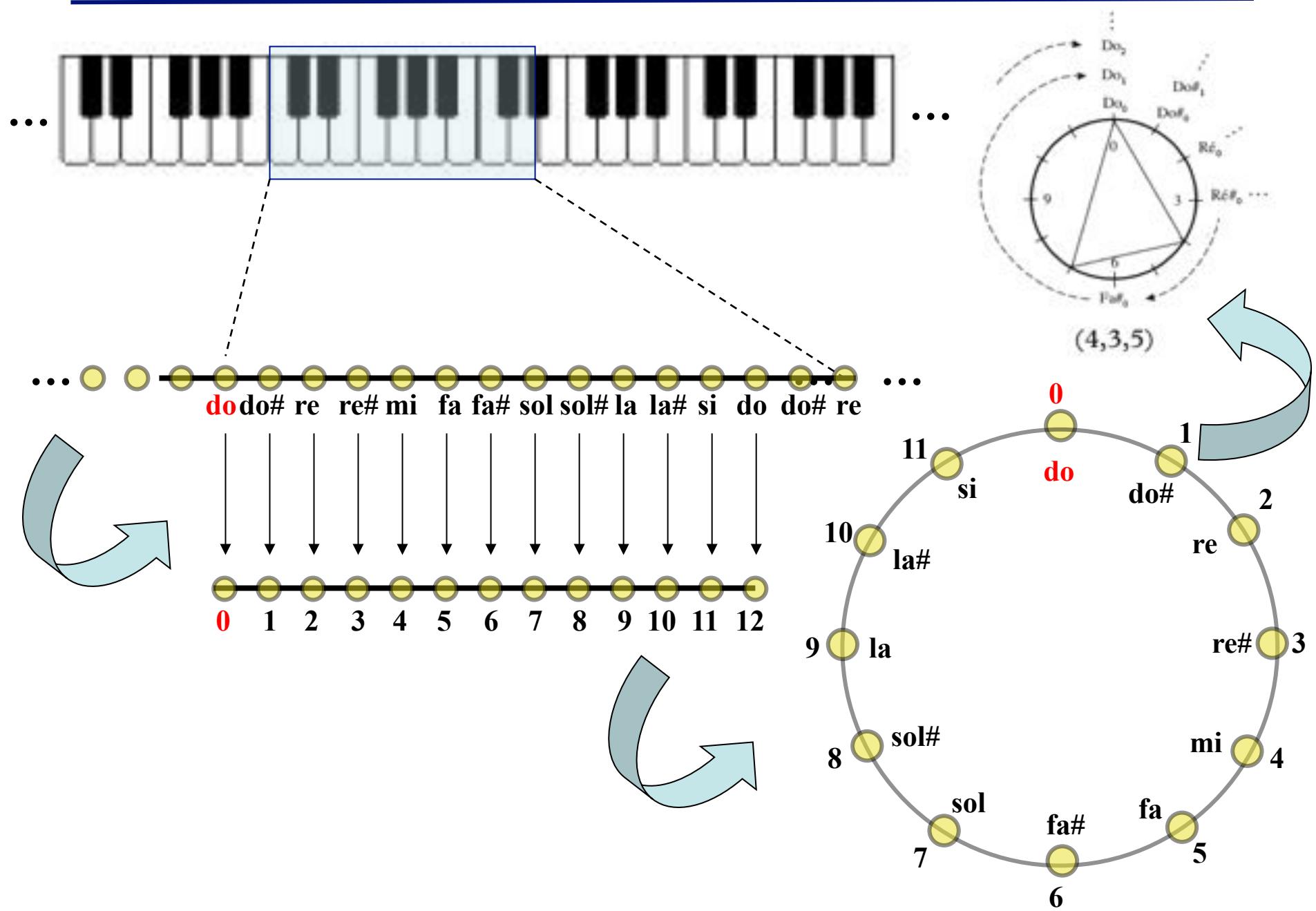
Congruence modulo 12 et structure de groupe (cyclique)



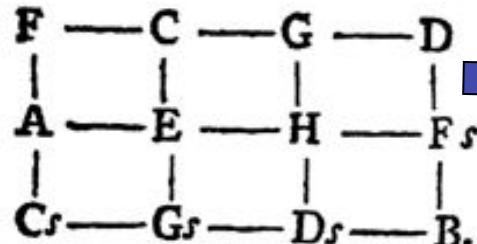
Ecriture par générateurs et relations



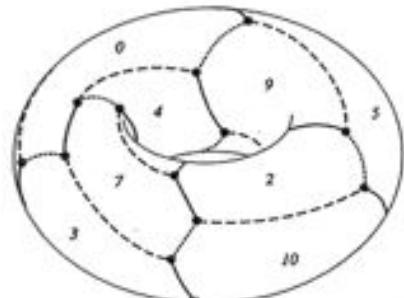
Premiers invariants algébriques : la structure intervallique



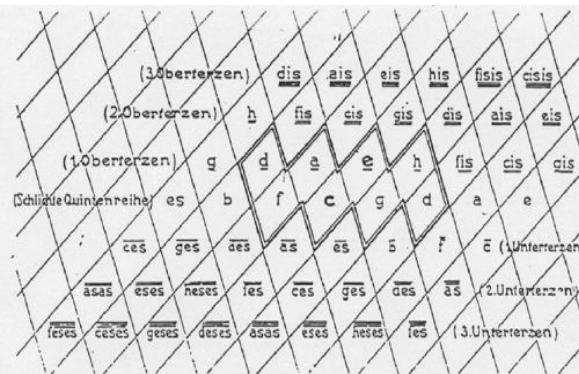
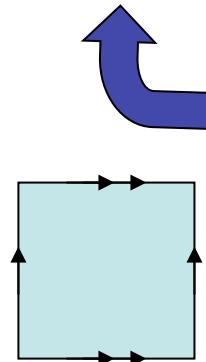
Du *Speculum musicum* d'Euler au Tonnetz de Riemann



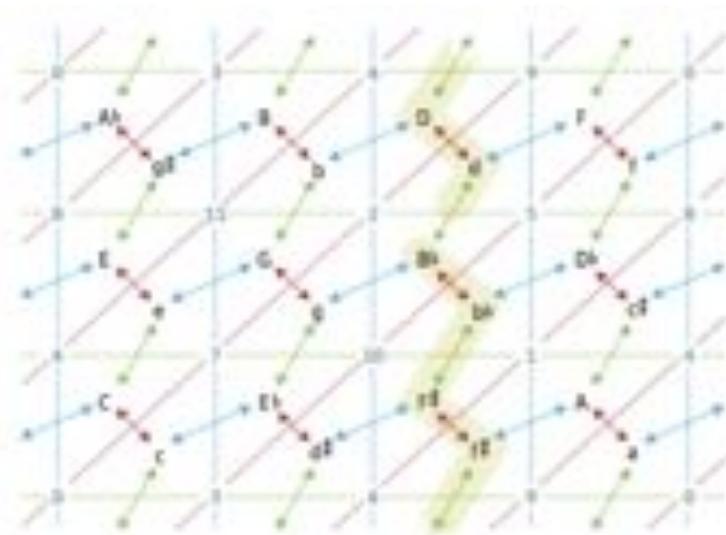
Euler : *Speculum musicum*, 1773



Douthett & Steinbach,
JMT, 1998



Hugo Riemann : « Ideen zu einer Lehre von den Tonvorstellung », 1914



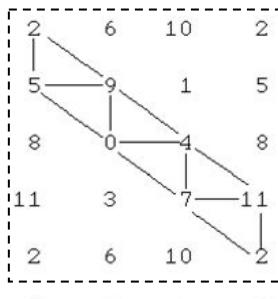
J. Hook, « Exploring Musical Space »,
Science, 2006

A	C*	F	A'	C#'	F'	A''	C#''	F''	A'''
B	E*	A#	D'	E#'	A#'	D''	E#''	A#''	D'''
G	B	D#	G'	B'	D#'	G''	B''	D#''	G'''
C	E	G#	C'	E'	G#'	C''	E''	G#''	C'''
F	A	C#	F'	A'	C#'	F''	A''	C#''	F'''
Bb	D	F*	B#'	D'	F#'	B#''	D''	F#''	B#'''
Eb	G	B	E#'	G'	B#'	E#''	G''	B#''	E#'''
Ab	C	E	A#'	C'	E#'	A#''	C''	E''	A#'''

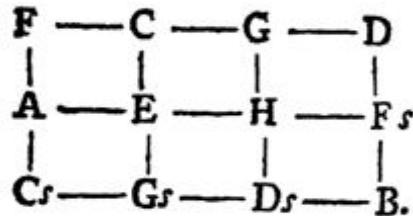
Longuet-Higgins (1962)

m3	0	8	0	4	8
M3 → 0	4	8	0	4	8
M3 → 3	7	11	3	7	11
M3 → 6	10	2	6	10	2
M3 → 9	1	5	9	1	5
M3 → 0	4	8	0	4	8
M3 → 3	7	11	3	7	11
M3 → 6	10	2	6	10	2
M3 → 9	1	5	9	1	5

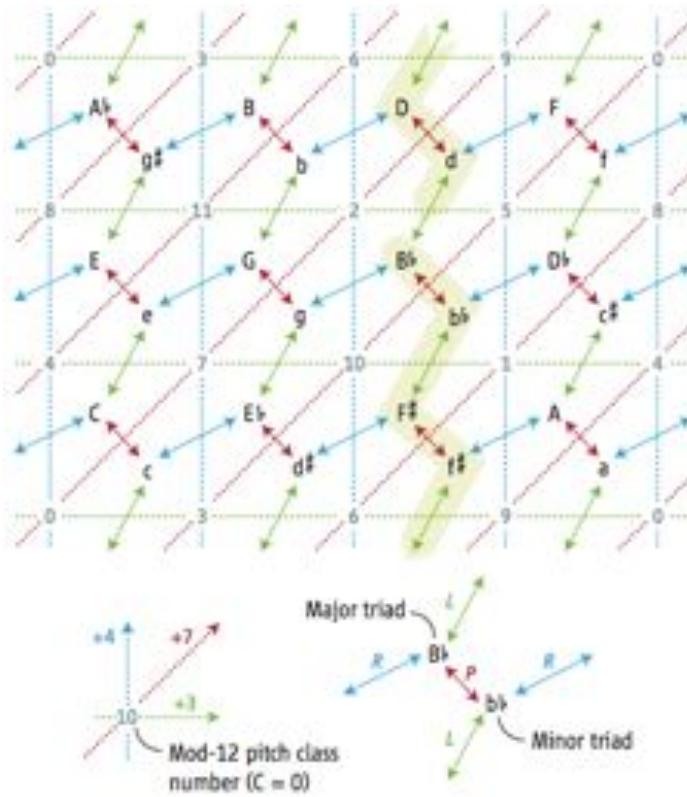
Balzano (1980)



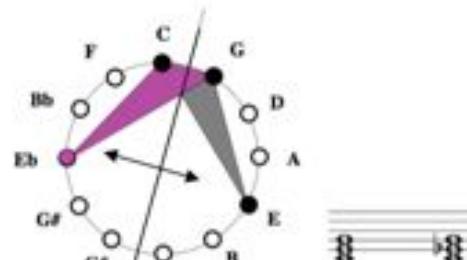
Du réseau d'Euler au Tonnetz



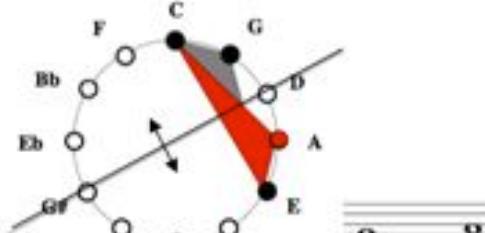
Euler : *Speculum musicum*, 1773



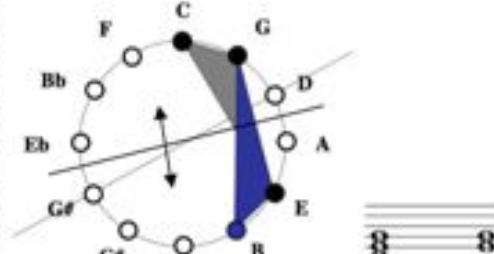
(Neo-)Riemannian Operation P = „Parallel“



(Neo-)Riemannian Operation R = „Relative“



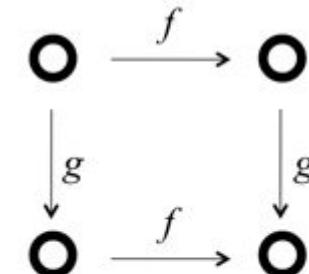
(Neo-)Riemannian Operation L = „Leading-Tone“



$$\rho = \langle L, R \mid L^2 = (LR)^{12} = 1 \\ LRL = L(LR)^{-1} \rangle$$

$$\rho \subseteq C_{\text{Sym}}(D_{12}) \\ D_{12} \subseteq C_{\text{Sym}}(\rho)$$

$$D_{12} = \langle I, T \mid I^2 = T^{12} = 1 \\ ITI = I(IT)^{-1} \rangle$$

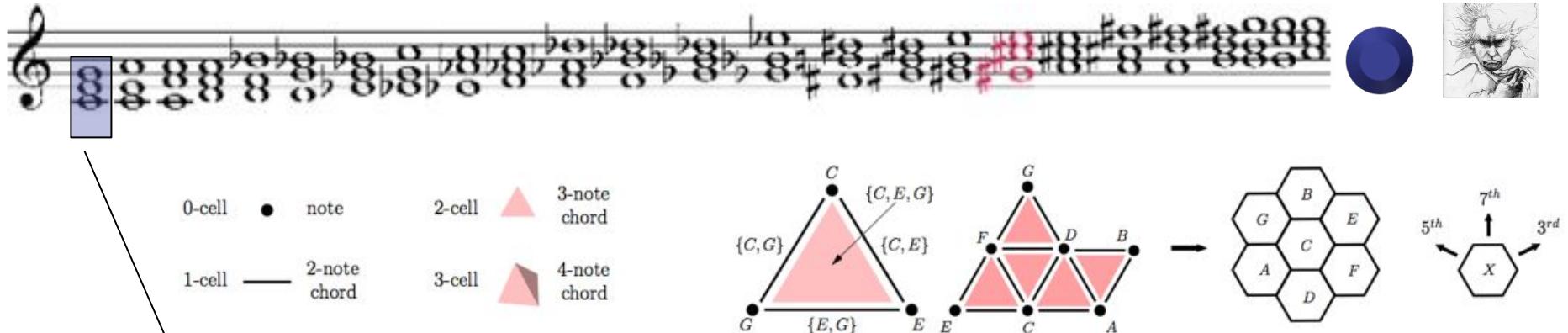


Tout diagramme commute

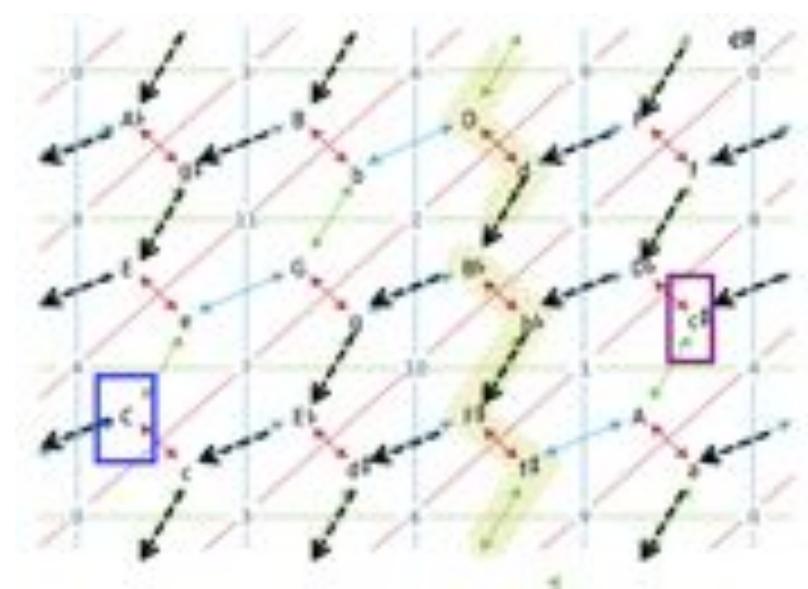
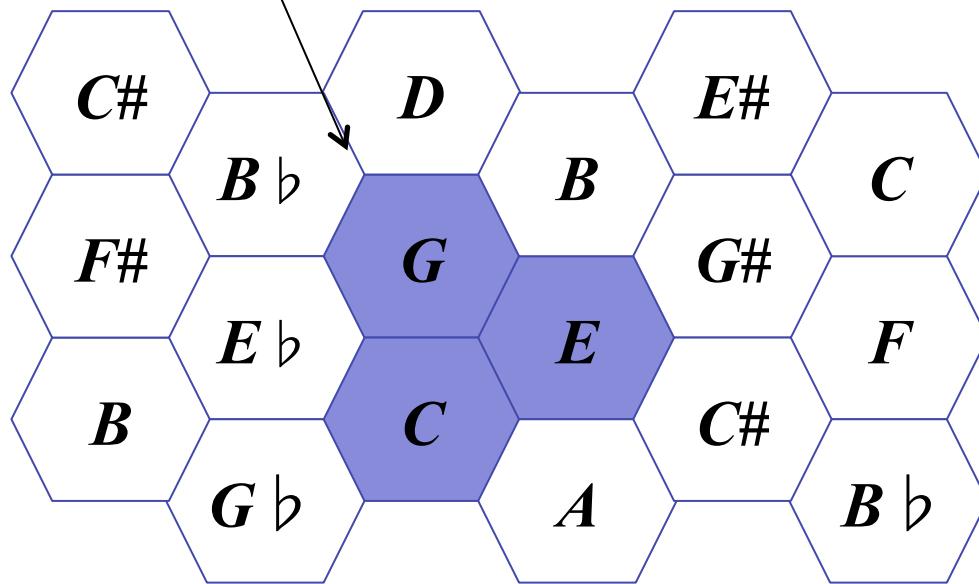
$$\forall f \in D_{12} \\ \forall g \in \rho$$

Crans A., Fiore T., and Satyendra R. "Musical Actions of Dihedral Groups." *The American Mathematical Monthly*, Vol. 116 (2009), No. 6: 479 – 495
(Winner of the The Mathematical Association of America's Merten M. Hasse Prize 2011)

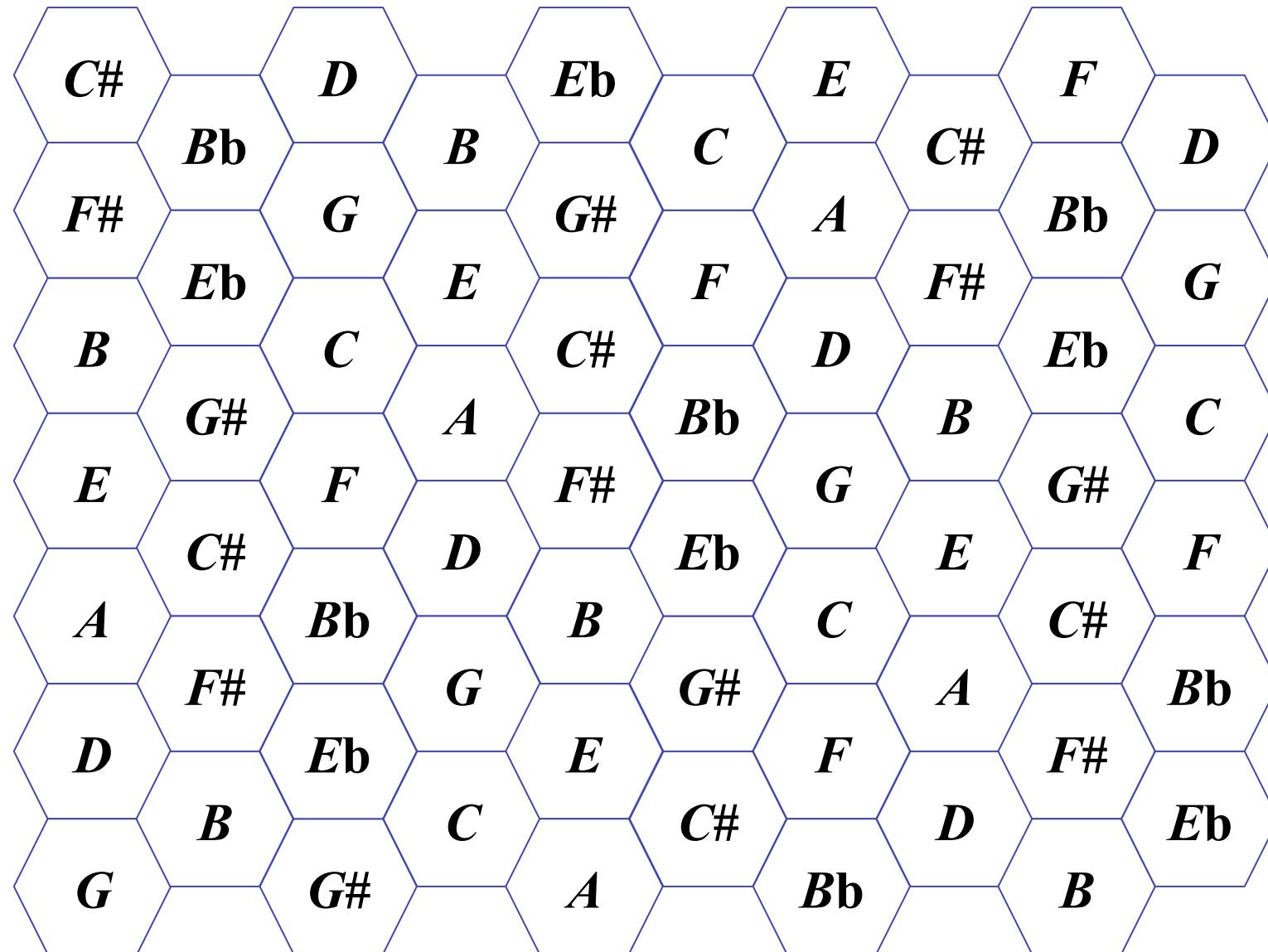
Du Tonnetz à la programmation spatiale



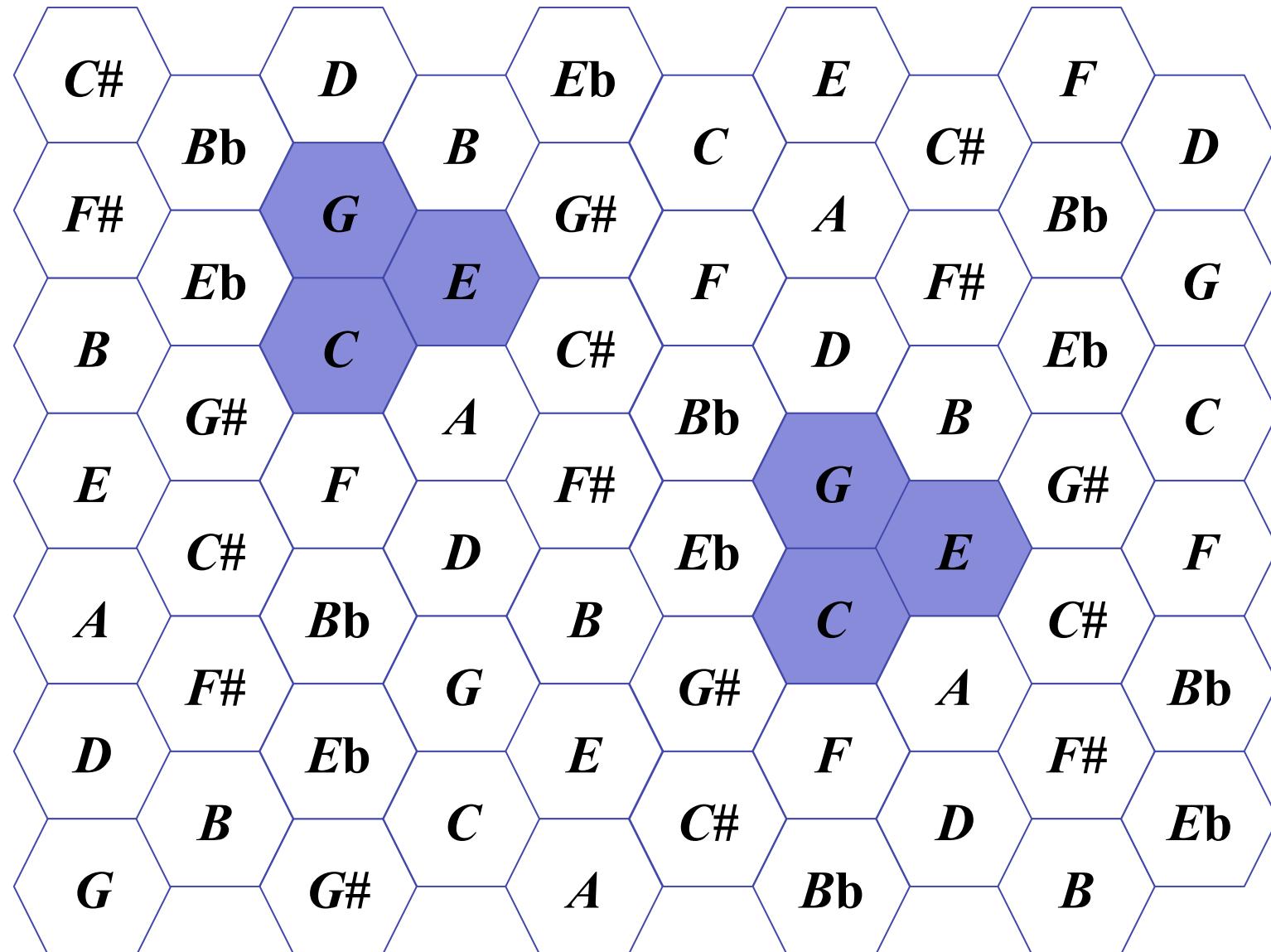
L. Bigo, J.-L. Giavitto, A. Spicher, "Building Topological Spaces for Musical Objects", MCM 2011
 L. Bigo, thèse (en cours), Ircam / Université Paris 12



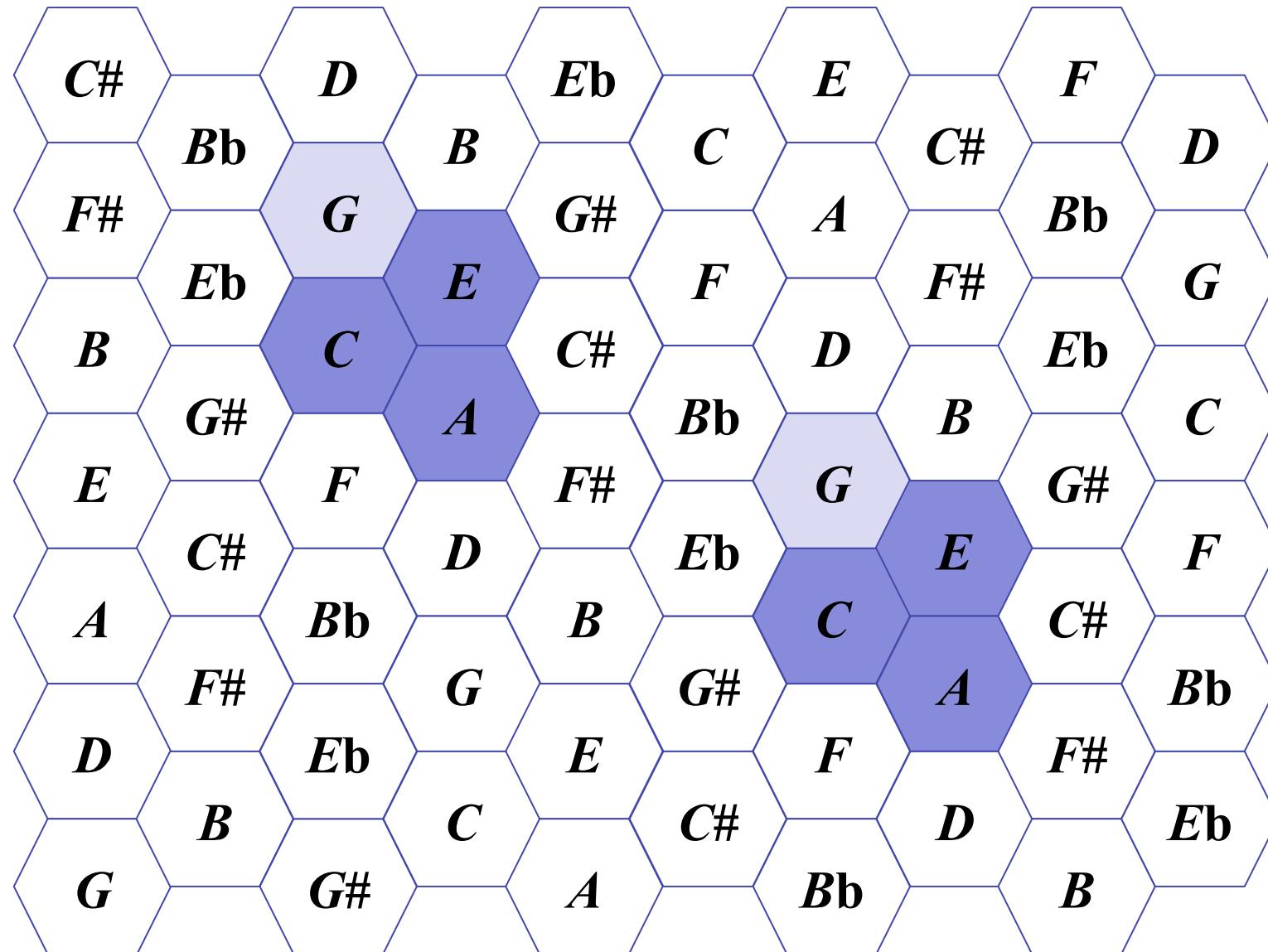
Extract of the 2nd movement of the Symphony No. 9 (L. van Beethoven)



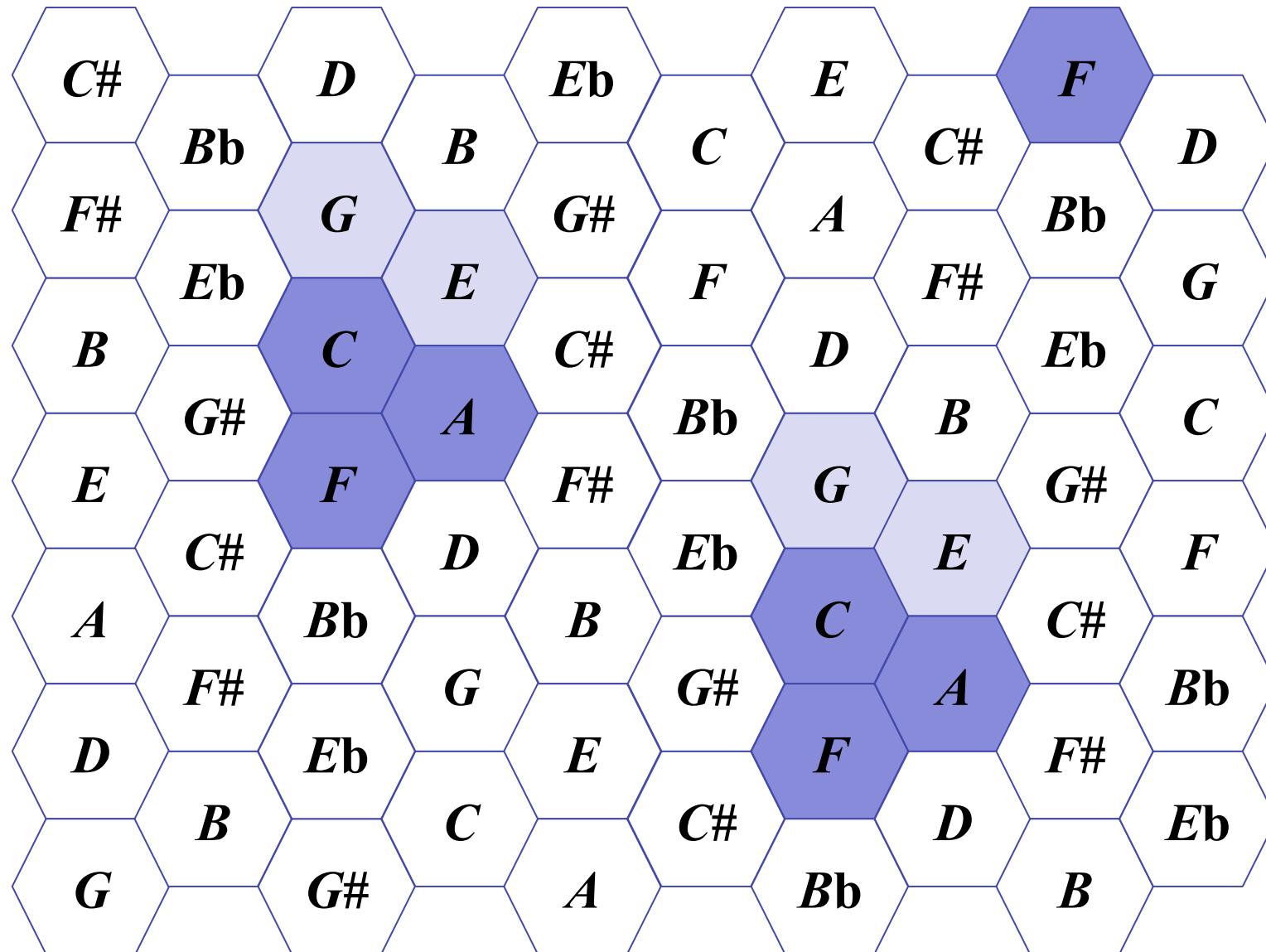
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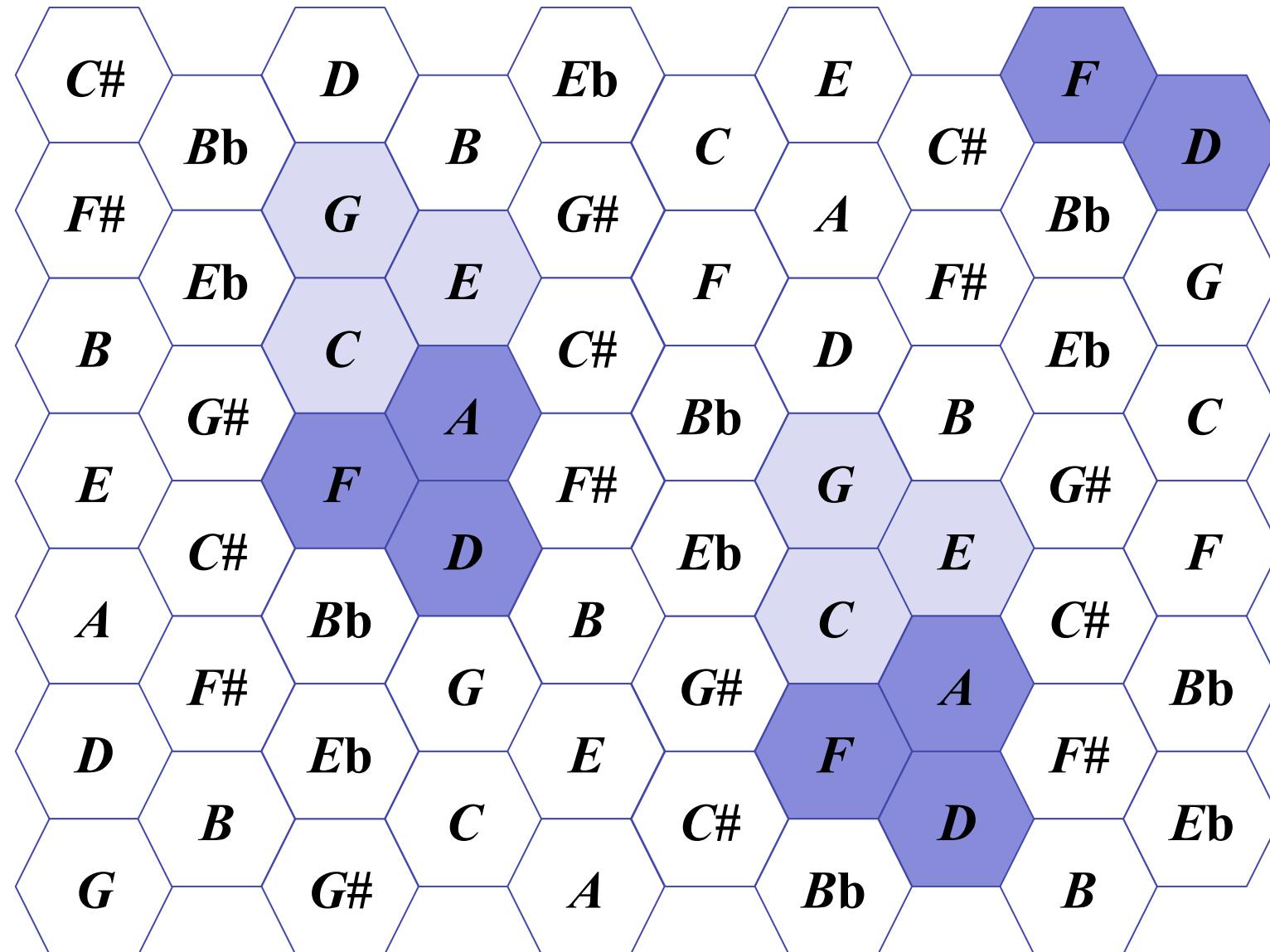
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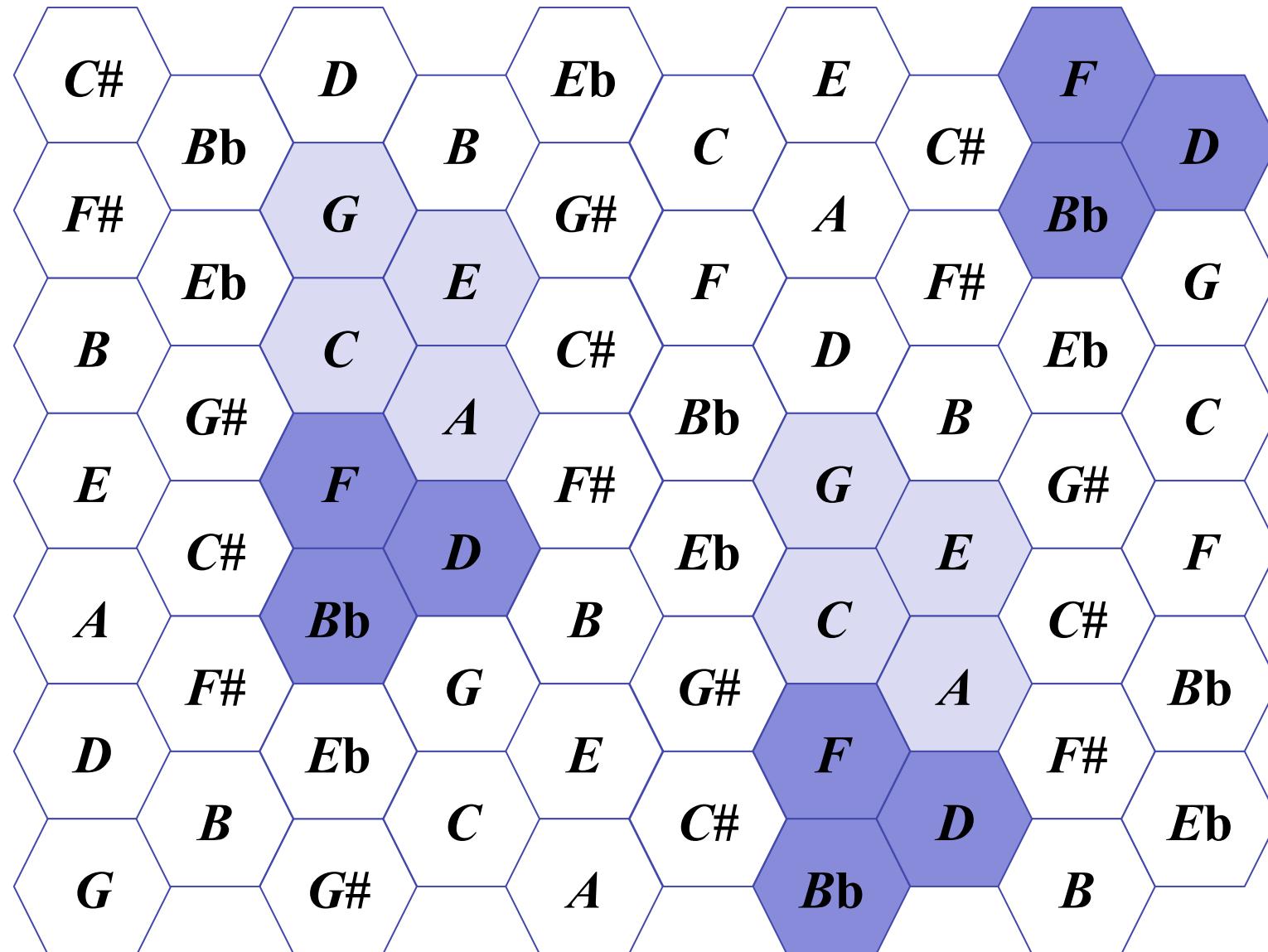
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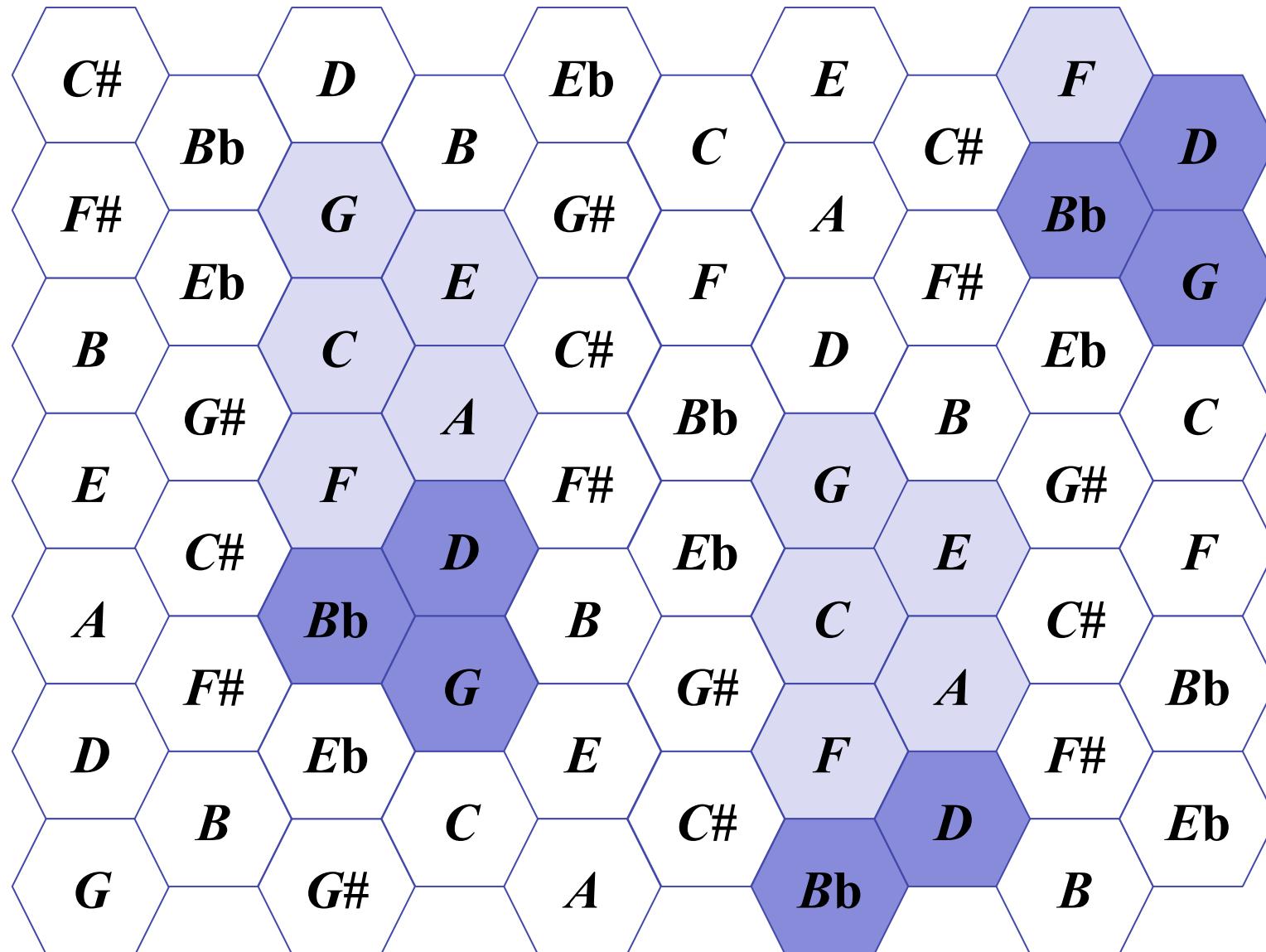
Extract of the 2nd movement of the Symphony No. 9 (L. van Beethoven)



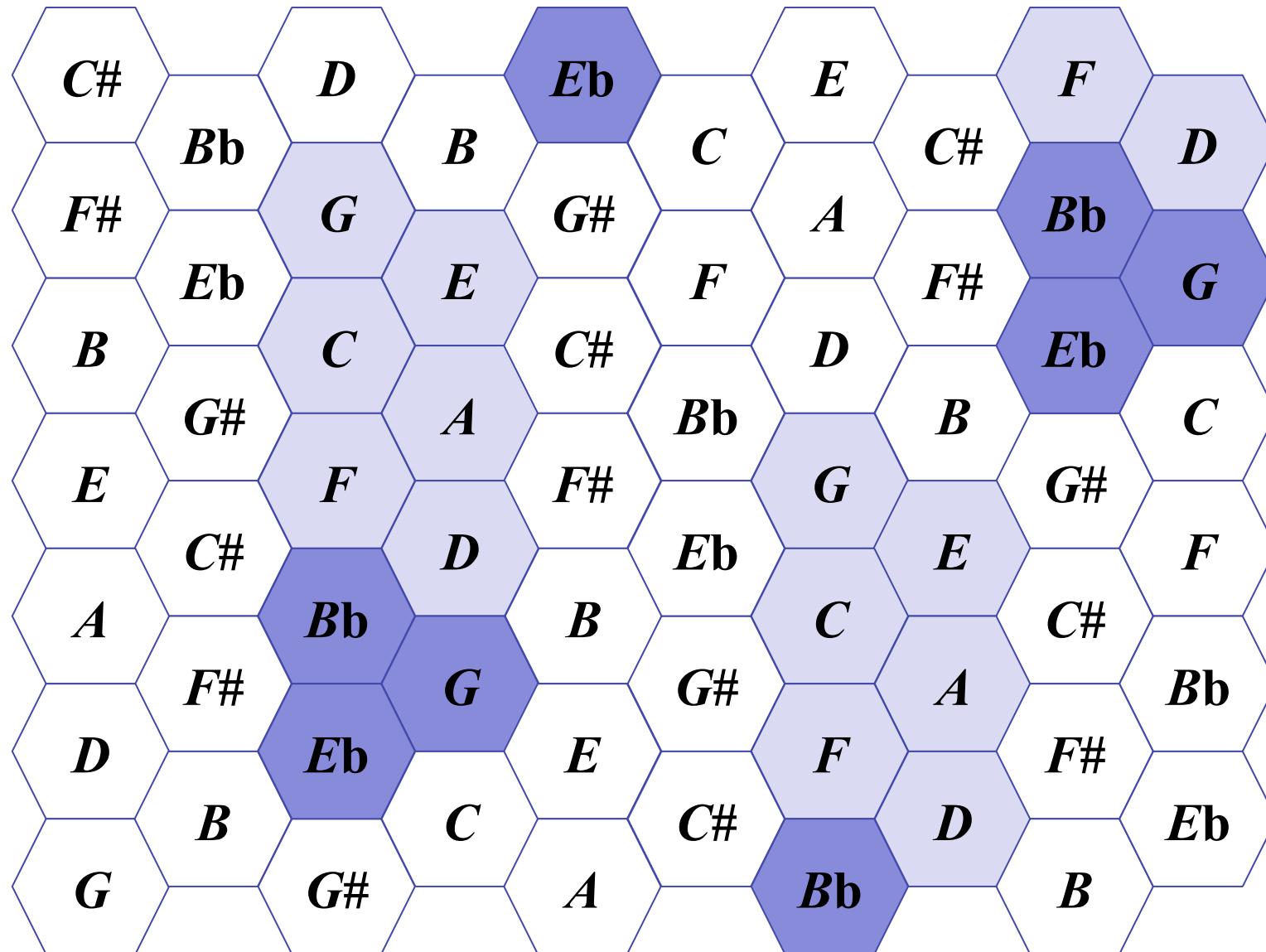
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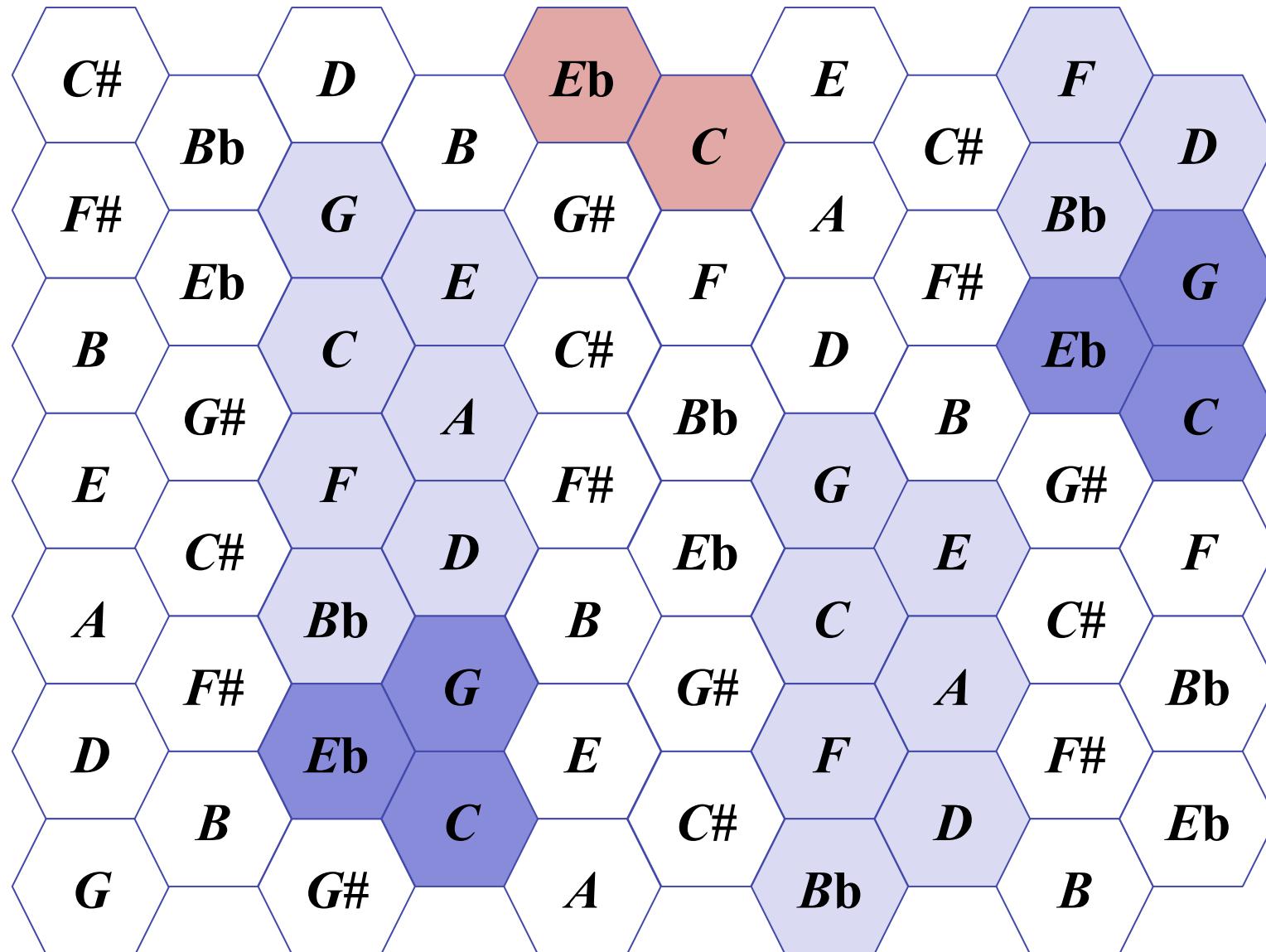
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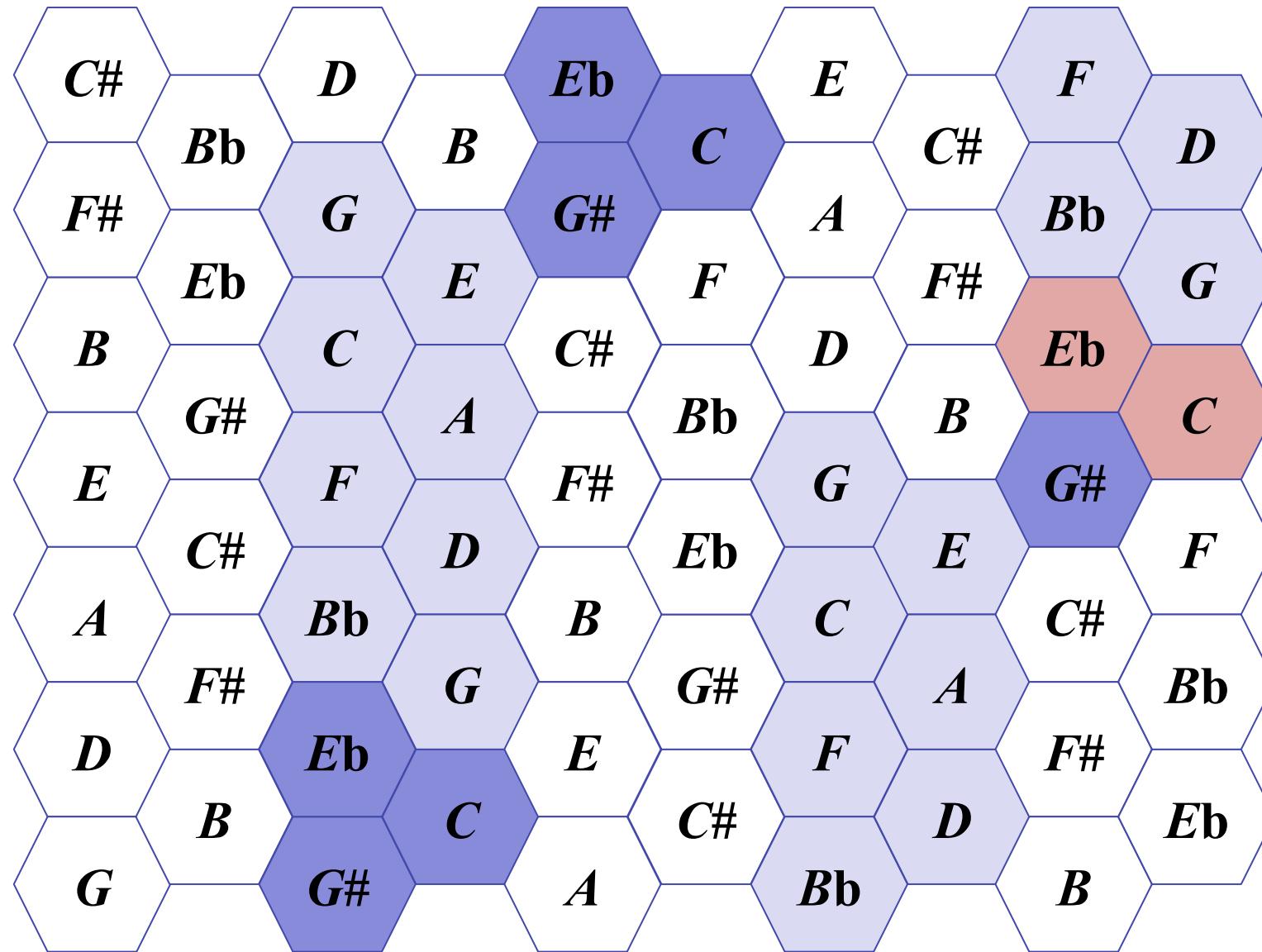
Extract of the 2nd movement of the Symphony No. 9 (L. van Beethoven)



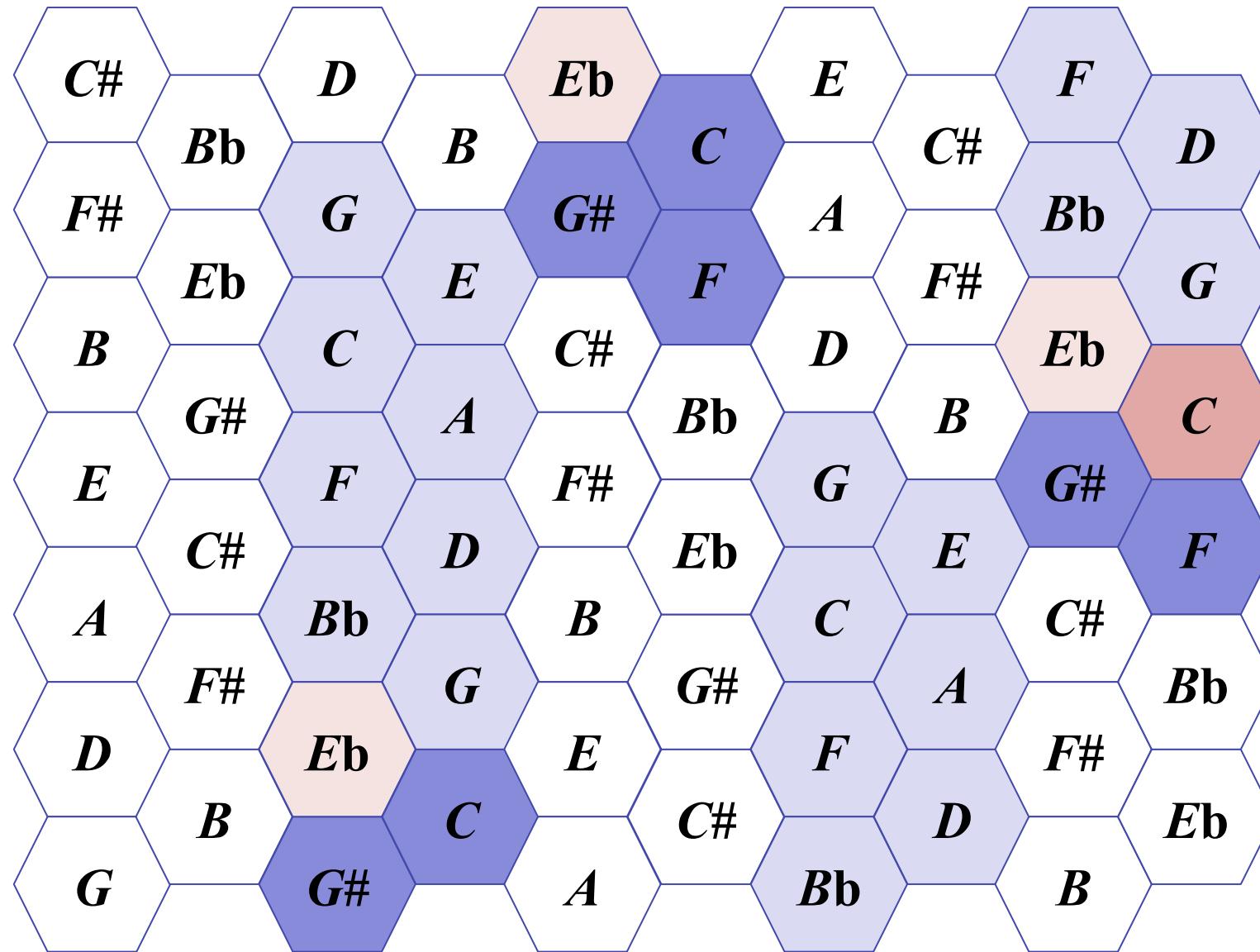
Extract of the 2nd movement of the Symphony No. 9 (L. van Beethoven)



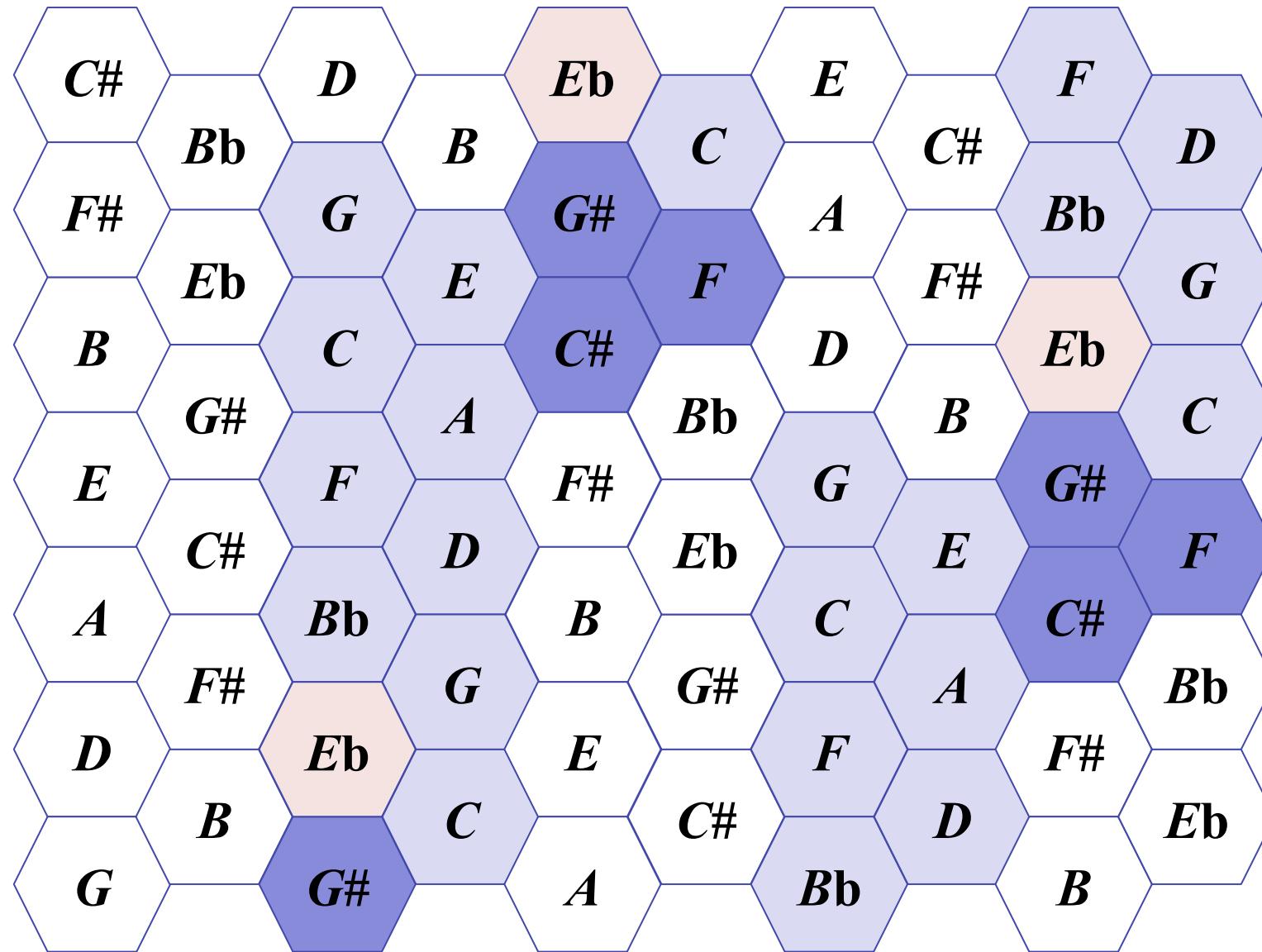
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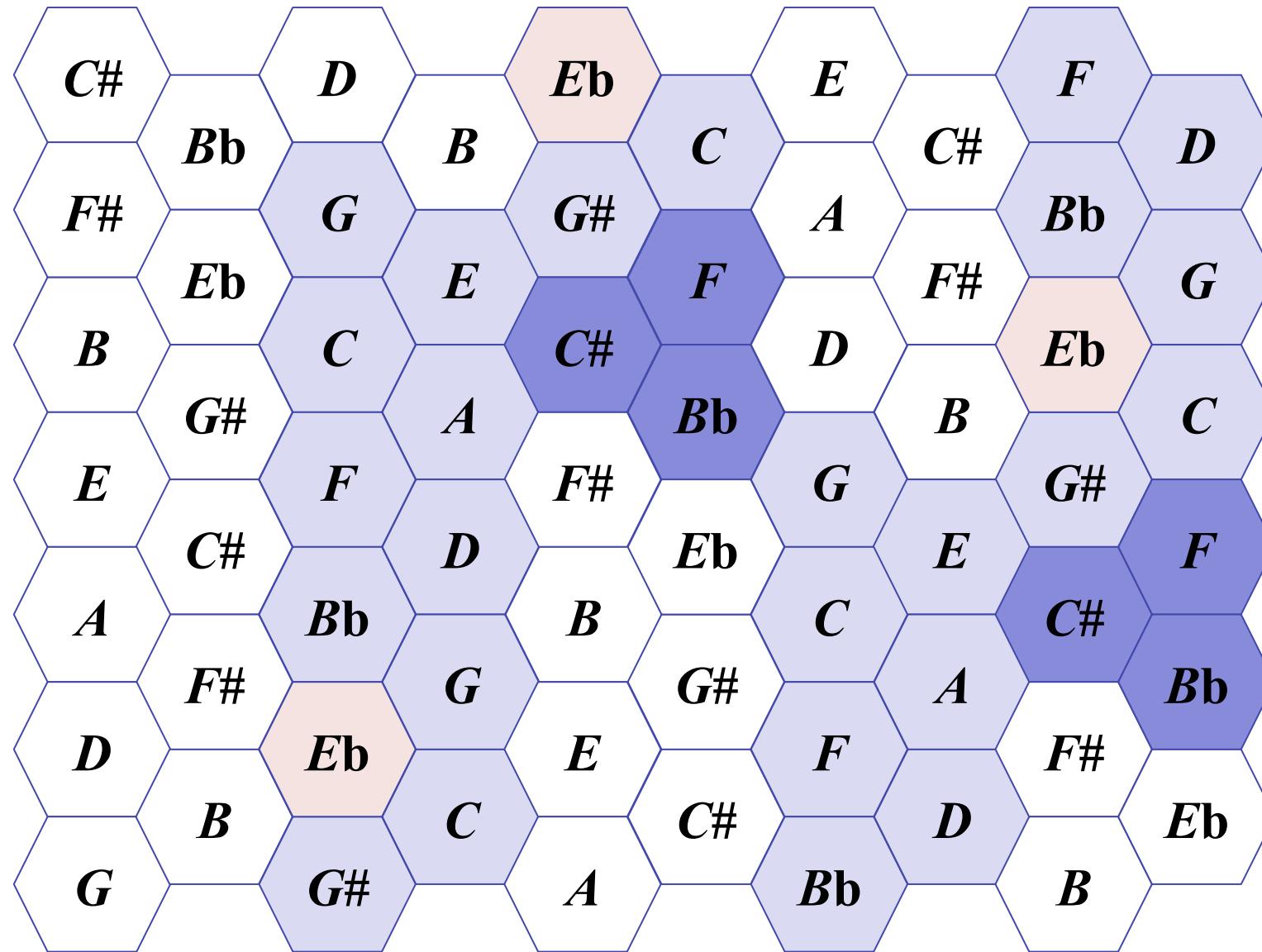
Extract of the 2nd movement of the Symphony No. 9 (L. van Beethoven)



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Vers l'émergence de la notion de groupe en musique

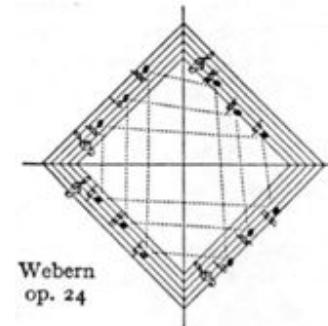
Ernst Krenek et l'approche axiomatique en musique



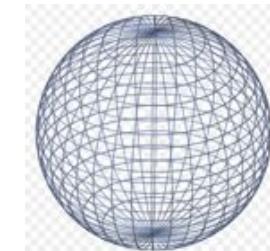
- *The Relativity of Scientific Systems*
- *The Significance of Axioms*
- *Axioms in music*
- *Musical Theory and Musical Practice*

Ernst Krenek : *Über Neue Musik*, 1937
(Engl. Transl. *Music here and now*, 1939)

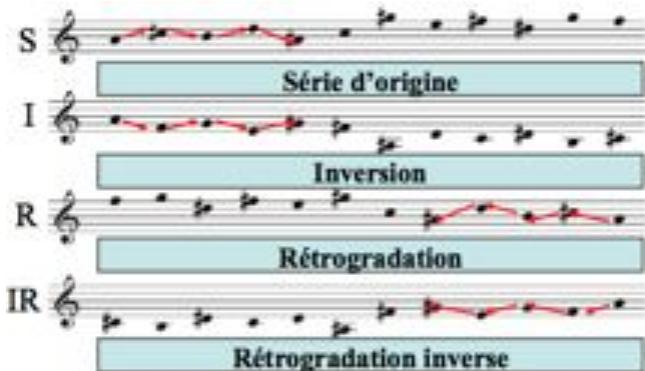
Physicists and mathematicians are far in advance of musicians in realizing that their respective sciences do not serve to establish a concept of the universe conforming to an objectively existent nature



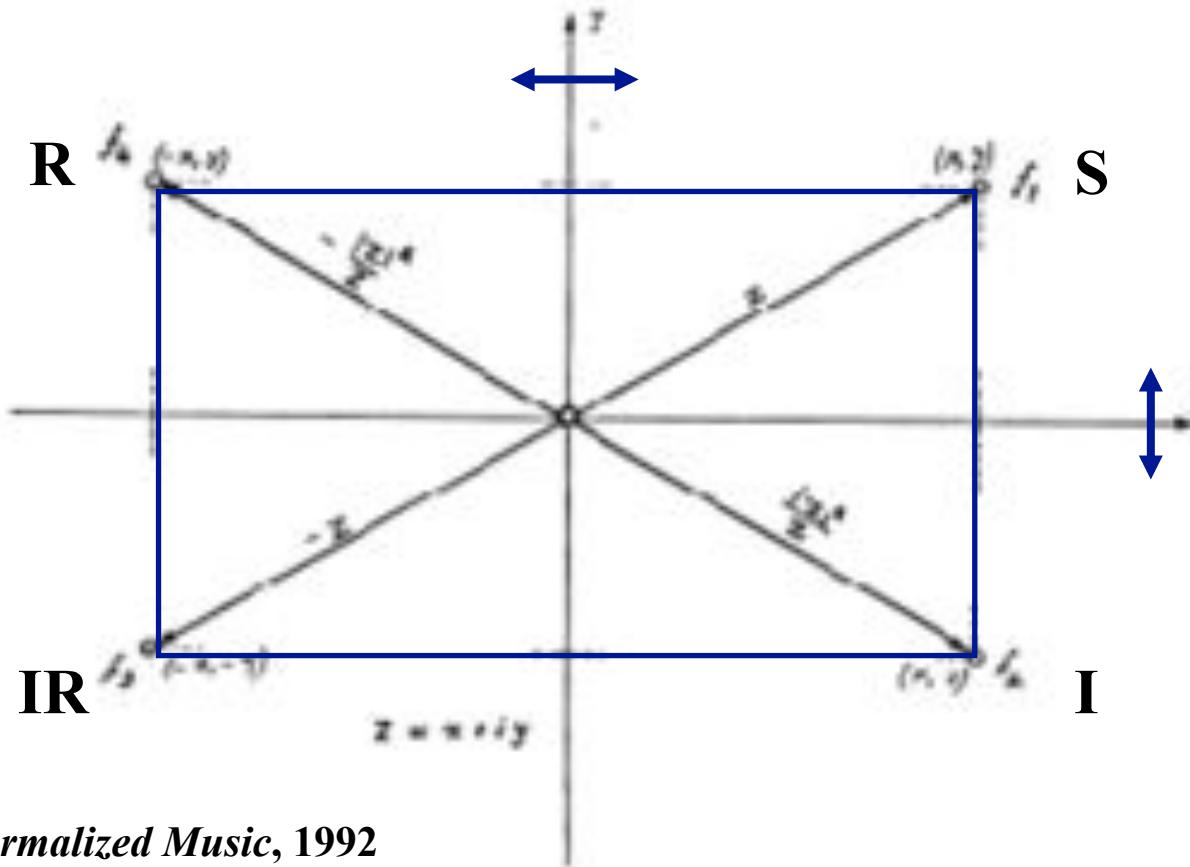
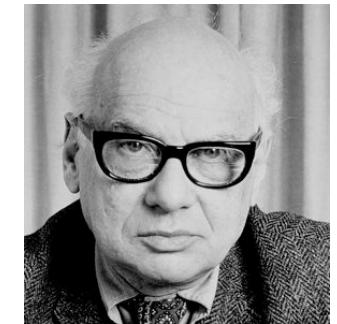
As the study of axioms eliminates the idea that axioms are something absolute, conceiving them instead as free propositions of the human mind, just so would this musical theory free us from the concept of major/minor tonality [...] as an irrevocable law of nature.



Opérations dodécaphoniques et structures algébriques



	S	I	R	RI
S	S	I	R	RI
I	I	S	RI	R
R	R	RI	S	I
RI	RI	R	I	S



Iannis Xenakis, *Formalized Music*, 1992

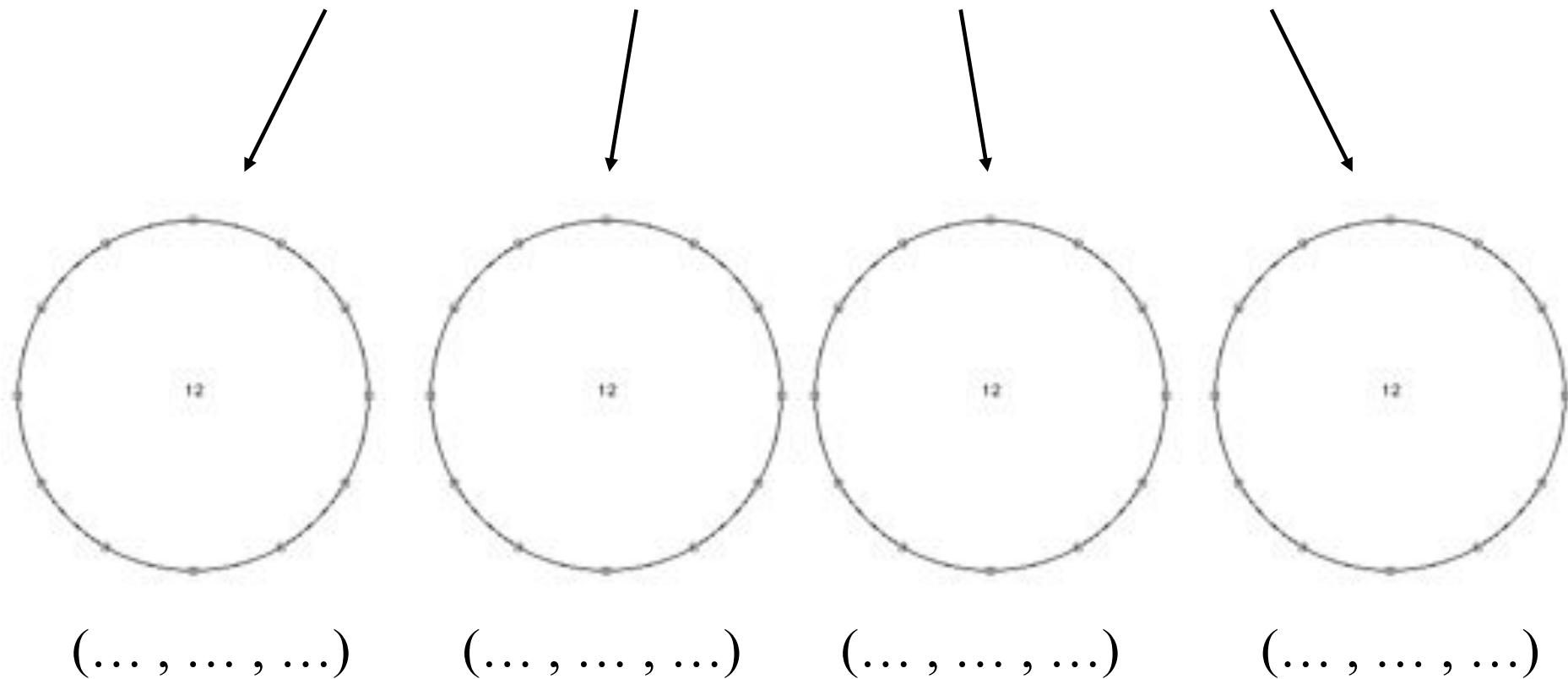
Felix Klein

Exercice : retrouver les symétries dans une série (I)

Schoenberg: Serenade Op.24, Mouvement 5

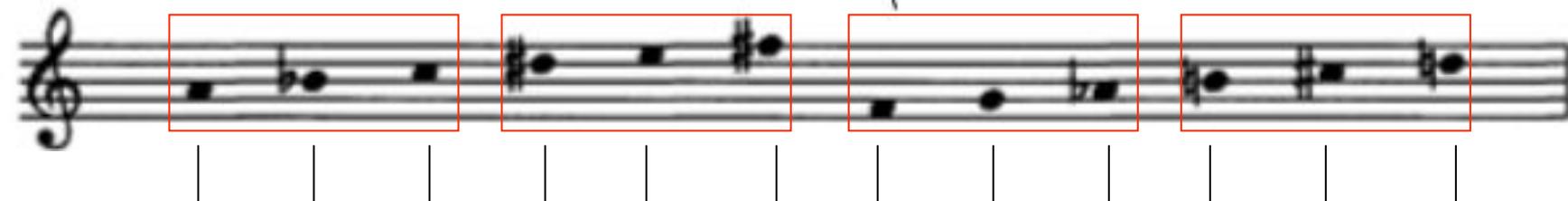
A musical score excerpt on a treble clef staff. Four groups of three notes each are highlighted with red boxes. Below the staff, four sets of three dots each are shown, corresponding to the groups highlighted by the red boxes.

{..., ..., ...} {..., ..., ...} {..., ..., ...} {..., ..., ...}



Exercice : retrouver les symétries dans une série (I)

Schoenberg: Serenade Op.24, Mouvement 5

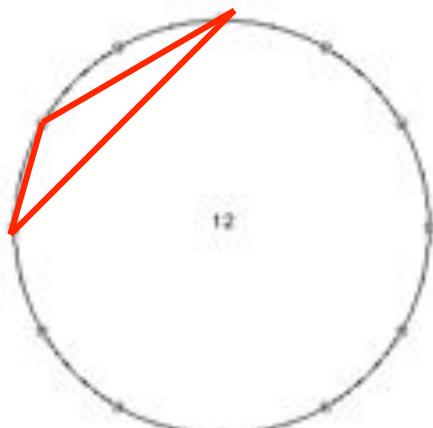


$\{9, 10, 0\}$

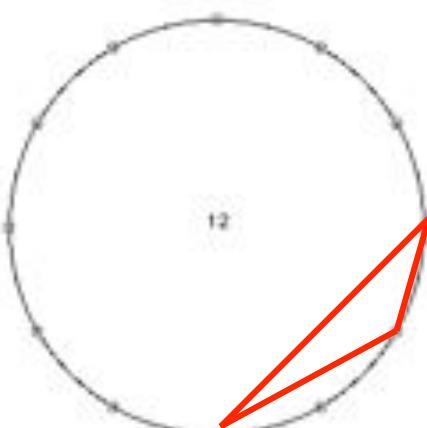
$\{3, 4, 6\}$

$\{5, 7, 8\}$

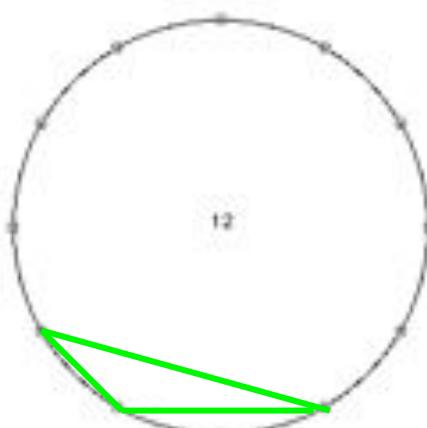
$\{11, 1, 2\}$



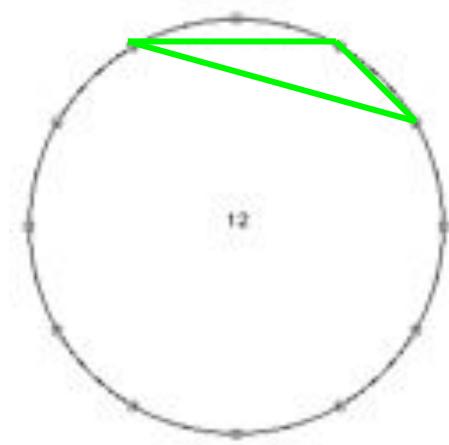
$(1, 2, 9)$



$(1, 2, 9)$



$(2, 1, 9)$



$(2, 1, 9)$

L'analyse formalisée ou les entités formelles en musique

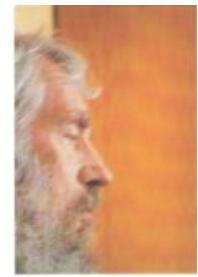
André Riotte e Marcel Mesnage

A. Schoenberg : *Klavierstück Op. 33a*, 1929

The image shows a musical score for piano by Arnold Schoenberg, specifically his Klavierstück Op. 33a, from 1929. The score is in 4/4 time, with two staves: treble and bass. The music consists of short, complex chords and patterns. Several regions of the score are highlighted with colored boxes and arrows, indicating formal entities or motivic structures. These highlighted areas include:

- A blue box highlights a cluster of notes in the upper treble staff.
- A green dashed box highlights a cluster of notes in the lower treble staff.
- A yellow diamond highlights a cluster of notes in the upper treble staff.
- A yellow diamond highlights a cluster of notes in the middle treble staff.
- A blue box highlights a cluster of notes in the middle bass staff.
- A green dashed box highlights a cluster of notes in the upper bass staff.

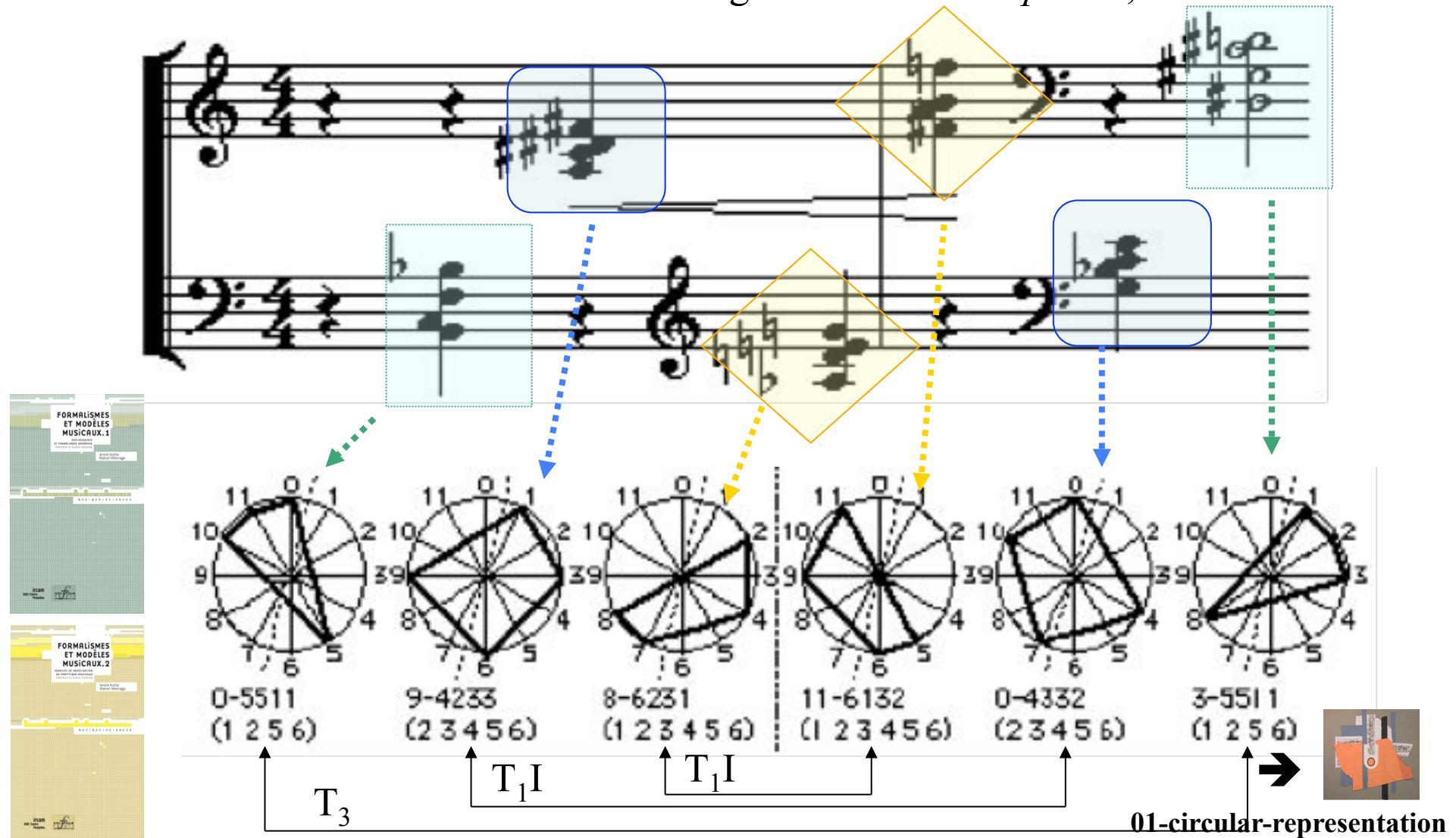
Arrows point from these highlighted areas to a series of circular diagrams below, each labeled with "12". These diagrams represent 12-note sets, likely referring to the twelve-tone technique. The labels at the bottom indicate repeating sequences of these 12-note sets, such as "(..., ..., ..., ...)".



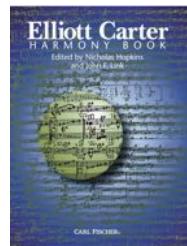
L'analyse formalisée ou les entités formelles en musique

André Riotte & Marcel Mesnage

A. Schoenberg : *Klavierstück Op. 33a*, 1929



Elliott Carter : 90+ (1994)



- Combinatoire d'accords
 - Hexacordes
 - Tétracordes
 - Triades
 - Relation Z
- Séries tous-intervalles
 - *Link-chords*



(piano: John Snijders)

mille e novanta auguri a caro Goffredo

90+

Elliott Carter
(1994)

+ Use pedal only to join one chord to another ligato, as in mm. 1-13, 18-21, 36-43, and 45-48.

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PIB 503

Printed in U.S.A.

Elliott Carter : 90+ (1994) : combinatoire tetra/tricordale

mille e novemila segni a sette Guglie

90+

Elliott Carter
(1994)

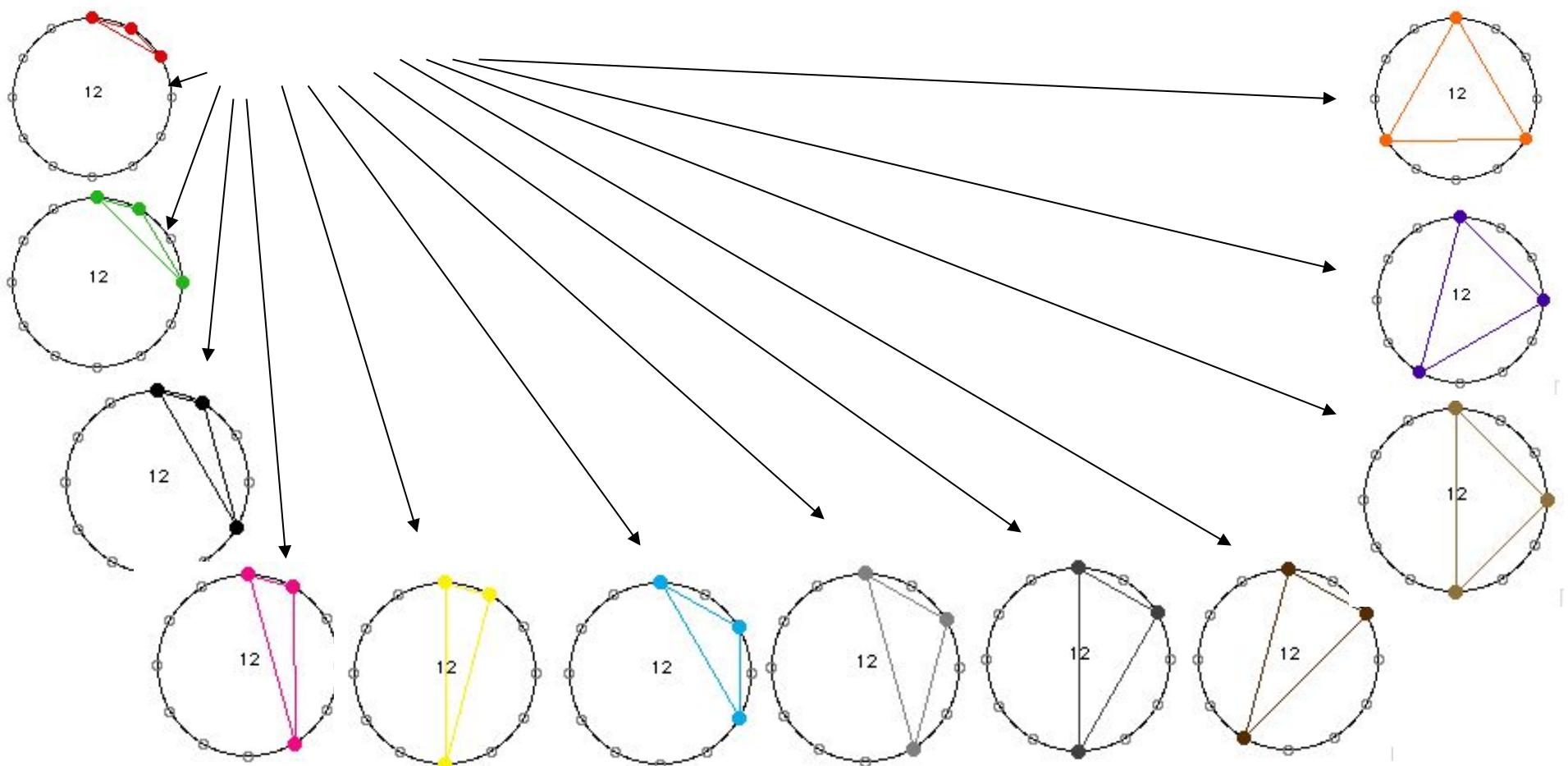
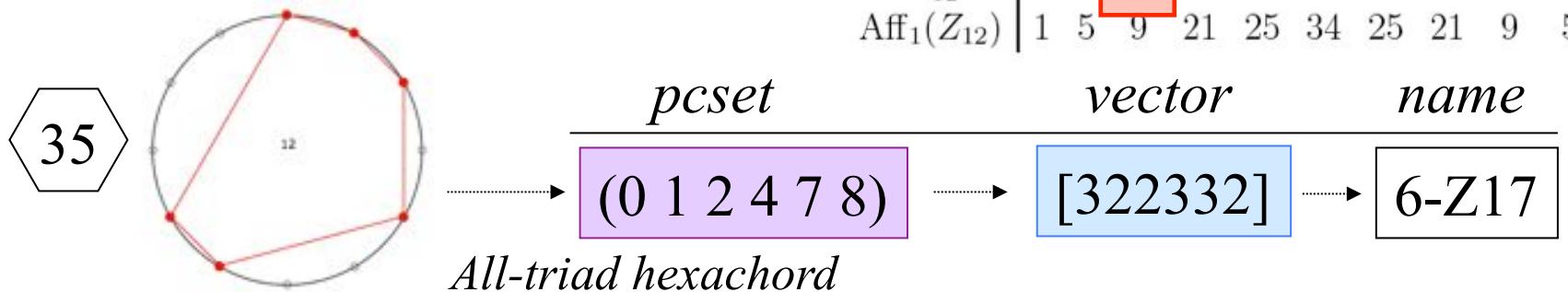
The musical score consists of five staves of music for piano, strings, and woodwinds. Various measures are highlighted with colored boxes: blue, green, orange, red, yellow, cyan, magenta, purple, and black. Some boxes are solid, while others are dashed or have a different stroke. The score is divided into sections by vertical red lines.

Diagram illustrating the combinatorial tetra/tricordale system:

- Top Row:** Six circles labeled "12" representing a 12-note set. The first five circles show various red line segments connecting points, while the last one is empty.
- Middle Row:** Six circles labeled "12". The first three are colored green, blue, and purple respectively, with red line segments connecting points. The next two are empty, and the last one has a large black question mark "?".
- Bottom Row:** Three pairs of circles labeled "12". Each pair is enclosed in a dashed box. The top pair shows a circle with a red line segment and another with a red line segment. The middle pair shows a circle with a red line segment and another with a red line segment. The bottom pair shows a circle with a red line segment and another with a red line segment.

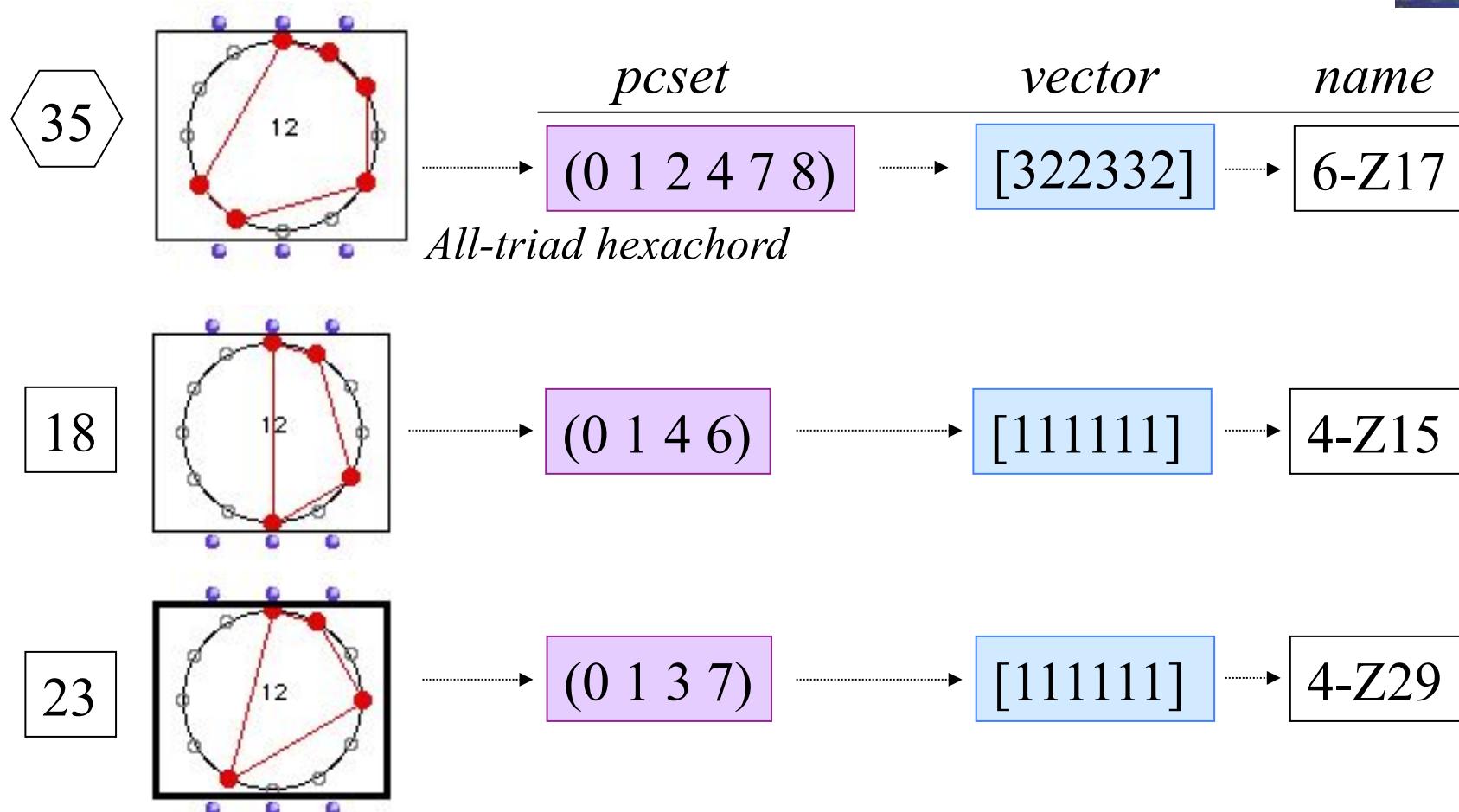
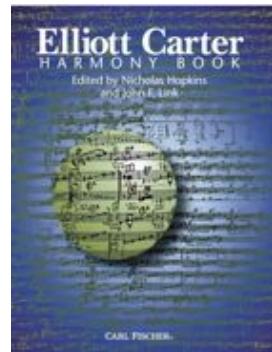
Elliott Carter: 90+ (1994)

$G \setminus k$	1	2	3	4	5	6	7	8	9	10	11	12
C_{12}	1	6	19	43	66	80	66	43	19	6	1	1
D_{12}	1	6	12	29	38	50	38	29	12	6	1	1
$\text{Aff}_1(Z_{12})$	1	5	9	21	25	34	25	21	9	5	1	1



Elliott Carter: 90+ (1994)

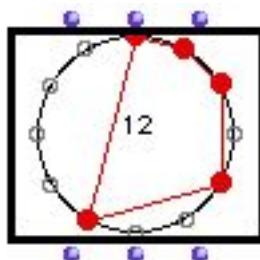
« From about 1990, I have reduced my vocabulary of chords more and more to the six note chord n° 35 and the four note chords n° 18 and 23, which encompass all the intervals » (Harmony Book, 2002, p. ix)



La Set Theory d'Allen Forte: catalogue des pitch-class sets

complementare					
name	pcs	vector	name	pcs	vector
5-30	0,1,4,6,8	121321	7-30	0,1,2,4,6,8,9	343542
5-31	0,1,3,6,9	114112	7-31	0,1,3,4,6,7,9	336333
5-32	0,1,4,6,9	113221	7-32	0,1,3,4,6,8,9	335442
5-33(12)	0,2,4,6,8	040402	7-33	0,1,2,4,6,8,10	262623
5-34(12)	0,2,4,6,9	032221	7-34	0,1,3,4,6,8,10	254442
5-35(12)	0,2,4,7,9	032140	7-35	0,1,3,5,6,8,10	254361
5-Z36	0,1,2,4,7	222121	7-Z36	0,1,2,3,5,6,8	444342
5-Z37(12)	0,3,4,5,8	212320	7-Z37	0,1,3,4,5,7,8	434541
5-Z38	0,1,2,5,8	212221	7-Z38	0,1,2,4,5,7,8	434442
6-1(12)	0,1,2,3,4,5	543210			
6-2	0 1 2 3 4 6	443211			
5-Z36	0,1,2,4,7	222121	7-Z36	0,1,2,3,5,6,8	444342
6-Z4(12)	0,1,2,4,5,6	432321	6-Z37(12)	0,1,2,3,4,8	
6-5	0,1,2,3,6,7	422232	6-Z38(12)	0,1,2,3,7,8	
6-Z6(12)	0,1,2,5,6,7	421242			
6-7(6)	0,1,2,6,7,8	420243			
6-8(12)	0,2,3,4,5,7	343230			
6-9	0,1,2,3,5,7	342231			
6-Z10	0,1,3,4,5,7	333321			
6-Z11	0,1,2,4,5,7	333231			
6-Z12	0,1,2,4,6,7	332232			
6-Z13(12)	0,1,3,4,6,7	324222			
<i>Relation Z</i>					

Vecteur d'intervalles et relation Z



5-30 0,1,4,6,8

5-31 0,1,3,6,9

5-32 0,1,4,6,9

5-33(12) 0,2,4,6,8

5-34(12) 0,2,4,6,9

5-35(12) 0,2,4,7,9

5-Z36 0,1,2,4,7

5-Z37(12) 0,3,4,5,8

5-Z38 0,1,2,5,8

6-1(12) 0,1,2,3,4,5

6-2 0,1,2,3,4,6

5-Z36 0,1,2,4,7

6-Z4(12) 0,1,2,4,5,6

6-5 0,1,2,3,6,7

6-Z6(12) 0,1,2,5,6,7

6-7(6) 0,1,2,6,7,8

6-8(12) 0,2,3,4,5,7

6-9 0,1,2,3,5,7

6-Z10 0,1,3,4,5,7

6-Z11 0,1,2,4,5,7

6-Z12 0,1,2,4,6,7

6-Z13(12) 0,1,3,4,6,7

121321

114112

113221

040402

032221

032140

222121

212320

212221

543210

443211

7-30 0,1,2,4,6,8,9

7-31 0,1,3,4,6,7,9

7-32 0,1,3,4,6,8,9

7-33 0,1,2,4,6,8,10

7-34 0,1,3,4,6,8,10

7-35 0,1,3,5,6,8,10

7-Z36 0,1,2,3,5,6,8

7-Z37 0,1,3,4,5,7,8

7-Z38 0,1,2,4,5,7,8

6-Z36 0,1,2,3,4,7

6-Z37(12) 0,1,2,3,4,8

6-Z38(12) 0,1,2,3,7,8

6-Z39 0,2,3,4,5,8

6-Z40 0,1,2,3,5,8

6-Z41 0,1,2,3,6,8

6-Z42(12) 0,1,2,3,6,9

343542

336333

335442

262623

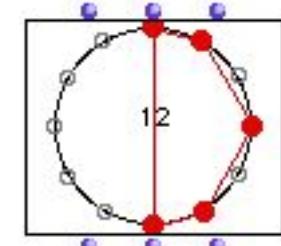
254442

254361

444342

434541

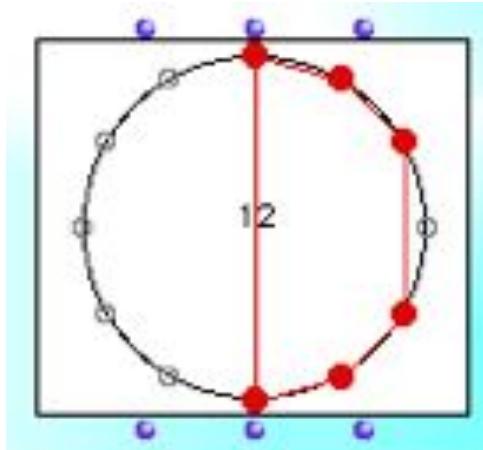
434442



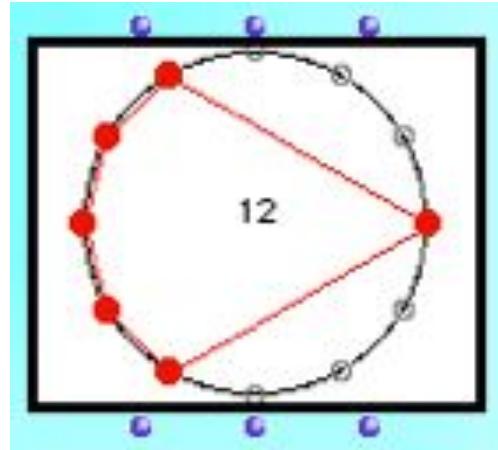
5-Z12

Théorème de l'hexacorde (ou théorème de Babbitt)

(Wilcox, Ralph Fox (?), Chemillier, Lewin, Mazzola, Schaub, ..., Amiot [2006])



A



A'

$$\text{IV}(A) = [4, 3, 2, 3, 2, 1] = [4, 3, 2, 3, 2, 1] = \text{IV}(A')$$

Un hexacorde et son complémentaire ont le même vecteur d'intervalle

Relation Z en musique et théorie de l'homométrie

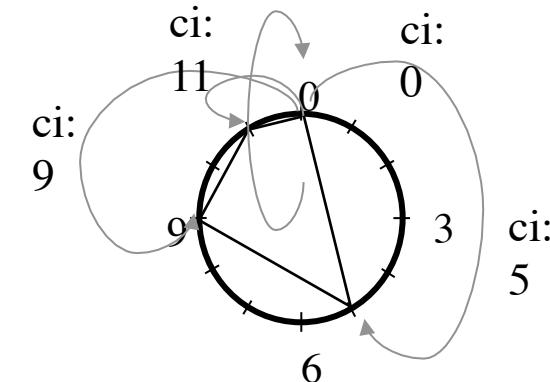
- Le contenu intervallique est équivalent à un produit de convolution de fonctions caractéristiques (Lewin, 1958)

$$\mathcal{F}_A : t \mapsto \sum_{k \in A} e^{-2i\pi kt/c}$$

$$IC_A(k) = (1_A \star 1_{-A})(k)$$

$$1_A * \tilde{1}_B(k) = \sum_i 1_A(i) \times 1_B(i - k) = \sum_{\substack{i \in A \\ i - k \in B}} 1$$

$$\mathcal{F}(1_A * \tilde{1}_B) = \mathcal{F}(1_A) \times \mathcal{F}(\tilde{1}_B)$$



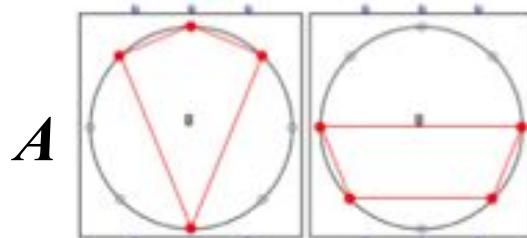
$$A = \{0, 5, 9, 11\} \Rightarrow IC_A(k) = 1 \forall k$$

$$\mathcal{F}(IC_A) = \mathcal{F}_A \times \mathcal{F}_{-A} = |\mathcal{F}_A|^2$$

→ Relation Z

$$\forall k \mathcal{F}(IC_{Z_c \setminus A})(k) = \mathcal{F}(IC_A)(k)$$

→ Théorème de l'hexacorde



$A' \rightarrow IC_A = IC_{A'}$

• P. Beauguitte, *Transformée de Fourier discrète et structures musicales*, Master ATIAM, 2011

FIGURE 1.4 – Plus petite Z-relation non triviale

Z-relation, homométrie et reconstruction de la phase

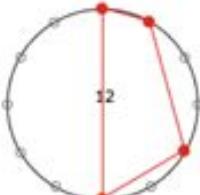
- Deux ensembles sont en Z-relations s'ils ont le même module de la DFT

$$IC_A(k) = (1_A \star 1_{-A})(k)$$

$$\mathcal{F}_A : t \mapsto \sum_{k \in A} e^{-2i\pi kt/c}$$

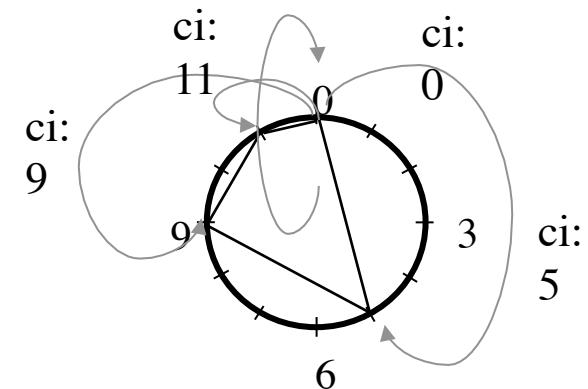
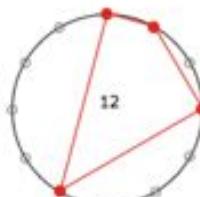
$$\mathcal{F}(IC_A) = \mathcal{F}_A \times \mathcal{F}_{-A} = |\mathcal{F}_A|^2$$

$\{0, 1, 4, 6\}_{12}$



$$\mathbf{iv} = [4, 1, 1, 1, 1, 1, 2, 1, 1, 1, 1, 1]$$

$\{0, 1, 3, 7\}_{12}$



$$\begin{aligned} A &= \{0, 5, 9, 11\} \\ IC_A(k) &= 1 \quad \forall k = 1 \dots 11 \end{aligned}$$

- Mandereau J., D. Ghisi, E. Amiot, M. Andreatta, C. Agon, (2011a), « Z-relation and homometry in musical distributions », *Journal of Mathematics and Music*, vol. 5, n° 2, p. 83-98.
- Mandereau J, D. Ghisi, E. Amiot, M. Andreatta, C. Agon (2011b), « Discrete phase retrieval in musical structures », *Journal of Mathematics and Music*, vol. 5, n° 2, p. 99-116.

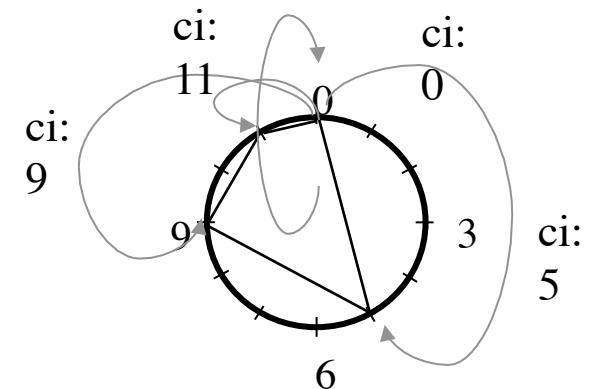
Z-relation, homométrie et reconstruction de la phase

- Deux ensembles sont en Z-relations s'ils ont le même module de la DFT

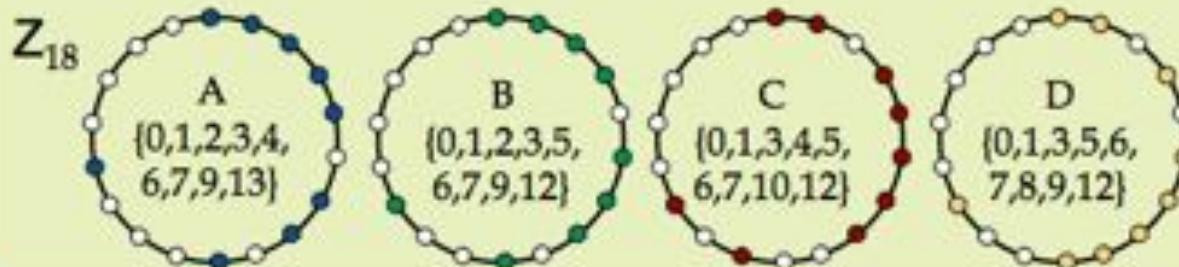
$$IC_A(k) = (1_A \star 1_{-A})(k)$$

$$\mathcal{F}_A : t \mapsto \sum_{k \in A} e^{-2i\pi kt/c}$$

$$\mathcal{F}(IC_A) = \mathcal{F}_A \times \mathcal{F}_{-A} = |\mathcal{F}_A|^2$$



Not only do we have Z-couples, but we also have Z-related t-uples for all t's.



$$A = \{0, 5, 9, 11\}$$
$$IC_A(k) = 1 \quad \forall k = 1 \dots 11$$

- Daniele Ghisi, "From Z-relation to homometry: an introduction and some musical examples", Séminaire MaMuX, 10 décembre 2010 [<http://repmus.ircam.fr/mamux/saisons/saison10-2010-2011/2010-12-10>]

Z-relation, homométrie et reconstruction de la phase

■ Contenu intervallaire et Fonction de Patterson

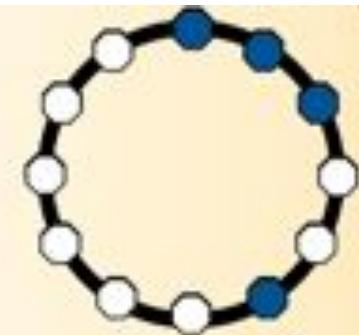
$$IC_A(k) = (1_A \star 1_{-A})(k)$$

Expression polynomiale :

$$D(x) = 1 + x + x^2 + x^5$$

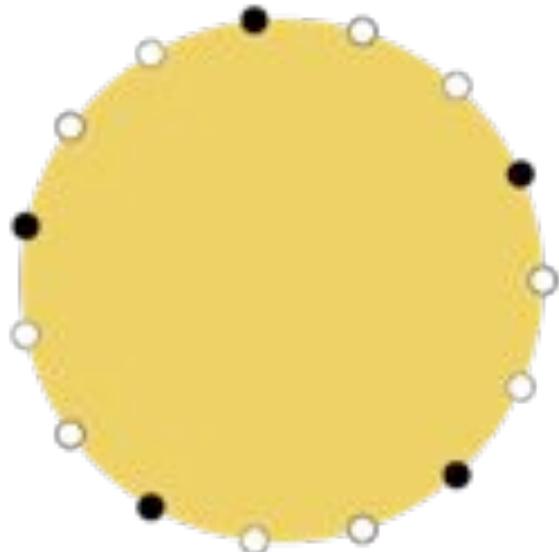
Fonction de Patterson (ou d'autocorrelation) :

$$\begin{aligned} D(x)D(x^{-1}) &= \\ &= (1 + x + x^2 + x^5)(1 + x^{11} + x^{10} + x^7) \\ &= 1 + x + x^2 + x^5 + x^7 + x^8 + x^9 + x^{10} + 2x^{11} + 3x^{12} + x^{13} + x^{15} + x^{16} \\ &= 4 + 2x + x^2 + x^3 + x^4 + x^5 + x^7 + x^8 + x^9 + x^{10} + 2x^{11} \end{aligned}$$



- Daniele Ghisi, "From Z-relation to homometry: an introduction and some musical examples", Séminaire MaMuX, 10 décembre 2010 [<http://repmus.ircam.fr/mamux/saisons/saison10-2010-2011/2010-12-10>]

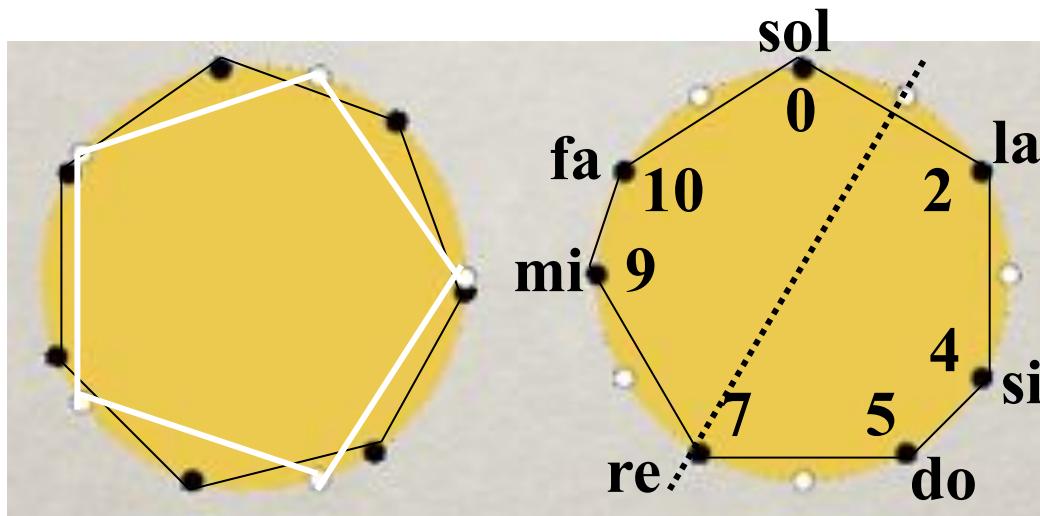
Maximally-Even Sets (ME-sets)



$$\mathcal{F}_A : t \mapsto \sum_{k \in A} e^{-2i\pi kt/c}$$

$$|F_A(5)| = 1 + 1 + 1 + 1 + 1 = 5$$

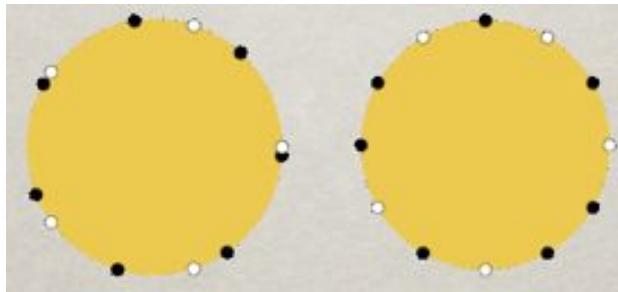
En général, $|F_A(t)| \leq \#A$



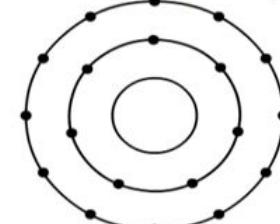
Gamme diatonique:
 $\{0, 2, 4, 5, 7, 9, 10\}$

Gamme pentatonique:
 $\{1, 3, 6, 8, 11\}$

Nouvelle définition de ME sets



Rotation into the Key of C-Major

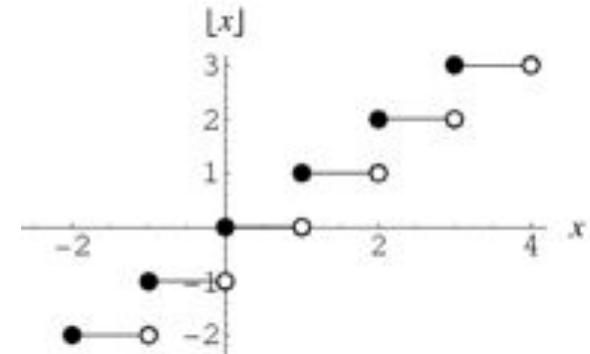


Definition (Clough-Myerson-Douthett) A set A with cardinality d in a given equal tempered space \mathbf{Z}_c is maximally even if $A = \{a_k\}$

$$a_k = J_{c,d}^\alpha(k) = \left\lfloor \frac{kc + \alpha}{d} \right\rfloor \quad \text{where } \alpha \in \mathbf{R}$$

$\lfloor x \rfloor$ is the integer part of x

$$J_{12,7}^5 = \left\{ \left\lfloor \frac{12k+5}{7} \right\rfloor \right\}_{k=0}^6 = \{0, 2, 4, 5, 7, 9, 11\}$$

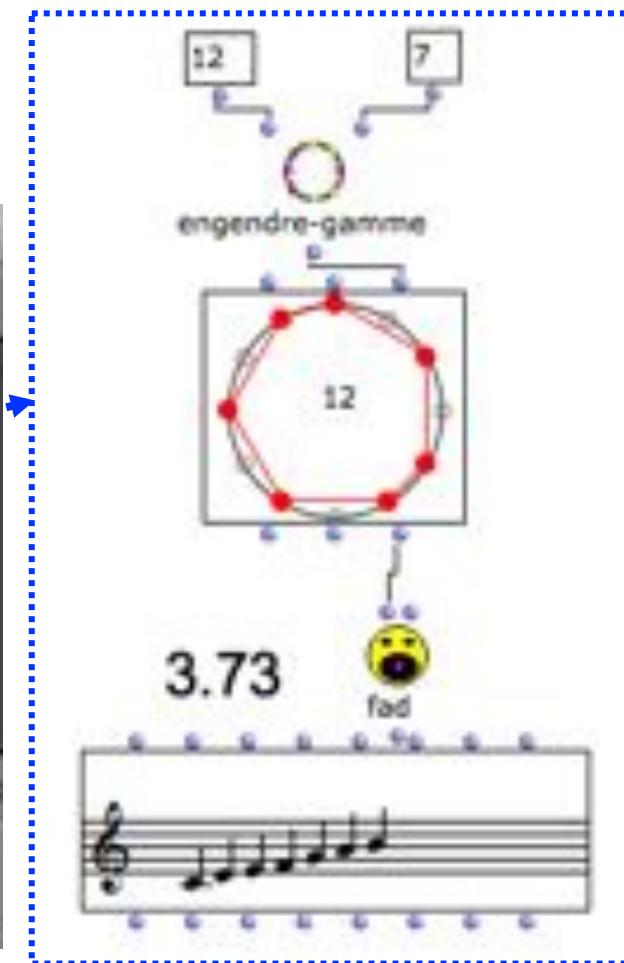
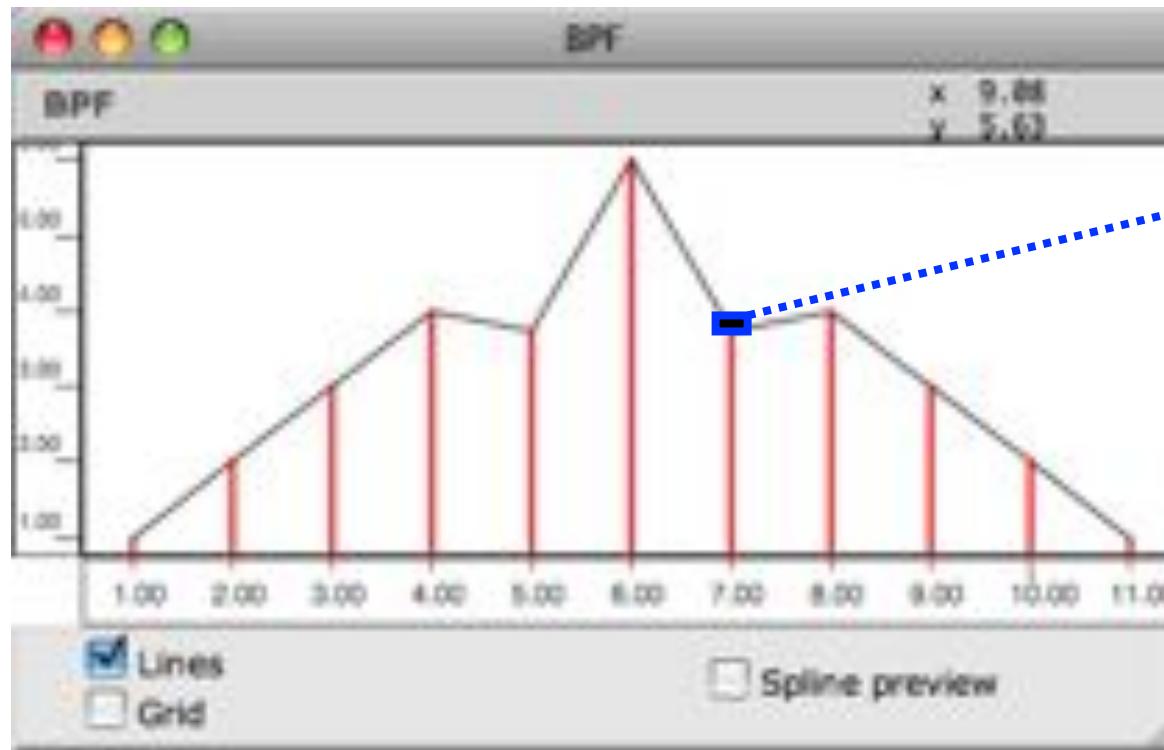


Definition (Amiot, 2005) A set A with cardinality d given equal tempered space \mathbf{Z}_c is maximally even if $|F_A(d)| \geq |F_B(d)|$ for all subsets B of cardinality d in \mathbf{Z}_c .

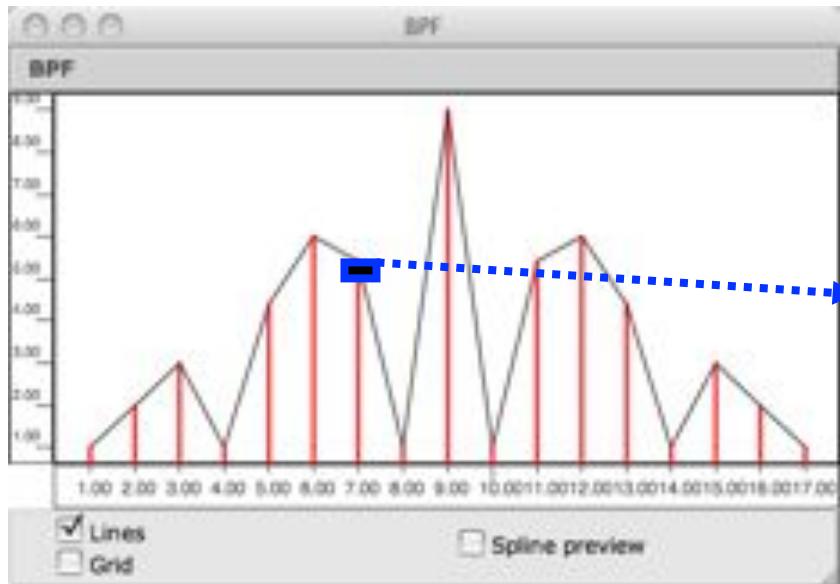
$$\text{where } F_{set}(t) := \sum_{k \in set} e^{2i\pi kt/12}$$

Nouvelle classification des structures musicales à l'aide de la DFT

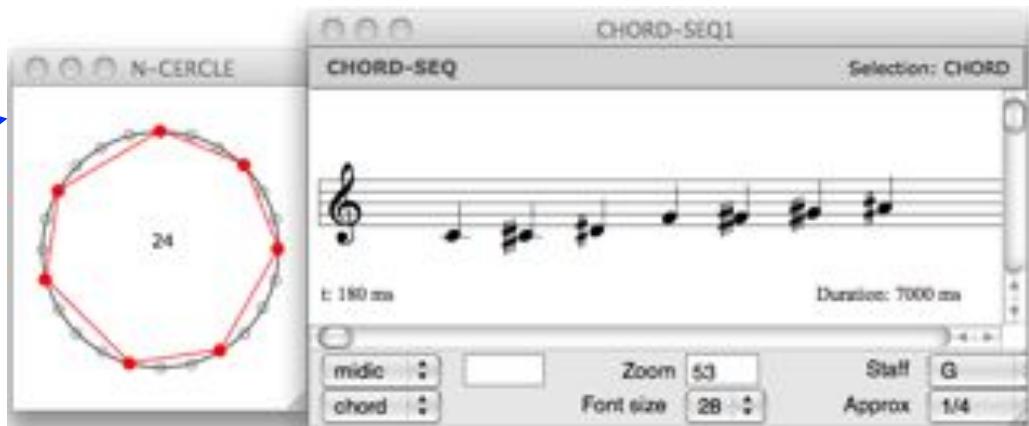
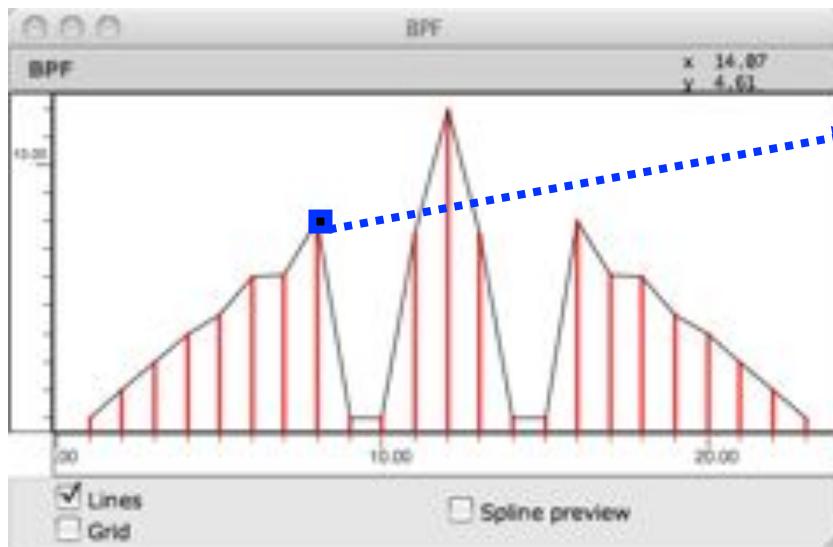
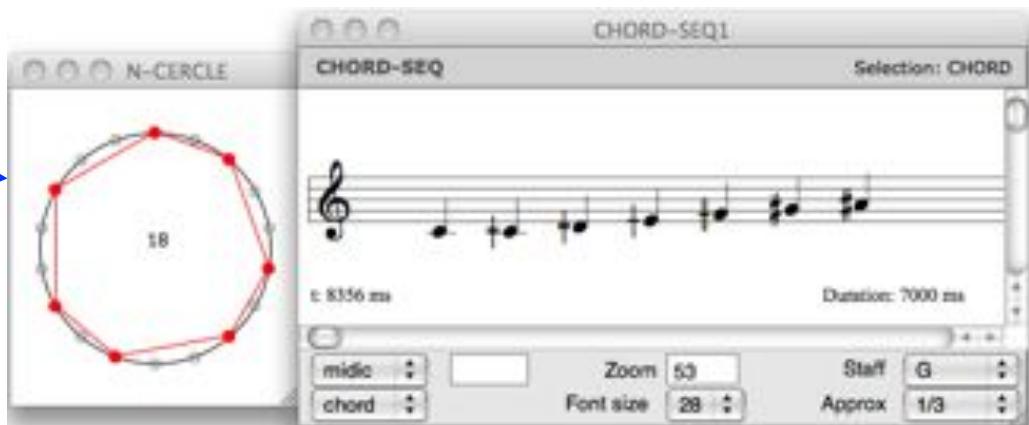
$$\mathcal{F}_A : t \mapsto \sum_{k \in A} e^{-2i\pi kt/c}$$



Classification des structures microtonales à l'aide de la DFT



Système en tiers de ton



Système en quarts de ton

The one-dimensional antiferromagnetic spin-1/2 Ising Model

Jack Douthett & Richard Krantz, "Energy extremes and spin configurations for the one-dimensional antiferromagnetic Ising model with arbitrary-range interaction", *J. Math. Phys.* 37 (7), July 1996

