Calcul d'orbites via le polynôme de Polya

Transformation	*Cycle representation*	*No. of cycles*	*Fixed configs.*	*Cycle type*	*Cycle index*
T_0	(0)(1)(2)(3)(4)(5)(6)(7)(8)(9)(A)(B)	12	$2^{12} = 4096$	1^{12}	t_1^{12}
T_1	(0 1 2 3 4 5 6 7 8 9 A B)	1	$2^1 = 2$	12^1	t_{12}^1
T_2	(0 2 4 6 8 A)(1 3 5 7 9 B)	2	$2^2 = 4$	6^2	t_6^2
T_3	(0 3 6 9)(1 4 7 A)(2 5 8 B)	3	$2^3 = 8$	4^3	t_4^3
T_4	(0 4 8)(1 5 9)(2 6 A)(3 7 B)	4	$2^4 = 16$	3^4	t_3^4
T_5	(0 5 A 3 8 1 6 B 4 9 2 7)	1	$2^1 = 2$	12^1	t_{12}^1
T_6	(0 6)(1 7)(2 8)(3 9)(4 A)(5 B)	6	$2^6 = 64$	2^6	t_2^6
T_7	(0 7 2 9 4 B 6 1 8 3 A 5)	1	$2^1 = 2$	12^1	t_{12}^1
T_8	(0 8 4)(1 9 5)(2 A 6)(3 B 7)	4	$2^4 = 16$	3^4	t_3^4
T_9	(0 9 6 3)(1 A 7 4)(2 B 8 5)	3	$2^3 = 8$	4^3	t_4^3
T_{10}	(0 A 8 6 4 2)(1 B 9 7 5 3)	2	$2^2 = 4$	6^2	t_6^2
T_{11}	(0 B A 9 8 7 6 5 4 3 2 1)	1	$2^1 = 2$	12^1	t_{12}^1
I	(0)(1 B)(2 A)(3 9)(4 8)(5 7)(6)	7	$2^7 = 128$	$1^2 2^5$	$t_1^2 t_2^5$
T_1I	(0 1)(2 B)(3 A)(4 9)(5 8)(6 7)	6	$2^6 = 64$	2^6	t_2^6
T_2I	(0 2)(1)(3 B)(4 A)(5 9)(6 8)(7)	7	$2^7 = 128$	$1^2 2^5$	$t_1^2 t_2^5$
T_3I	(0 3)(1 2)(4 B)(5 A)(6 9)(7 8)	6	$2^6 = 64$	2^6	t_2^6
T_4I	(0 4)(1 3)(2)(5 B)(6 A)(7 9)(8)	7	$2^7 = 128$	$1^2 2^5$	$t_1^2 t_2^5$
T_5I	(0 5)(1 4)(2 3)(6 B)(7 A)(8 9)	6	$2^6 = 64$	2^6	t_2^6
T_6I	(0 6)(1 5)(2 4)(3)(7 B)(8 A)(9)	7	$2^7 = 128$	$1^2 2^5$	$t_1^2 t_2^5$
T_7I	(0 7)(1 6)(2 5)(3 4)(8 B)(9 A)	6	$2^6 = 64$	2^6	t_2^6
T_8I	(0 8)(1 7)(2 6)(3 5)(4)(9 B)(A)	7	$2^7 = 128$	$1^2 2^5$	$t_1^2 t_2^5$
T_9I	(0 9)(1 8)(2 7)(3 6)(4 5)(A B)	6	$2^6 = 64$	2^6	t_2^6
$T_{10}I$	(0 A)(1 9)(2 8)(3 7)(4 6)(5)(B)	7	$2^7 = 128$	$1^2 2^5$	$t_1^2 t_2^5$
$T_{11}I$	(0 B)(1 A)(2 9)(3 8)(4 7)(5 6)	6	$2^6 = 64$	2^6	t_2^6

Lemme de Burnside

$$n = \frac{1}{|G|}\sum_{g\in G}|X^g|.$$

$X^g = \{x \in X : gx = x\}$

?

d'accords = 1/24[4224+1152] = 224

$G\backslash k$	1	2	3	4	5	6	7	8	9	10	11	12
C_{12}	1	6	19	43	66	80	66	43	19	6	1	1
D_{12}	1	6	12	29	38	50	38	29	12	6	1	1
$\mathrm{Aff}_1(Z_{12})$	1	5	9	21	25	34	25	21	9	5	1	1

Calcul d'orbites via le polynôme de Polya

Transformation	*Cycle representation*	*No. of cycles*	*Fixed configs.*	*Cycle type*	*Cycle index*
T_0	(0)(1)(2)(3)(4)(5)(6)(7)(8)(9)(A)(B)	12	$2^{12} = 4096$	1^{12}	t_1^{12}
T_1	(0 1 2 3 4 5 6 7 8 9 A B)	1	$2^1 = 2$	12^1	t_{12}^1
T_2	(0 2 4 6 8 A)(1 3 5 7 9 B)	2	$2^2 = 4$	6^2	t_6^2
T_3	(0 3 6 9)(1 4 7 A)(2 5 8 B)	3	$2^3 = 8$	4^3	t_4^3
T_4	(0 4 8)(1 5 9)(2 6 A)(3 7 B)	4	$2^4 = 16$	3^4	t_3^4
T_5	(0 5 A 3 8 1 6 B 4 9 2 7)	1	$2^1 = 2$	12^1	t_{12}^1
T_6	(0 6)(1 7)(2 8)(3 9)(4 A)(5 B)	6	$2^6 = 64$	2^6	t_2^6
T_7	(0 7 2 9 4 B 6 1 8 3 A 5)	1	$2^1 = 2$	12^1	t_{12}^1
T_8	(0 8 4)(1 9 5)(2 A 6)(3 B 7)	4	$2^4 = 16$	3^4	t_3^4
T_9	(0 9 6 3)(1 A 7 4)(2 B 8 5)	3	$2^3 = 8$	4^3	t_4^3
T_{10}	(0 A 8 6 4 2)(1 B 9 7 5 3)	2	$2^2 = 4$	6^2	t_6^2
T_{11}	(0 B A 9 8 7 6 5 4 3 2 1)	1	$2^1 = 2$	12^1	t_{12}^1
I	(0)(1 B)(2 A)(3 9)(4 8)(5 7)(6)	7	$2^7 = 128$	$1^2 2^5$	$t_1^2 t_2^5$
T_1I	(0 1)(2 B)(3 A)(4 9)(5 8)(6 7)	6	$2^6 = 64$	2^6	t_2^6
T_2I	(0 2)(1)(3 B)(4 A)(5 9)(6 8)(7)	7	$2^7 = 128$	$1^2 2^5$	$t_1^2 t_2^5$
T_3I	(0 3)(1 2)(4 B)(5 A)(6 9)(7 8)	6	$2^6 = 64$	2^6	t_2^6
T_4I	(0 4)(1 3)(2)(5 B)(6 A)(7 9)(8)	7	$2^7 = 128$	$1^2 2^5$	$t_1^2 t_2^5$
T_5I	(0 5)(1 4)(2 3)(6 B)(7 A)(8 9)	6	$2^6 = 64$	2^6	t_2^6
T_6I	(0 6)(1 5)(2 4)(3)(7 B)(8 A)(9)	7	$2^7 = 128$	$1^2 2^5$	$t_1^2 t_2^5$
T_7I	(0 7)(1 6)(2 5)(3 4)(8 B)(9 A)	6	$2^6 = 64$	2^6	t_2^6
T_8I	(0 8)(1 7)(2 6)(3 5)(4)(9 B)(A)	7	$2^7 = 128$	$1^2 2^5$	$t_1^2 t_2^5$
T_9I	(0 9)(1 8)(2 7)(3 6)(4 5)(A B)	6	$2^6 = 64$	2^6	t_2^6
$T_{10}I$	(0 A)(1 9)(2 8)(3 7)(4 6)(5)(B)	7	$2^7 = 128$	$1^2 2^5$	$t_1^2 t_2^5$
$T_{11}I$	(0 B)(1 A)(2 9)(3 8)(4 7)(5 6)	6	$2^6 = 64$	2^6	t_2^6

$$\begin{aligned}
\tfrac{1}{24}\Big[&(x+y)^{12} && \text{(from } T_0\text{)}\\
&+4(x^{12}+y^{12}) && \text{(from } T_1, T_5, T_7, T_{11}\text{)}\\
&+2(x^6+y^6)^2 && \text{(from } T_2, T_{10}\text{)}\\
&+2(x^4+y^4)^3 && \text{(from } T_3, T_9\text{)}\\
&+2(x^3+y^3)^4 && \text{(from } T_4, T_8\text{)}\\
&+7(x^2+y^2)^6 && \text{(from } T_6, T_1I, T_3I, T_5I, T_7I, T_9I, T_{11}I\text{)}\\
&+6(x+y)^2(x^2+y^2)^5\Big], && \text{(from } I, T_2I, T_4I, T_6I, T_8I, T_{10}I\text{)}
\end{aligned}$$

$$\begin{aligned}
&x^{12}+x^{11}y+6x^{10}y^2+12x^9y^3+29x^8y^4+38x^7y^5+50x^6y^6\\
&+38x^5y^7+29x^4y^8+12x^3y^9+6x^2y^{10}+xy^{11}+y^{12}.
\end{aligned}$$

$G\backslash k$	1	2	3	4	5	6	7	8	9	10	11	12
C_{12}	1	6	19	43	66	80	66	43	19	6	1	1
D_{12}	1	6	12	29	38	50	38	29	12	6	1	1
$\mathrm{Aff}_1(Z_{12})$	1	5	9	21	25	34	25	21	9	5	1	1

Calcul d'orbites via le polynôme de Polya

Transformation	Cycle representation	No. of cycles	Fixed configs.	Cycle type	Cycle index
T_0	(0)(1)(2)(3)(4)(5)(6)(7)(8)(9)(A)(B)	12	$2^{12} = 4096$	1^{12}	t_1^{12}
T_1	(0 1 2 3 4 5 6 7 8 9 A B)	1	$2^1 = 2$	12^1	t_{12}^1
T_2	(0 2 4 6 8 A)(1 3 5 7 9 B)	2	$2^2 = 4$	6^2	t_6^2
T_3	(0 3 6 9)(1 4 7 A)(2 5 8 B)	3	$2^3 = 8$	4^3	t_4^3
T_4	(0 4 8)(1 5 9)(2 6 A)(3 7 B)	4	$2^4 = 16$	3^4	t_3^4
T_5	(0 5 A 3 8 1 6 B 4 9 2 7)	1	$2^1 = 2$	12^1	t_{12}^1
T_6	(0 6)(1 7)(2 8)(3 9)(4 A)(5 B)	6	$2^6 = 64$	2^6	t_2^6
T_7	(0 7 2 9 4 B 6 1 8 3 A 5)	1	$2^1 = 2$	12^1	t_{12}^1
T_8	(0 8 4)(1 9 5)(2 A 6)(3 B 7)	4	$2^4 = 16$	3^4	t_3^4
T_9	(0 9 6 3)(1 A 7 4)(2 B 8 5)	3	$2^3 = 8$	4^3	t_4^3
T_{10}	(0 A 8 6 4 2)(1 B 9 7 5 3)	2	$2^2 = 4$	6^2	t_6^2
T_{11}	(0 B A 9 8 7 6 5 4 3 2 1)	1	$2^1 = 2$	12^1	t_{12}^1

?

d'accords = 1/12[4096+2+4+8+16+2+64+2+16+8+4+2]=4224/12=352

$G\backslash k$	1	2	3	4	5	6	7	8	9	10	11	12
C_{12}	1	6	19	43	66	80	66	43	19	6	1	1
D_{12}	1	6	12	29	38	50	38	29	12	6	1	1
$\mathrm{Aff}_1(Z_{12})$	1	5	9	21	25	34	25	21	9	5	1	1

Calcul d'orbites via le polynôme de Polya

Transformation	*Cycle representation*	*No. of cycles*	*Fixed configs.*	*Cycle type*	*Cycle index*
T_0	(0)(1)(2)(3)(4)(5)(6)(7)(8)(9)(A)(B)	12	$2^{12}=4096$	1^{12}	t_1^{12}
T_1	(0 1 2 3 4 5 6 7 8 9 A B)	1	$2^1=2$	12^1	t_{12}^1
T_2	(0 2 4 6 8 A)(1 3 5 7 9 B)	2	$2^2=4$	6^2	t_6^2
T_3	(0 3 6 9)(1 4 7 A)(2 5 8 B)	3	$2^3=8$	4^3	t_4^3
T_4	(0 4 8)(1 5 9)(2 6 A)(3 7 B)	4	$2^4=16$	3^4	t_3^4
T_5	(0 5 A 3 8 1 6 B 4 9 2 7)	1	$2^1=2$	12^1	t_{12}^1
T_6	(0 6)(1 7)(2 8)(3 9)(4 A)(5 B)	6	$2^6=64$	2^6	t_2^6
T_7	(0 7 2 9 4 B 6 1 8 3 A 5)	1	$2^1=2$	12^1	t_{12}^1
T_8	(0 8 4)(1 9 5)(2 A 6)(3 B 7)	4	$2^4=16$	3^4	t_3^4
T_9	(0 9 6 3)(1 A 7 4)(2 B 8 5)	3	$2^3=8$	4^3	t_4^3
T_{10}	(0 A 8 6 4 2)(1 B 9 7 5 3)	2	$2^2=4$	6^2	t_6^2
T_{11}	(0 B A 9 8 7 6 5 4 3 2 1)	1	$2^1=2$	12^1	t_{12}^1

$$\tfrac{1}{12}\left[(x+y)^{12}+4(x^{12}+y^{12})+2(x^6+y^6)^2+2(x^4+y^4)^3+2(x^3+y^3)^4+(x^2+y^2)^6\right],$$

$$x^{12}+x^{11}y+6x^{10}y^2+19x^9y^3+43x^8y^4+66x^7y^5+80x^6y^6$$
$$+66x^5y^7+43x^4y^8+19x^3y^9+6x^2y^{10}+xy^{11}+y^{12}.$$

$G\backslash k$	1	2	3	4	5	6	7	8	9	10	11	12
C_{12}	1	6	19	43	66	80	66	43	19	6	1	1

Enumération d'accords et dans un système tempéré

D. Reiner: «Enumeration in Music Theory», *Amer. Math. Month*. 92:51-54, 1985

Z_n

$$\#\text{ of } k\text{-chords} = \frac{1}{n}\sum_{j|(n,k)} \phi(j)\binom{n/j}{k/j} = \frac{1}{n}\Phi_n(k),$$

D_n

$$\#\text{ of } k\text{-chords} = \begin{cases} \frac{1}{2n}\left[\Phi_n(k) + n\binom{(n-1)/2}{[k/2]}\right], & \text{if } n \text{ is odd,} \\ \frac{1}{2n}\left[\Phi_n(k) + n\binom{n/2}{k/2}\right], & \text{if } n \text{ is even and } k \text{ is even,} \\ \frac{1}{2n}\left[\Phi_n(k) + n\binom{(n/2)-1}{[k/2]}\right], & \text{if } n \text{ is even and } k \text{ is odd.} \end{cases}$$

$G\backslash k$	1	2	3	4	5	6	7	8	9	10	11	12
C_{12}	1	6	19	43	66	80	66	43	19	6	1	1
D_{12}	1	6	12	29	38	50	38	29	12	6	1	1
$\text{Aff}_1(Z_{12})$	1	5	9	21	25	34	25	21	9	5	1	1

Enumération des modes à transpositions limitées de Messiaen

R.C. Read: « Combinatorial problems in the theory of music », *Discrete Math.*, 1997

M. Broué : « Les tonalités musicales vues par un mathématicien », 2002

$$A_n = \sum_{k|n} \mu\left(\frac{n}{k}\right) 2^k$$

$$s_d(n) = \sum_{\{e\,;\,(e|(n/d))\}} \mu\left(\frac{n/d}{e}\right) 2^e$$

$$\begin{cases} \mu(k) = 0 & \text{si } k \text{ est divisible par un carré,} \\ \mu(k) = (-1)^m & \text{si } k \text{ est produit de } m \text{ nombres premiers distincts.} \end{cases}$$

$A_6 = \mu(6)2 + \mu(3)2^2 + \mu(2)2^3 + \mu(1)2^6 =$

$= (-1)^2 2 + (-1)2^2 + (-1)\ 2^3 + 2^2 =$

$= 2 - 4 - 8 + 64 =$

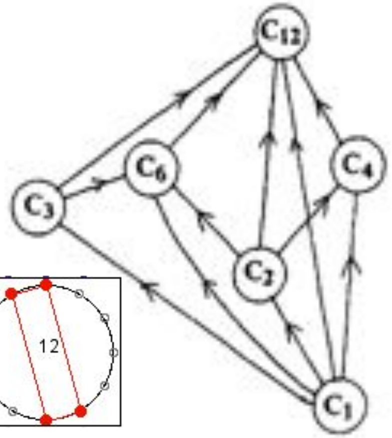
$= 54$

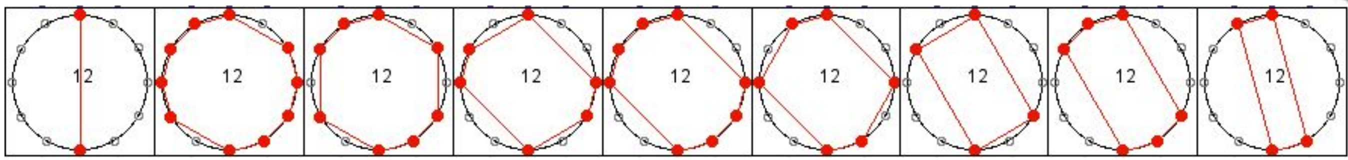
Table 1

Number of notes Symmetry	0	1	2	3	4	5	6	7	8	9	10	11	12
1		1	5	18	40	66	75	66	40	18	5	1	
2			1		2		3		2		1		
3				1			1			1			
4					1				1				
6							1						
12	1												1
All scales	1	1	6	19	43	66	80	66	43	19	6	1	1

$54/6 = 9$

$12/6 = 2$





Enumération des séries dodécaphoniques (Lemme de Burnside)

D. Reiner: «Enumeration in Music Theory»,
Amer. Math. Month. 92:51-54, 1985

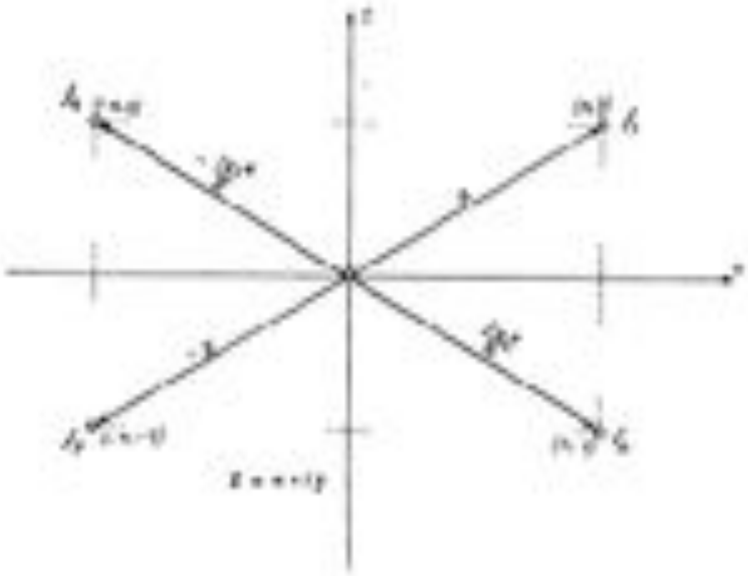
Lemme de Burnside

$$n = \frac{1}{|G|}\sum_{g\in G} |X^g|.$$

$$X^g = \{x\in X : gx=x\}$$

↑ ↑ ↑ ↑ ↑ ↑ ↑ ↑ ↑ ↑ ↑ ↑

0 1 2 3 4 5 6 7 8 9 10 11



Z = x + yi

f_1 = Z = x + yi = Z = $f_1(Z)$ = original form

f_2 = x - yi = … = $f_2(Z)$ = inversion

f_3 = -x - yi = -Z = $f_3(Z)$ = inverted retrogradation

f_4 = -x + yi = … = $f_4(Z)$ = retrogradation

Thus the number of *n*-tone rows is

$$\tfrac{1}{4}[(n-1)! + (n-1)(n-3)\cdots(2)] \qquad \text{if } n \text{ is odd;}$$
$$\tfrac{1}{4}[(n-1)! + (n-2)(n-4)\cdots(2)(1+n/2)] \qquad \text{if } n \text{ is even.}$$

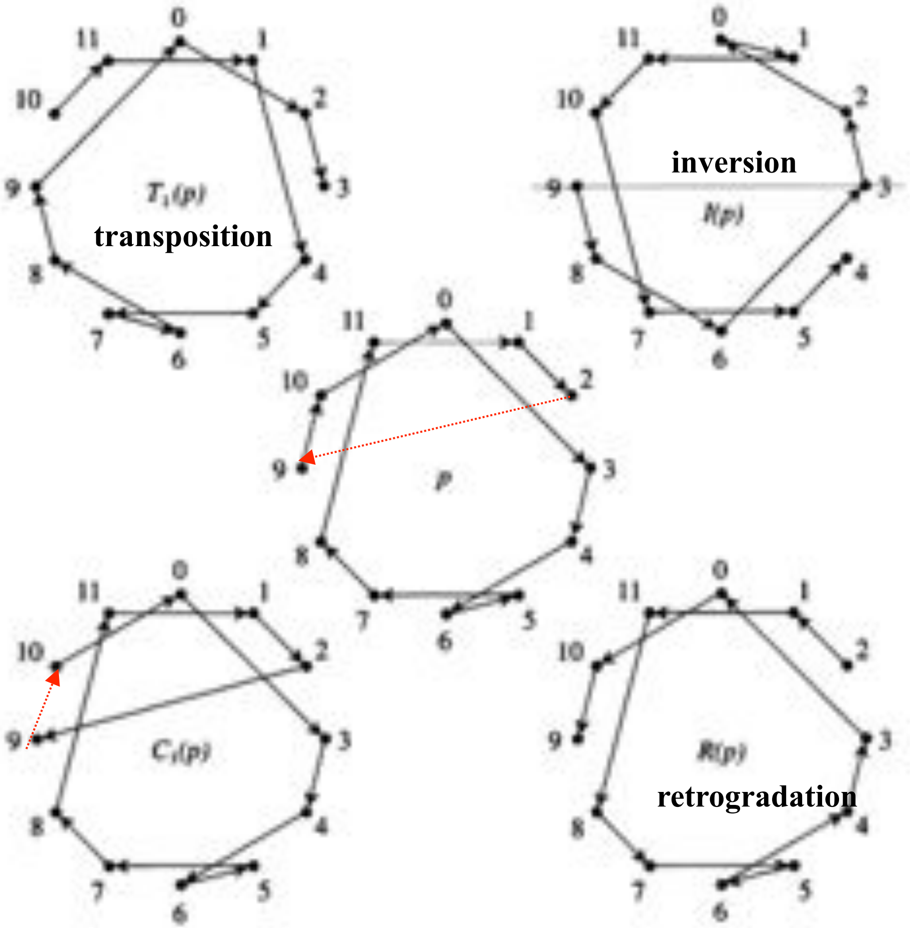
For example, there are 9985920 twelve tone rows, a fact which does not seem to be in the literature.

Enumération des séries dodécaphoniques (Lemme de Burnside)

R.C. Read: « Combinatorial problems in the theory of music », *Discrete Math.*, 1997

 # orbites = # superpositions de deux graphes cycliques de longueur 12.

836.017 séries

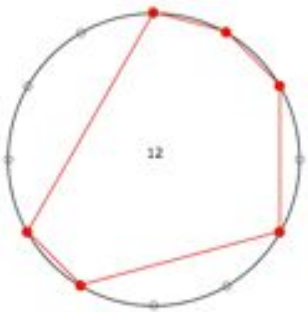


« rotation »
(permutation circulaire)

Elliott Carter: 90+ (1994)

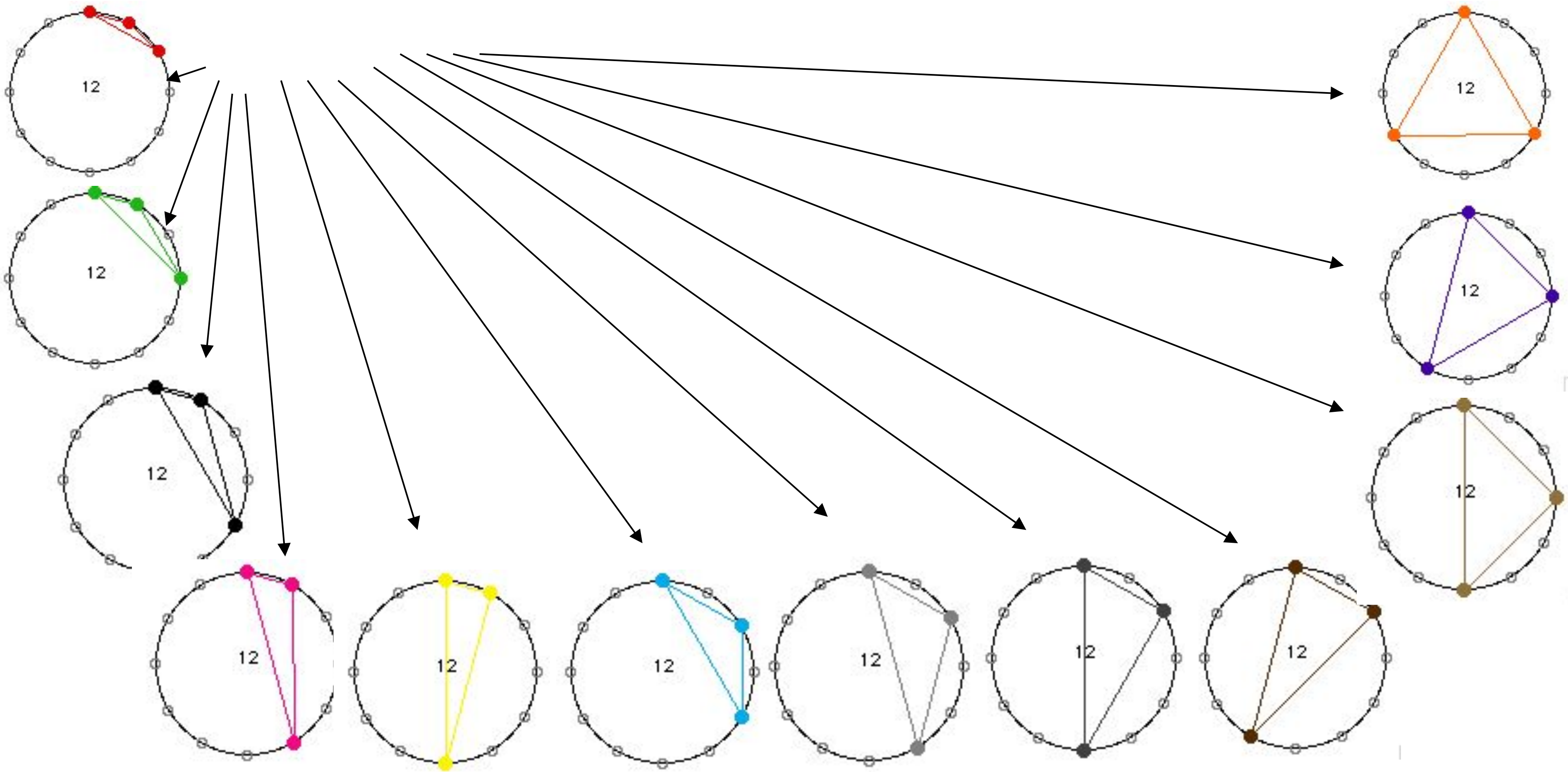
$G \backslash k$	1	2	3	4	5	6	7	8	9	10	11	12
C_{12}	1	6	19	43	66	80	66	43	19	6	1	1
D_{12}	1	6	12	29	38	50	38	29	12	6	1	1
$\mathrm{Aff}_1(Z_{12})$	1	5	9	21	25	34	25	21	9	5	1	1

35



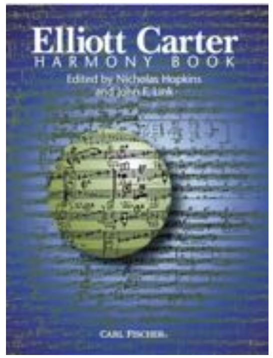
pcset	*vector*	*name*
(0 1 2 4 7 8)	[322332]	6-Z17

All-triad hexachord

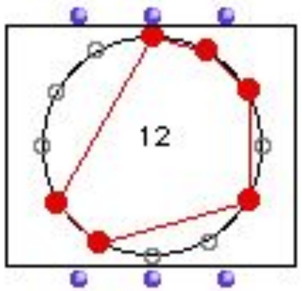


Elliott Carter: 90+ (1994)

« From about 1990, I have reduced my vocabulary of chords more and more to the six note chord n° 35 and the four note chords n° 18 and 23, which encompass all the intervals » (*Harmony Book*, 2002, p. ix)

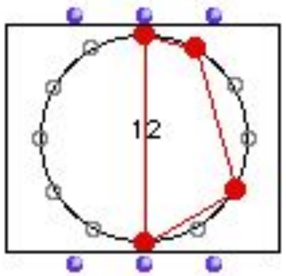


35

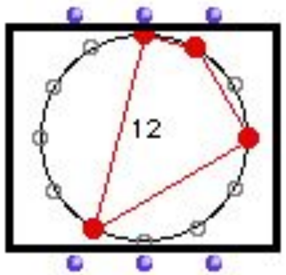


All-triad hexachord

18



23



	pcset	*vector*	*name*
35	(0 1 2 4 7 8)	[322332]	6-Z17
18	(0 1 4 6)	[111111]	4-Z15
23	(0 1 3 7)	[111111]	4-Z29

➔ 

07-carter_athexa

La *Set Theory* d'Allen Forte: catalogue des *pitch-class sets*

complementare

name	pcs	vector
5-30	0,1,4,6,8	121321
5-31	0,1,3,6,9	114112
5-32	0,1,4,6,9	113221
5-33(12)	0,2,4,6,8	040402
5-34(12)	0,2,4,6,9	032221
5-35(12)	0,2,4,7,9	032140
5-Z36	0,1,2,4,7	222121
5-Z37(12)	0,3,4,5,8	212320
5-Z38	0,1,2,5,8	212221
6-1(12)	0,1,2,3,4,5	543210
6-2	0,1,2,3,4,6	443211
6-Z4(12)	0,1,2,4,5,6	432321
6-5	0,1,2,3,6,7	422232
6-Z6(12)	0,1,2,5,6,7	421242
6-7(6)	0,1,2,6,7,8	420243
6-8(12)	0,2,3,4,5,7	343230
6-9	0,1,2,3,5,7	342231
6-Z10	0,1,3,4,5,7	333321
6-Z11	0,1,2,4,5,7	333231
6-Z12	0,1,2,4,6,7	332232
6-Z13(12)	0,1,3,4,6,7	324222

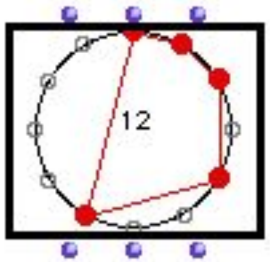
name	pcs	vector
7-30	0,1,2,4,6,8,9	343542
7-31	0,1,3,4,6,7,9	336333
7-32	0,1,3,4,6,8,9	335442
7-33	0,1,2,4,6,8,10	262623
7-34	0,1,3,4,6,8,10	254442
7-35	0,1,3,5,6,8,10	254361
7-Z36	0,1,2,3,5,6,8	444342
7-Z37	0,1,3,4,5,7,8	434541
7-Z38	0,1,2,4,5,7,8	434442
6-Z37(12)	0,1,2,3,4,8	
6-Z38(12)	0,1,2,3,7,8	
6-Z39	0,2,3,4,5,8	
6-Z40	0,1,2,3,5,8	
6-Z41	0,1,2,3,6,8	
6-Z42(12)	0,1,2,3,6,9	

5-Z36	0,1,2,4,7	222121

7-Z36	0,1,2,3,5,6,8	444342

Relation Z

Vecteur d'intervalles et relation Z

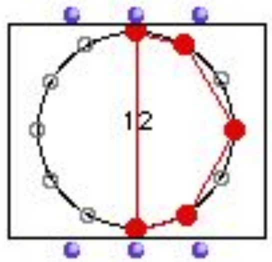


5-30	0,1,4,6,8	121321
5-31	0,1,3,6,9	114112
5-32	0,1,4,6,9	113221
5-33(12)	0,2,4,6,8	040402
5-34(12)	0,2,4,6,9	032221
5-35(12)	0,2,4,7,9	032140
5-Z36	0,1,2,4,7	222121
5-Z37(12)	0,3,4,5,8	212320
5-Z38	0,1,2,5,8	212221
6-1(12)	0,1,2,3,4,5	543210
6-2	0,1,2,3,4,6	443211
6-Z4(12)	0,1,2,4,5,6	432321
6-5	0,1,2,3,6,7	422232
6-Z6(12)	0,1,2,5,6,7	421242
6-7(6)	0,1,2,6,7,8	420243
6-8(12)	0,2,3,4,5,7	343230
6-9	0,1,2,3,5,7	342231
6-Z10	0,1,3,4,5,7	333321
6-Z11	0,1,2,4,5,7	333231
6-Z12	0,1,2,4,6,7	332232
6-Z13(12)	0,1,3,4,6,7	324222

5-Z36 0,1,2,4,7 222121

7-30	0,1,2,4,6,8,9	343542
7-31	0,1,3,4,6,7,9	336333
7-32	0,1,3,4,6,8,9	335442
7-33	0,1,2,4,6,8,10	262623
7-34	0,1,3,4,6,8,10	254442
7-35	0,1,3,5,6,8,10	254361
7-Z36	0,1,2,3,5,6,8	444342
7-Z37	0,1,3,4,5,7,8	434541
7-Z38	0,1,2,4,5,7,8	434442

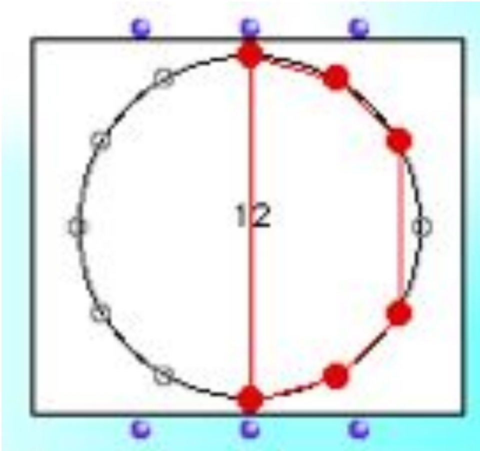
6-Z36	0,1,2,3,4,7
6-Z37(12)	0,1,2,3,4,8
6-Z38(12)	0,1,2,3,7,8
6-Z39	0,2,3,4,5,8
6-Z40	0,1,2,3,5,8
6-Z41	0,1,2,3,6,8
6-Z42(12)	0,1,2,3,6,9



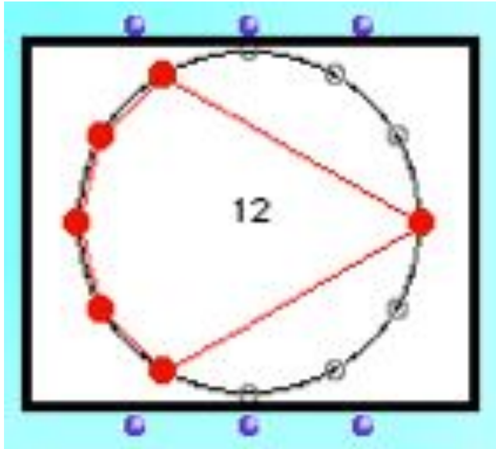
5-Z12

Théorème de l'hexacorde (ou théorème de Babbitt)

(Wilcox, Ralph Fox (?), Chemillier, Lewin, Mazzola, Schaub, …, Amiot [2006])



A

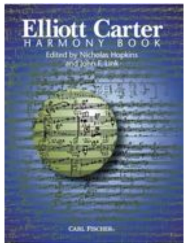


A'

$$\mathrm{IV}(A) = [4, 3, 2, 3, 2, 1\ ] = [4, 3, 2, 3, 2, 1\ ] = \mathrm{IV}(A')$$

Un hexacorde et son complementaire ont le même vecteur d'intervalles

Elliott Carter : 90+ (1994)

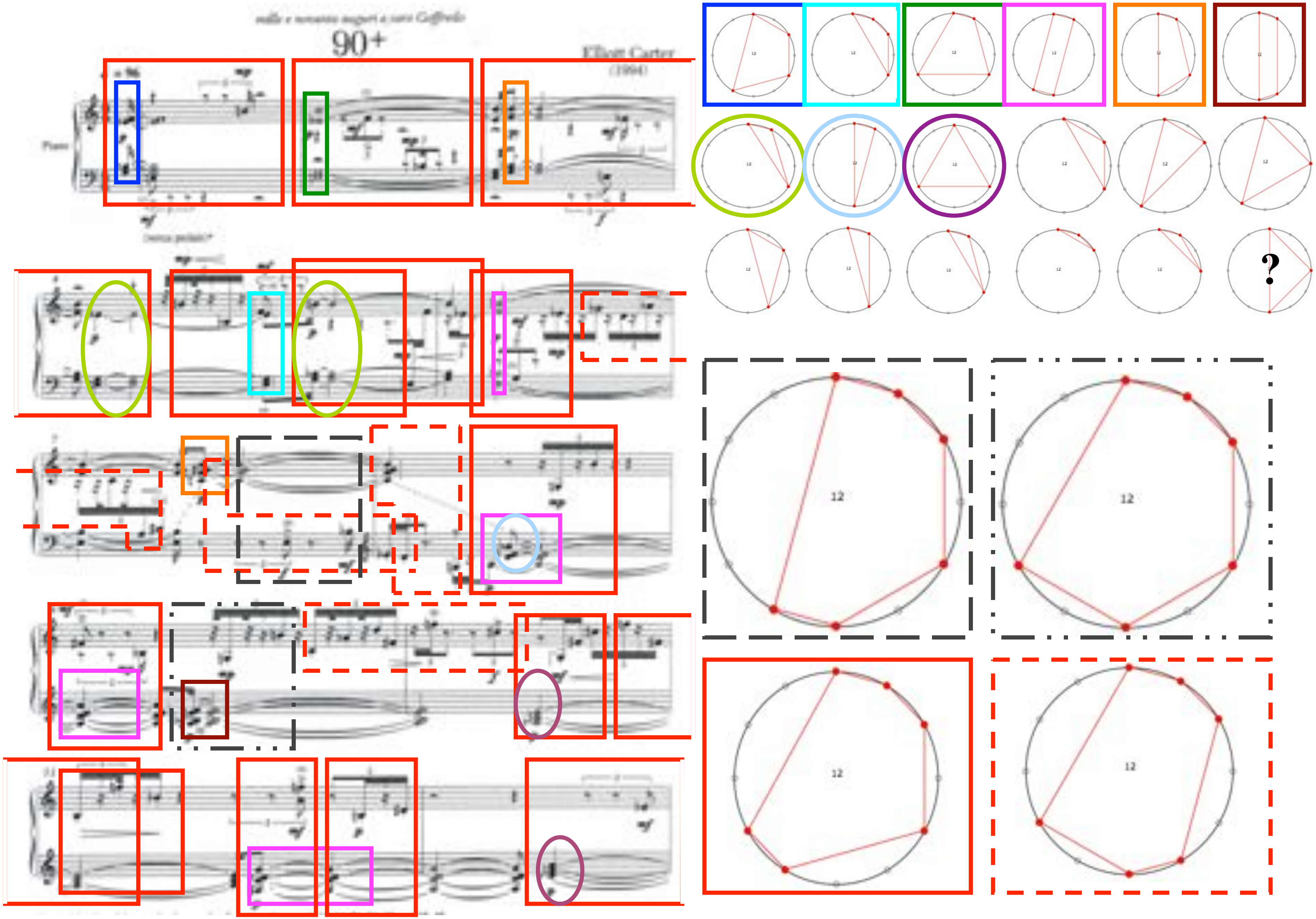


- **Combinatoire d'accords**
  - **Hexacordes**
  - **Tétracordes**
  - **Triades**
  - **Relation Z**

- **Séries tous–intervalles**
  - ***Link–chords***

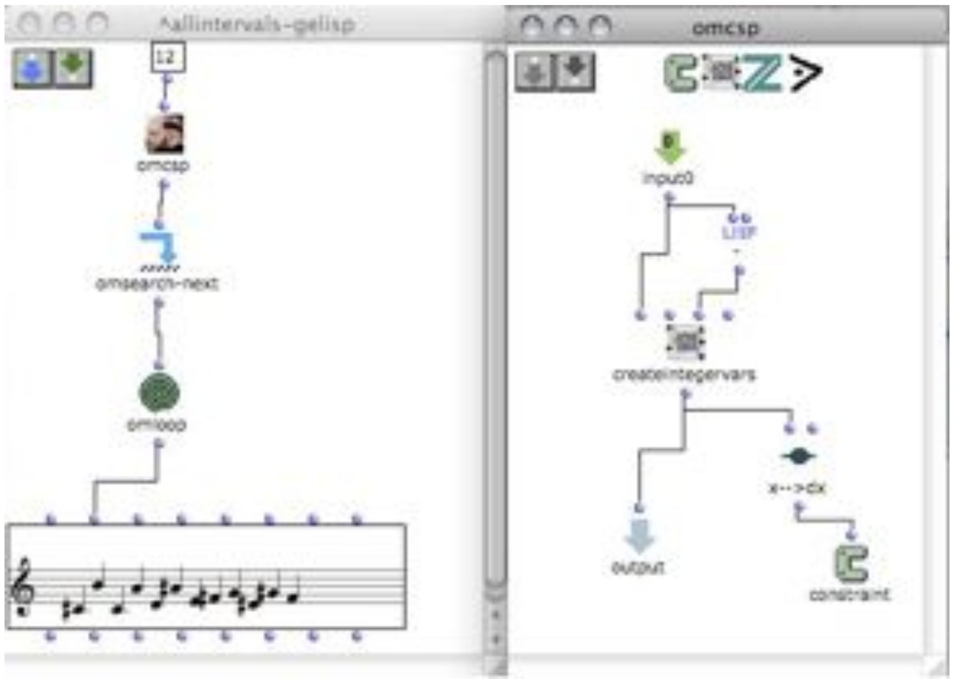
(piano: John Snijders)

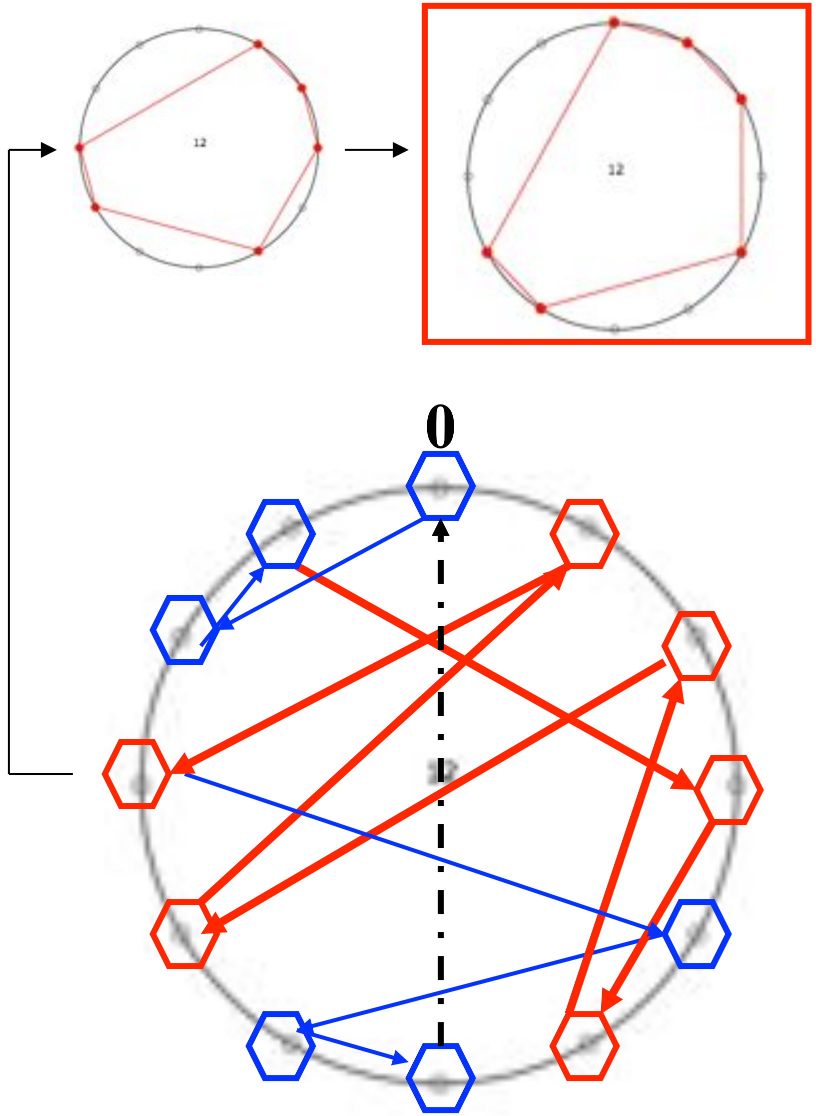
Elliott Carter : 90+ (1994) : combinatoire tetra/tricordale



Elliott Carter : 90+ (1994) : Link–chords (mm. 49–68)

OM-> ((**0 10 11 3 5 2 8 1 9 4 7 6)** (1 2 3 5 8 9))
OM-> ((0 10 11 1 5 2 9 3 8 4 7 6) (1 2 3 5 8 9))
OM-> ((0 10 3 5 2 8 9 1 4 11 7 6) (1 2 3 5 8 9))
OM-> ((0 9 4 8 2 3 5 10 1 11 7 6) (0 2 3 4 8 9))
OM-> ((0 9 4 2 3 8 10 1 5 11 7 6) (0 2 3 4 8 9))
OM-> ((0 9 3 11 4 5 7 10 2 1 8 6) (3 4 5 7 10 11))
OM-> ((0 9 1 4 2 8 3 5 10 11 7 6) (3 5 6 7 10 11))
…





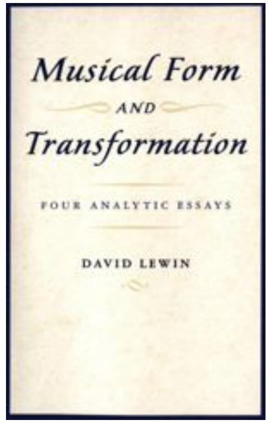
Mauricio Toro Universidad Javeriana, Colombia / IRCAM
http://gelisp.sourceforge.net/

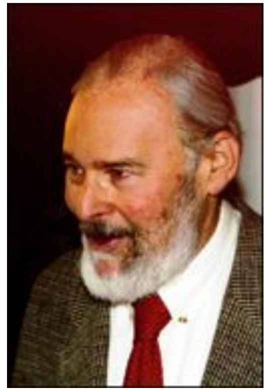
S = (0 10 11 3 5 2 8 1 9 4 7 6)
S*= (10 1 4 2 9 6 5 8 7 3 11)

« *Making* and *Using* a Pcset Network for Stockhausen's Klavierstück III »

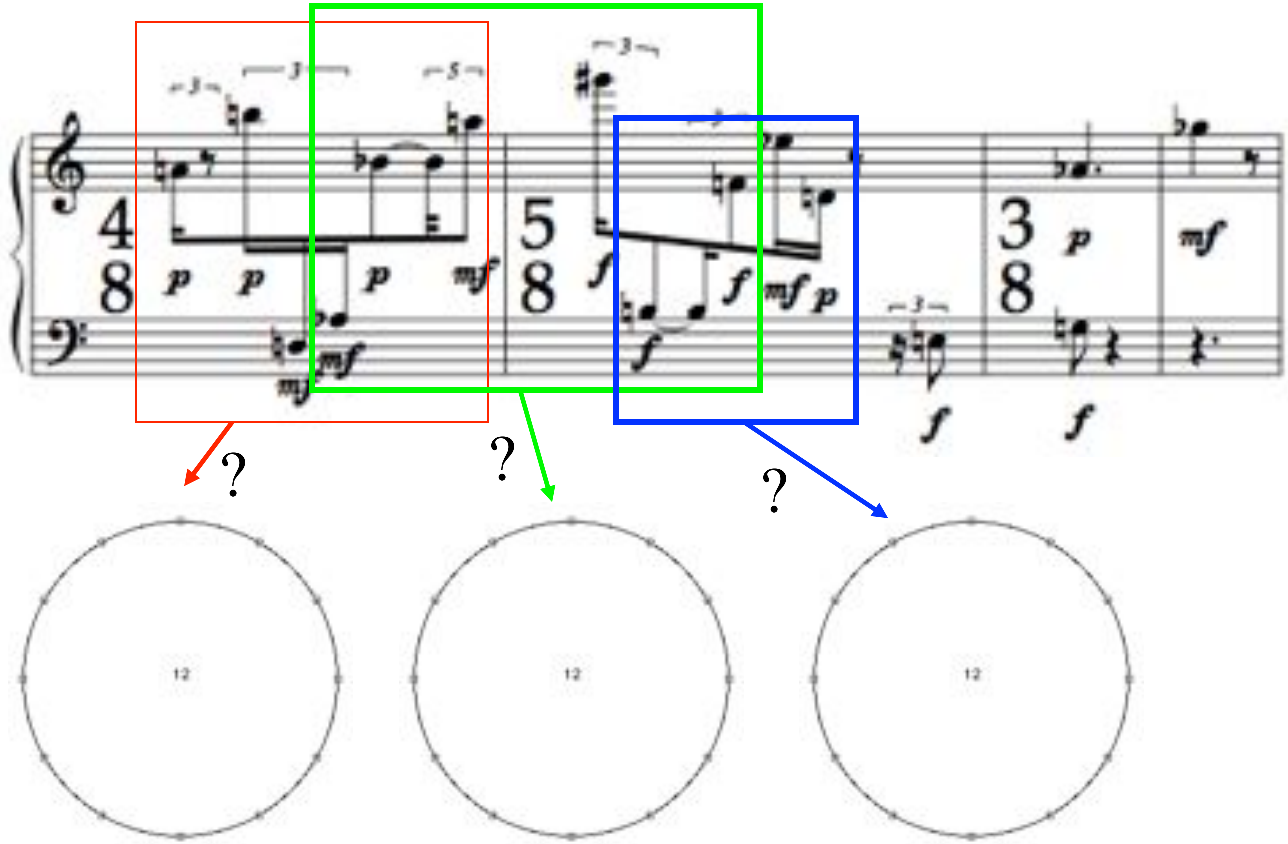
Trois interprétations :

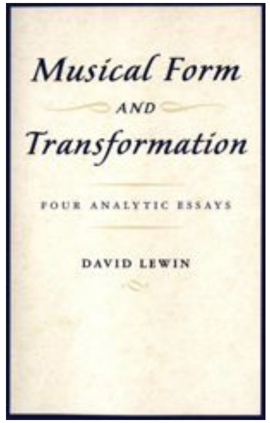
- **Henck**
- **Kontarsky**
- **Tudor**

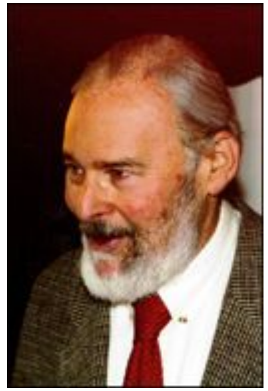




« *Making* and *Using* a Pcset Network for Stockhausen's Klavierstück III »





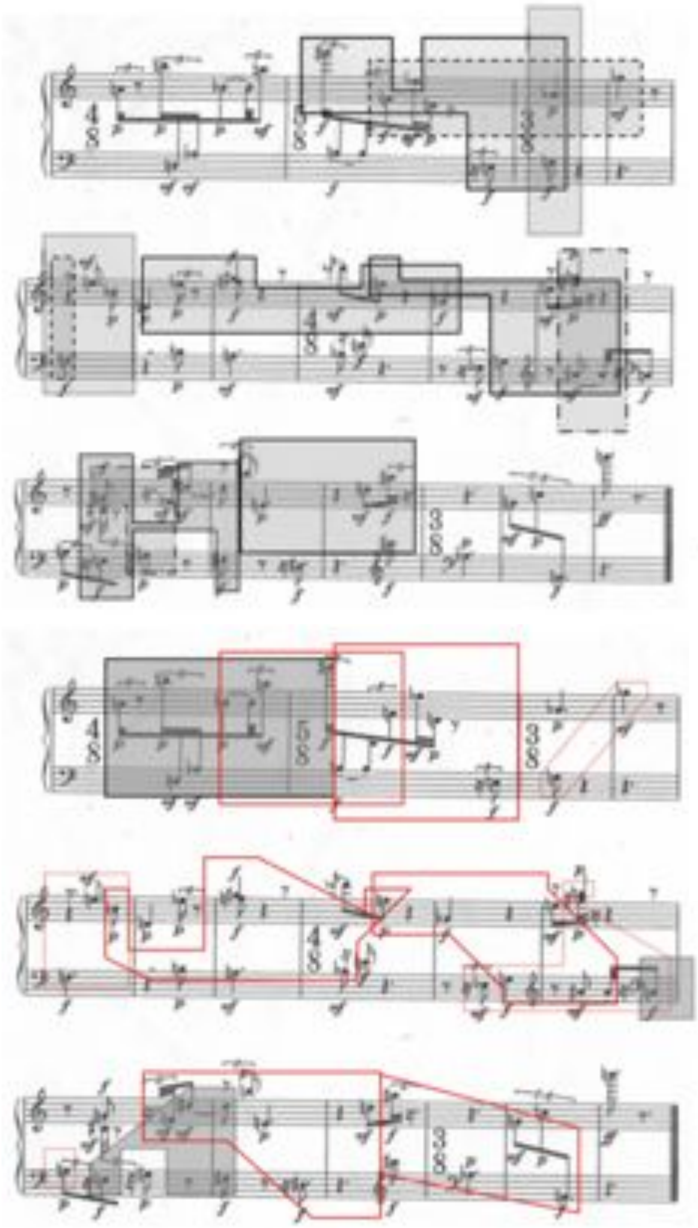


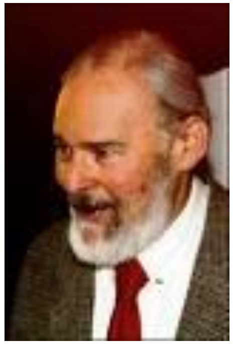
« The most ‘theoretical’ of the four essays, it focuses on the forms of one pentachord reasonably ubiquitous in the piece. A special **group of transformations** is developed, one suggested by the musical interrelations of the pentachord forms. Using that group, the essay arranges **all pentachord forms** of the music into a **spatial configuration** that illustrates network structure, for this particular phenomenon, over the entire piece. »

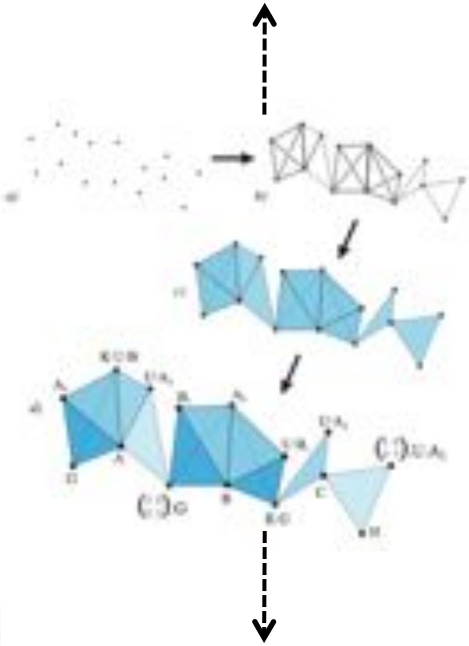
David Lewin, *Musical Form and Transformation,* YUP 1993

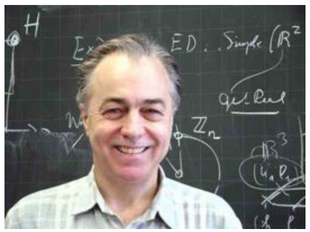
Vers une modélisation informatique de l'analyse transformationnelle

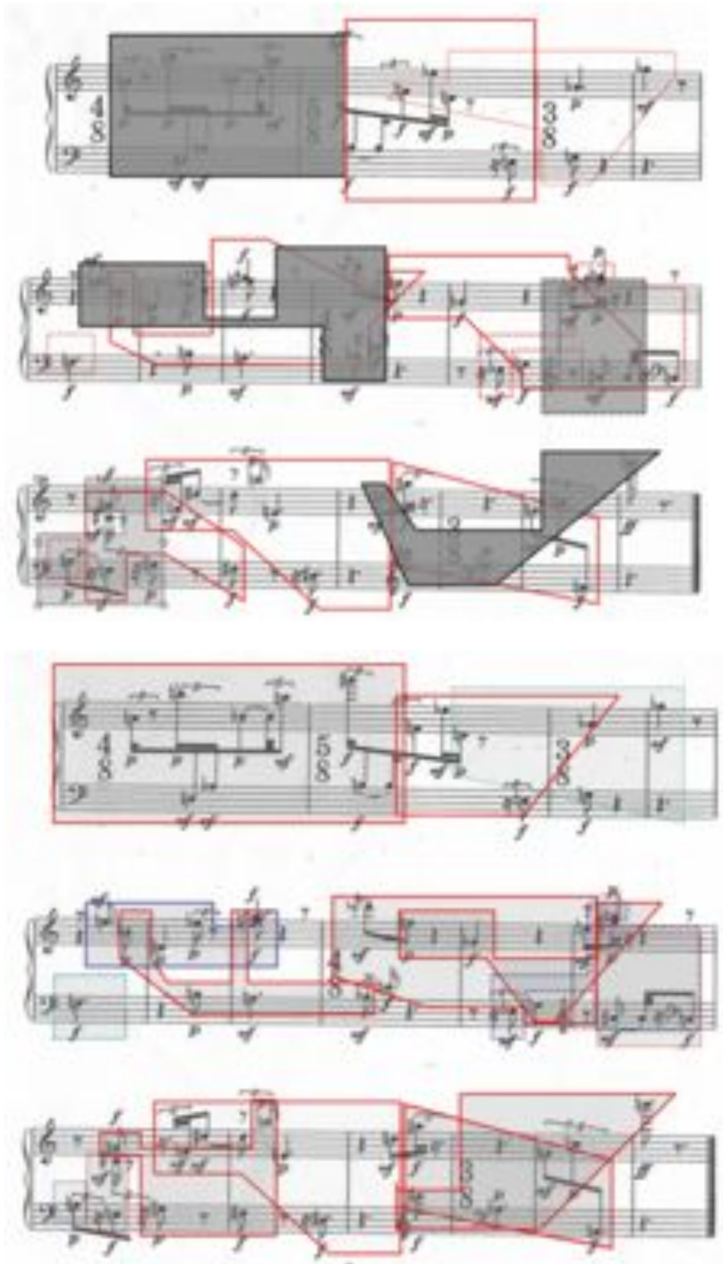
YunKang Ahn, L'analyse musicale computationnelle, thèse, Université de Paris VI / Ircam, déc 2009



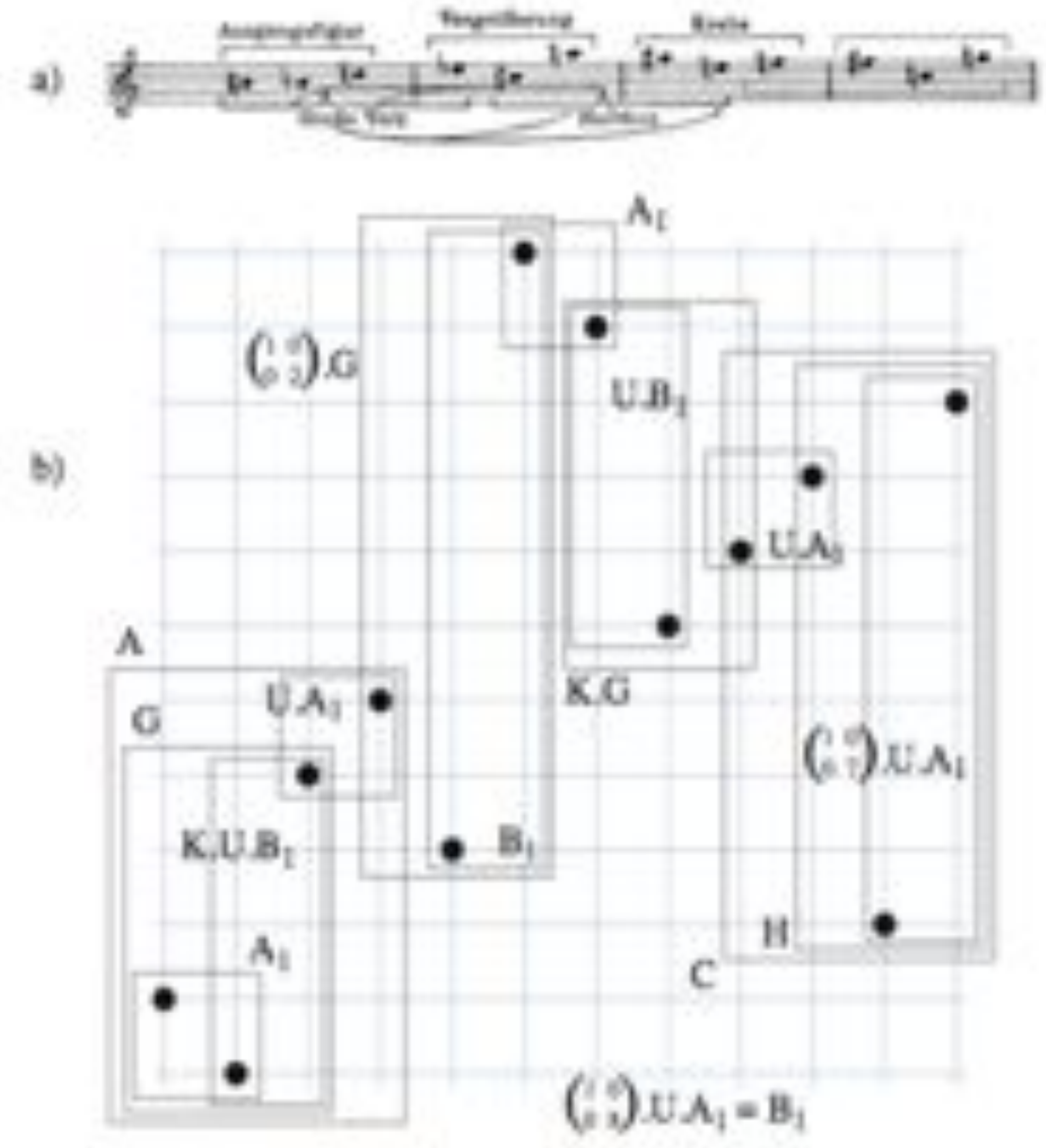


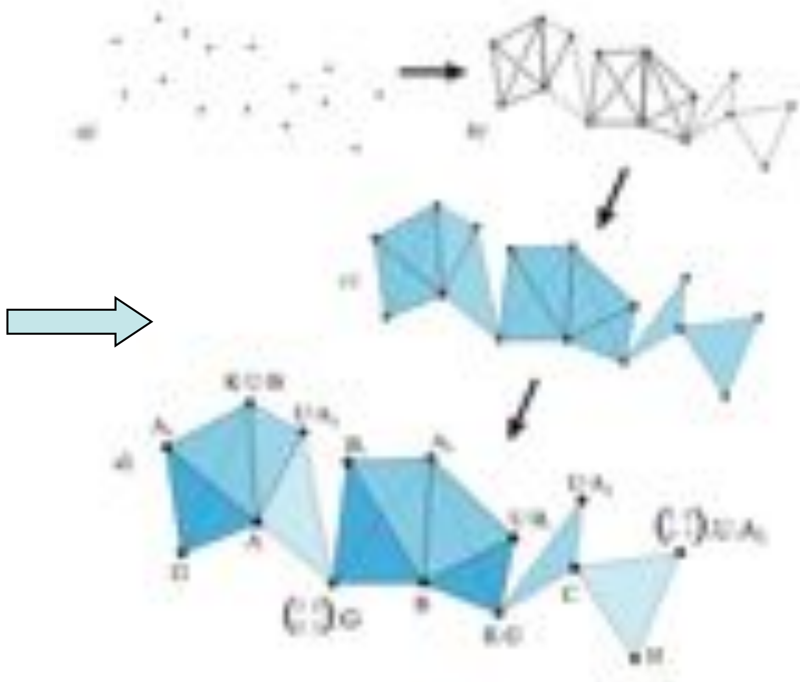




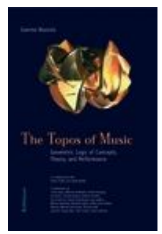


Nerf topologique et analyse musicale

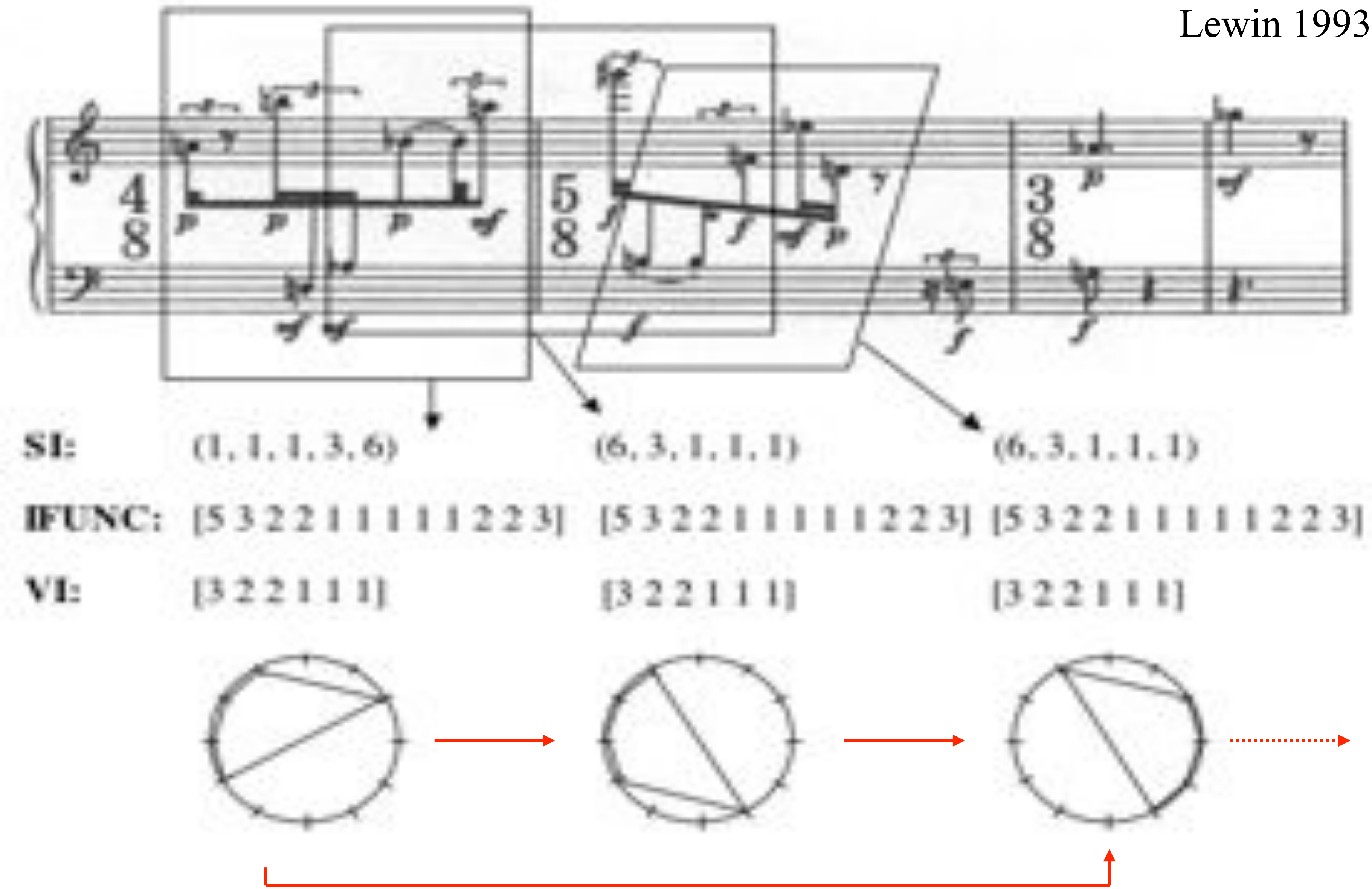




G. Mazzola : *The Topos of Music*, ch. 13 - “What are global compositions ?”

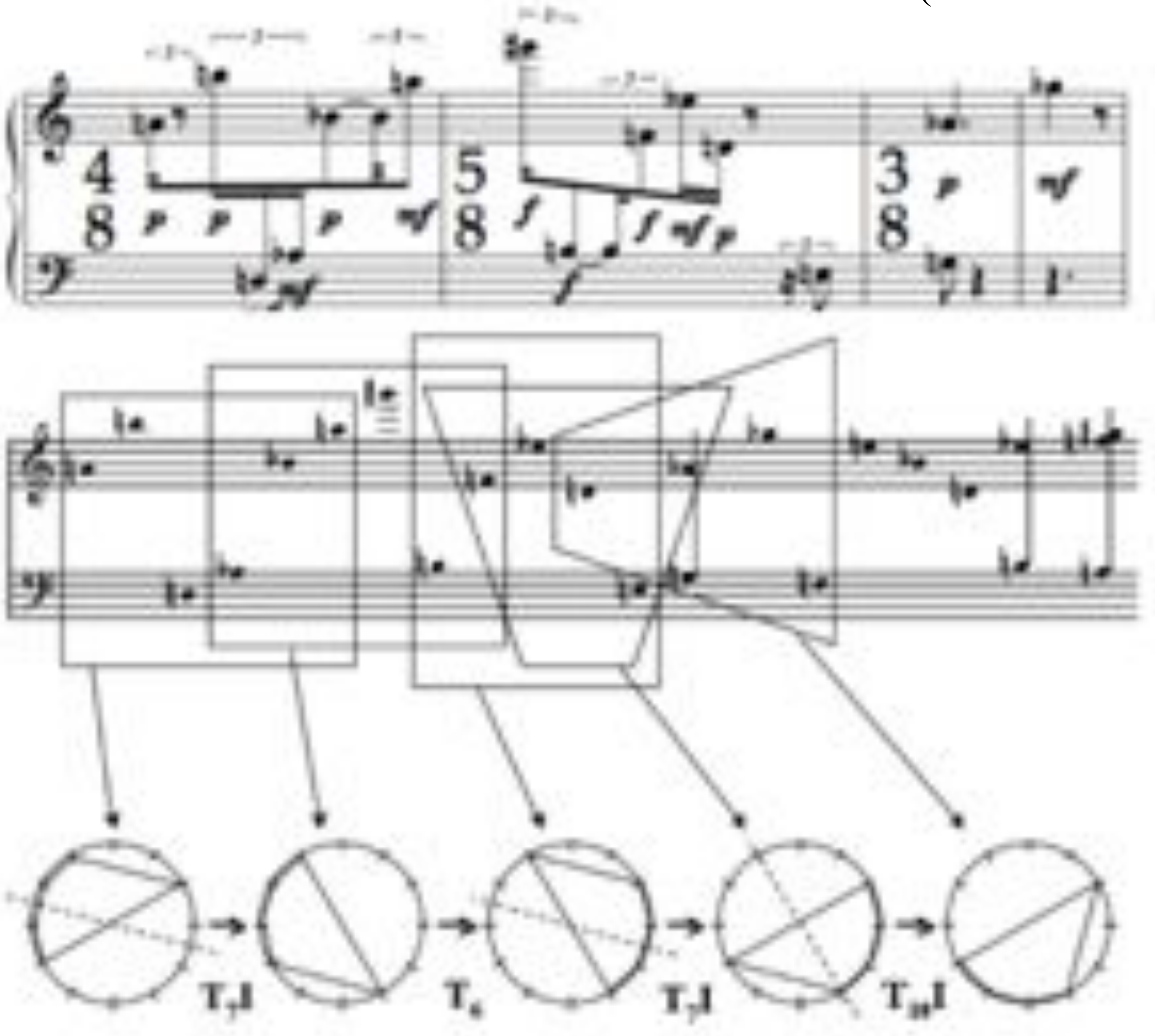


« *Making* and *Using* a Pcset Network for Stockhausen's Klavierstück III »

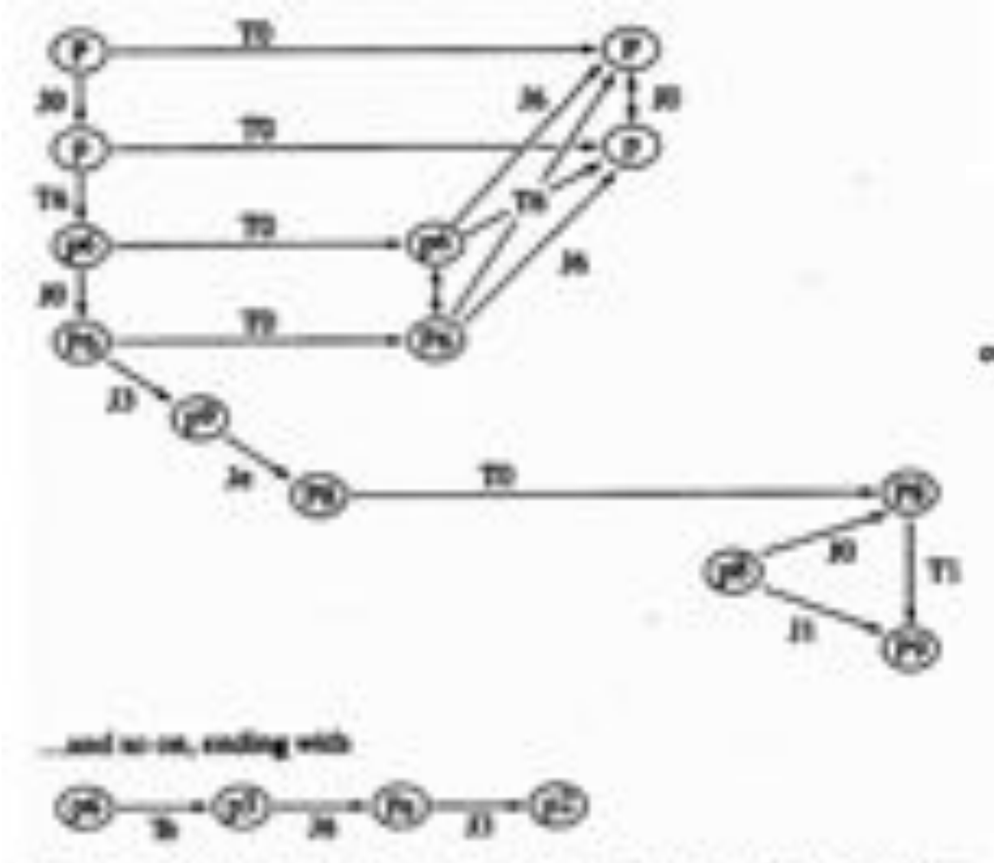


Segmentation par « imbriction »: progression transformationnelle

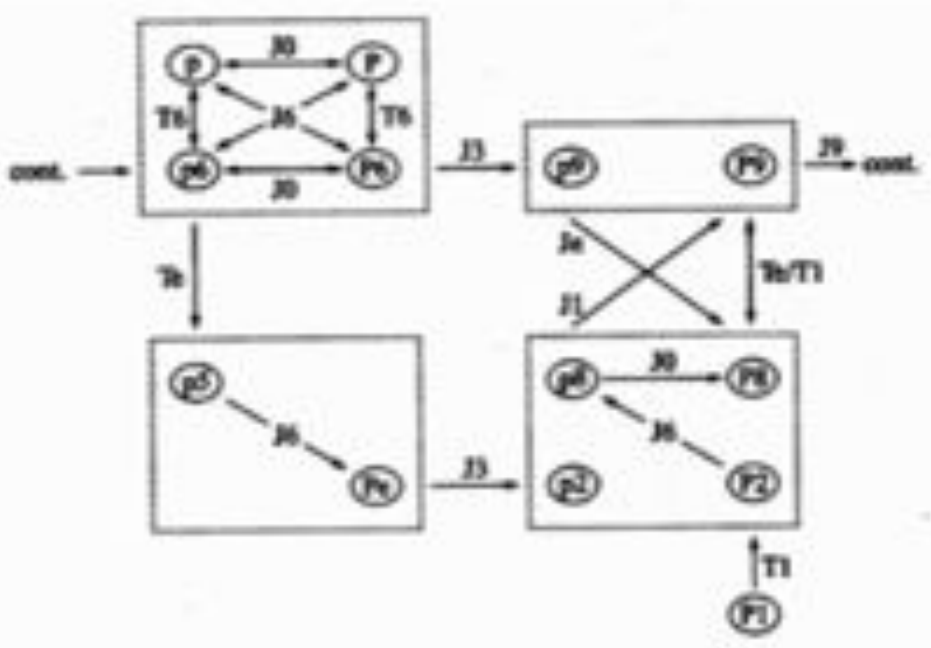
Stockhausen: *Klavierstück III* (Analisi di D. Lewin)



Progression transformationnelle *vs* réseau transformationnel



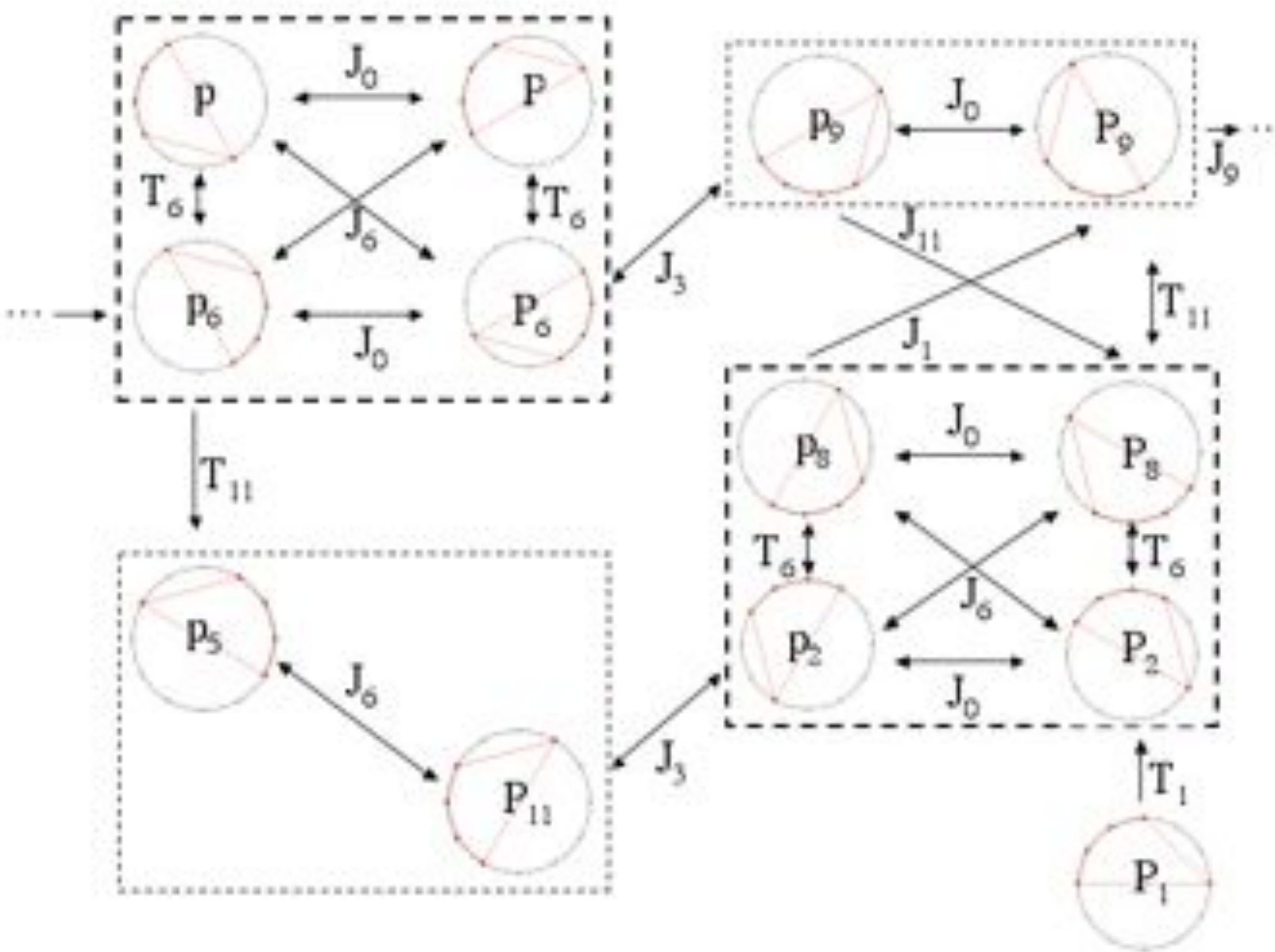
Example 2.4. A network whose left-to-right layout reflects the chronological progress of the piece through Pp forms.



« Rather than asserting a network that follows pentachord relations one at a time, according to the chronology of the piece, I shall assert instead a network that displays all the pentachord forms used and all their **potentially functional interrelationships**, in a very compactly organized little **spatial configuration**. »

Reseau transformationnel

Stockhausen: *Klavierstück III* (Analyse de D. Lewin)

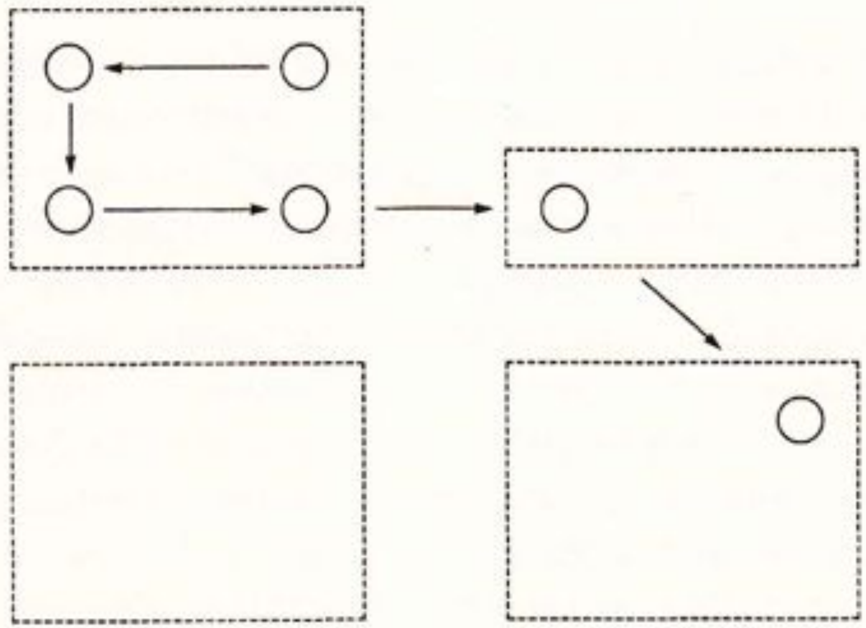


« […] the sequence of events moves within a clearly defined world of possible relationships, and because - in so moving - **it makes the abstract space of such a world accessible to our sensibilities**. That is to say that the story projects what one would traditionally call *form*. »

Parcours multiples d’écoute dans un réseau transformationnel

Stockhausen: *Klavierstück III* (Analyse de D. Lewin)

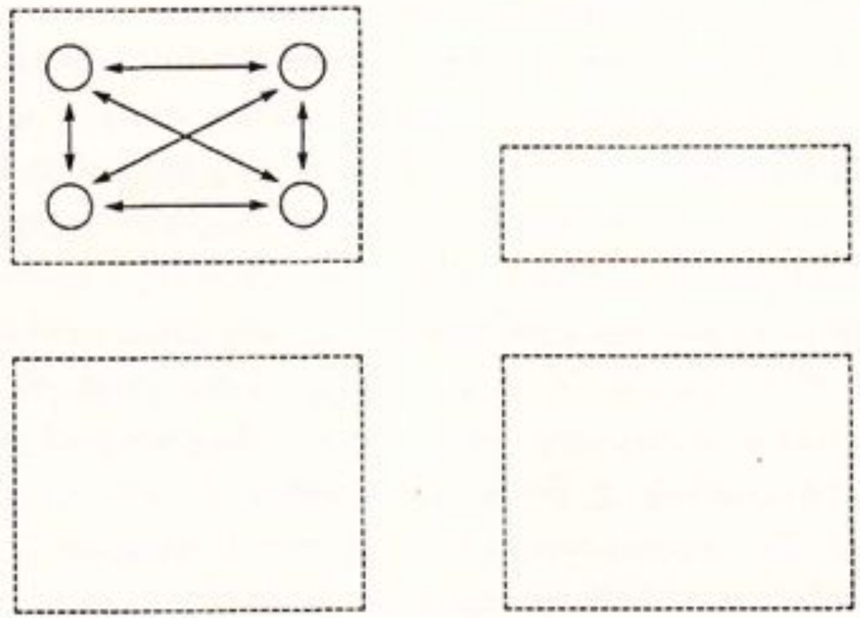
Pass 1 (mm. 1-5).



a

horizontal arrows within boxes = J0; between boxes = J3 or J9
vertical arrows within boxes = T6; between boxes = Te or T1
diagonal arrows within boxes = J6; between boxes = Je or J1

Pass 2 (mm. 5-8) goes back and elaborates the beginning area of pass 1.

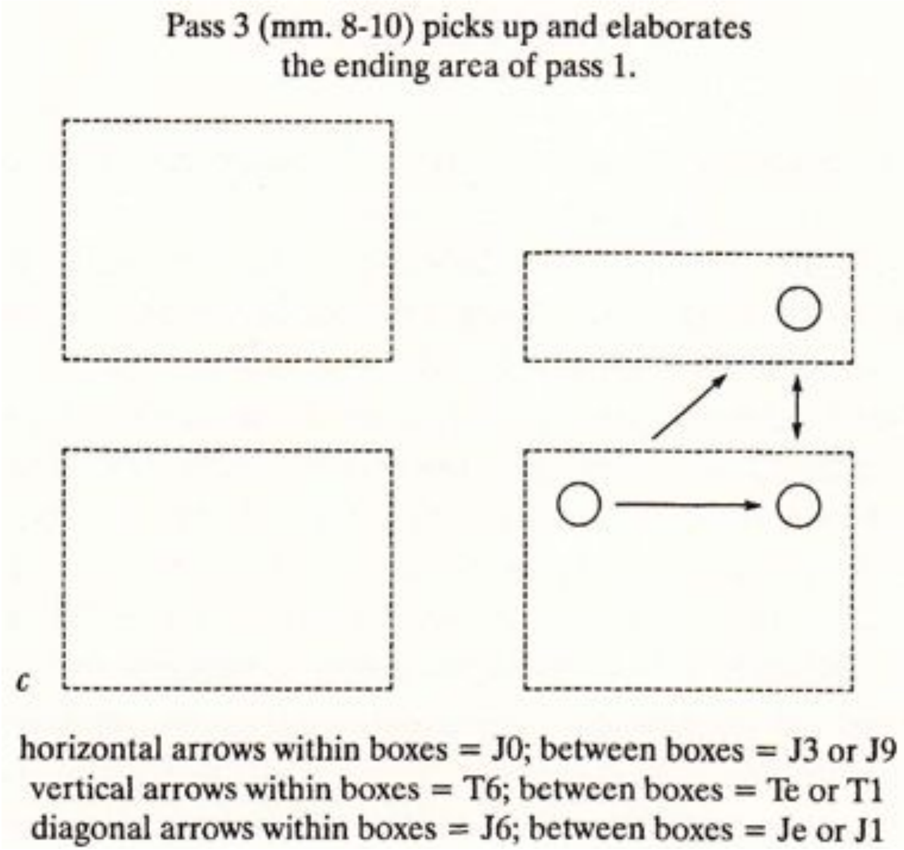


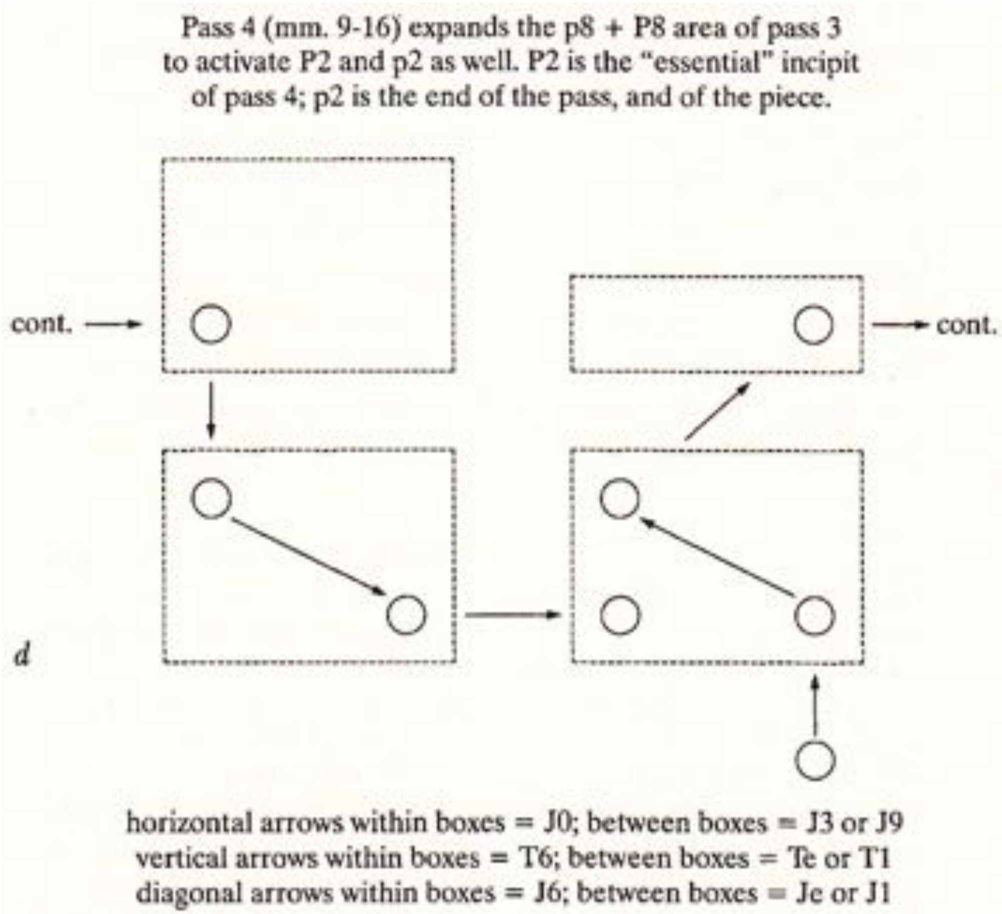
b

horizontal arrows within boxes = J0; between boxes = J3 or J9
vertical arrows within boxes = T6; between boxes = Te or T1
diagonal arrows within boxes = J6; between boxes = Je or J1

Parcours multiples d’écoute dans un réseau transformationnel

Stockhausen: *Klavierstück III* (Analyse de D. Lewin)





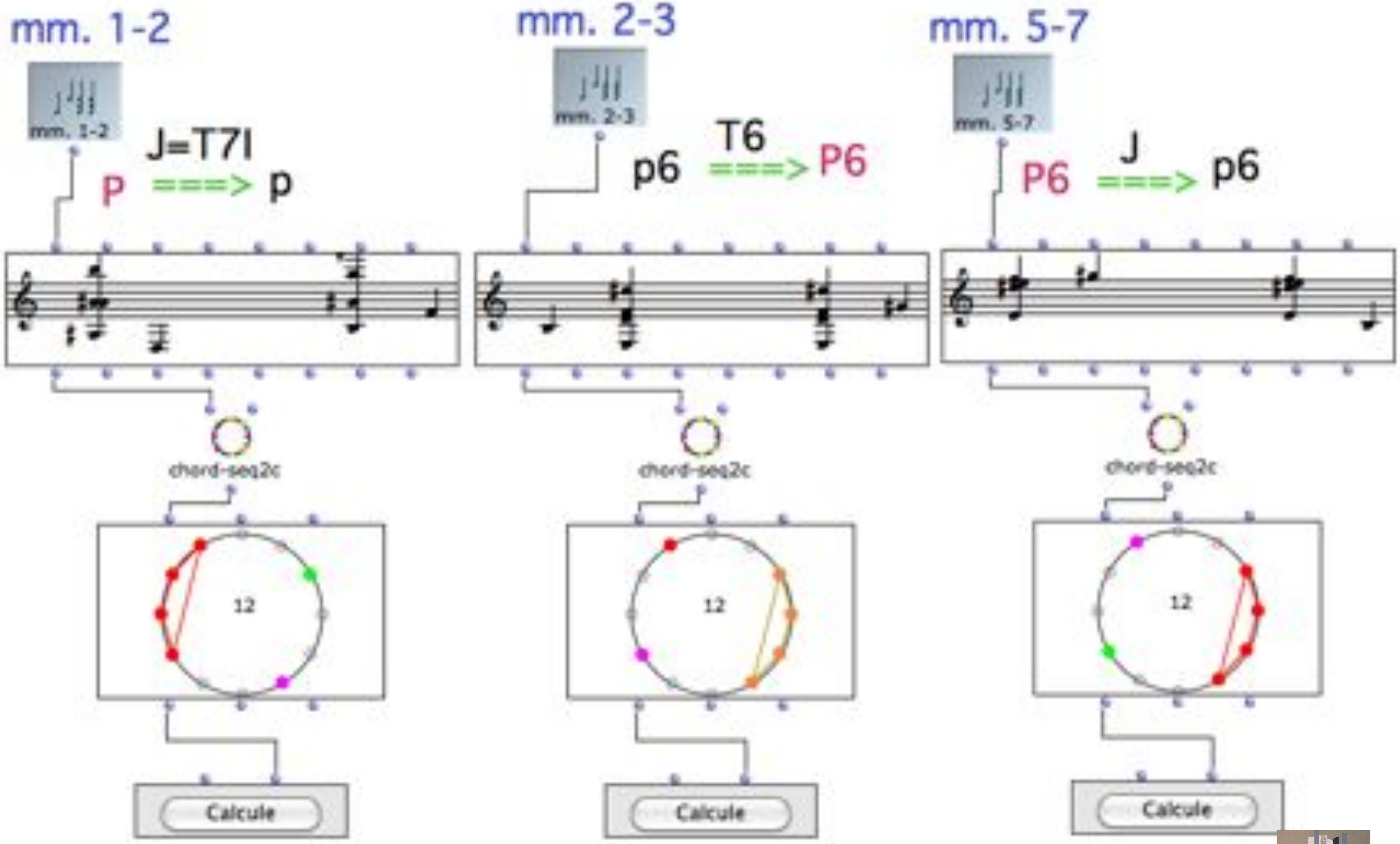
Exercices d'écoute : « do you hear it? » *vs* « can you hear it? »

Stockhausen: *Klavierstück III* (Analyse de D. Lewin)

Example 2.7. An ear-training aid for listening to P/p forms and their interrelations.

« I take the question 'Can you hear it » to mean something like this: After studying the analysis in examples 2.5 and 2.6, do you find it possible to focus your **aural attention** upon aspects of the acoustic signal that seem to engage the signifiers of that analysis? […] For me, the interesting questions involve the extent and ways in which I am satisfied and dissatisfied when **focusing my aural attention** in that manner. It is important to ask those questions about any systematic analysis of any musical composition ».

Computer-Aided Transformational Analysis in OpenMusic



08-lewin-stock

Visualisations multimédia de l'analyse transformationnelle

R. Attas : Metaphors in Motion: Agents and Representation in Transformational Analysis, *MTO,* 15(1), 2009
http://mto.societymusictheory.org/issues/mto.09.15.1/mto.09.15.1.attas.html

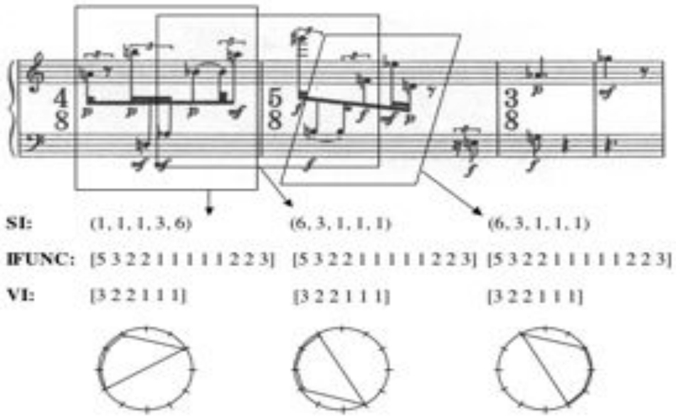
Animation 1. Klavierstück III

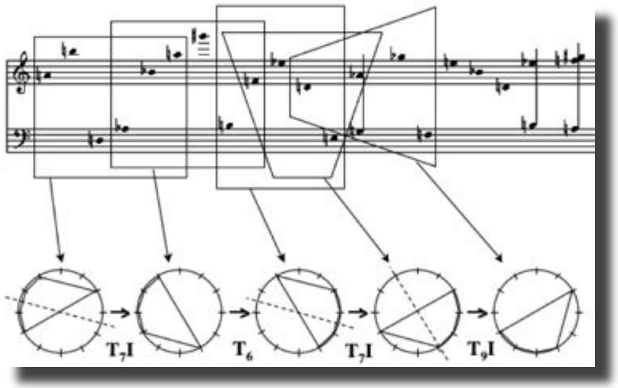
Visualisations multimédia de l'analyse transformationnelle

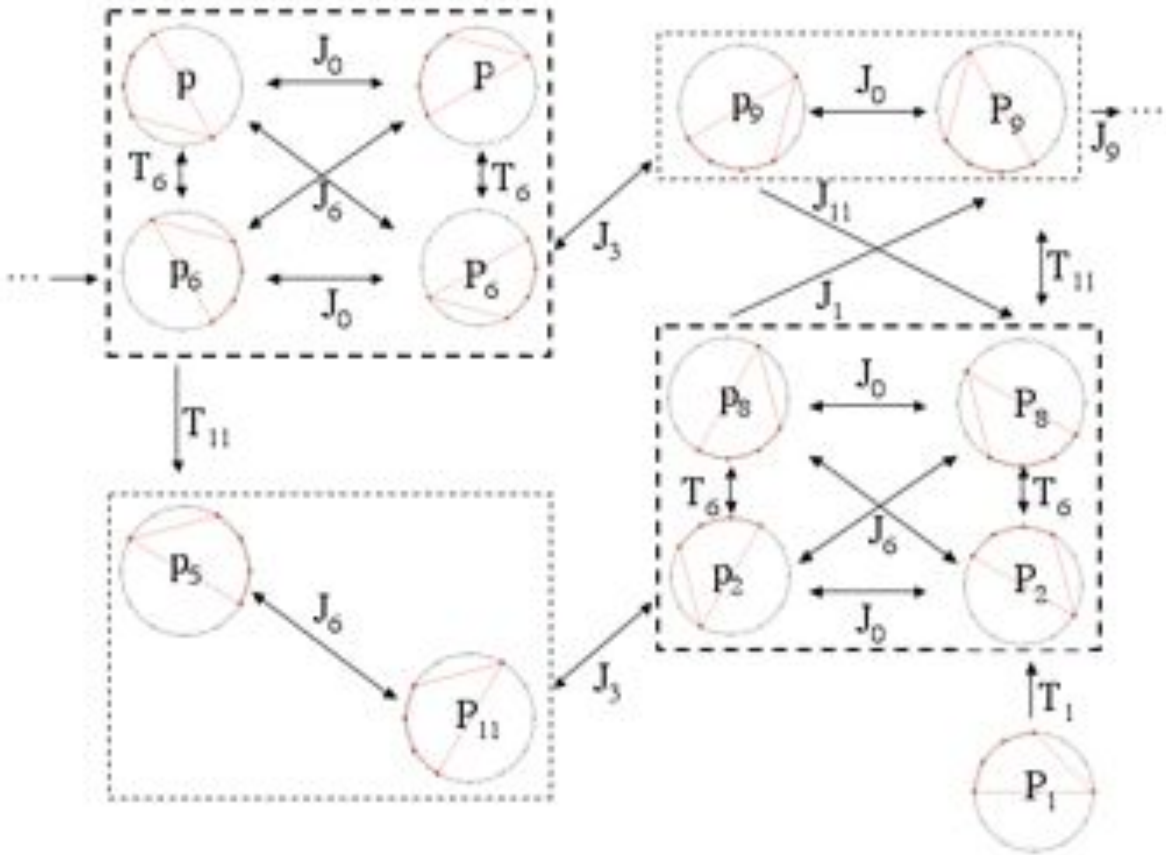
R. Attas : Metaphors in Motion: Agents and Representation in Transformational Analysis, *MTO,* 15(1), 2009
http://mto.societymusictheory.org/issues/mto.09.15.1/mto.09.15.1.attas.html

Animation 2. Grow Your Own Pentachord

Progression transformationnelle *vs* réseau transformationnel





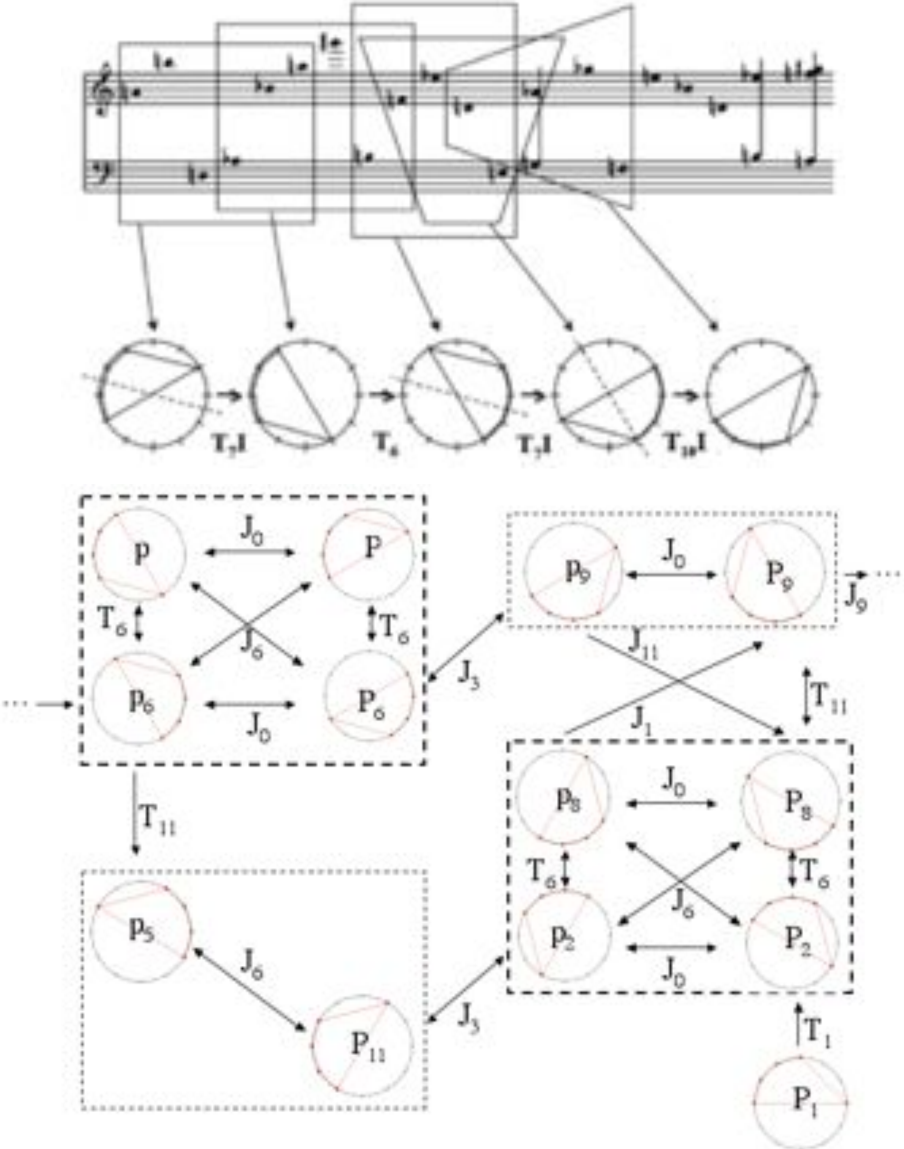


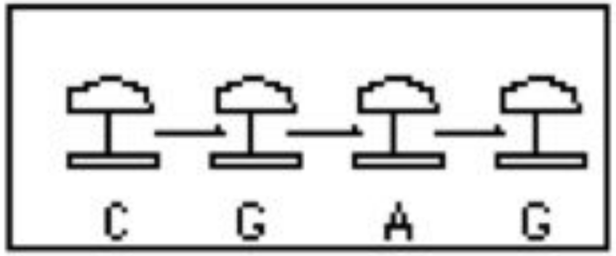
« A rational reconstruction of a work or works, which is a theory of the work or works, is an explanation not, assuredly, of the 'actual' process of construction, but of ***how the work or works may be construed by a hearer****, how the 'given' may be 'taken' »*

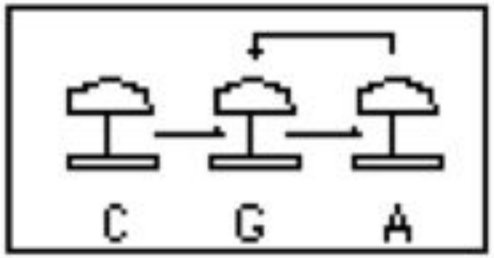
M. Babbitt : « Contemporary Music Composition and Music Theory as Contemporary Intellectual History », 1972

Réseau transformationnel et cognition (musicale)

Bamberger, J. (1986). Cognitive issues in the development of musically gifted children. In *Conceptions of giftedness (eds., R. J. Sternberg, & J. E. Davidson), pp. 388-413. Cambridge University Press, Cambridge*





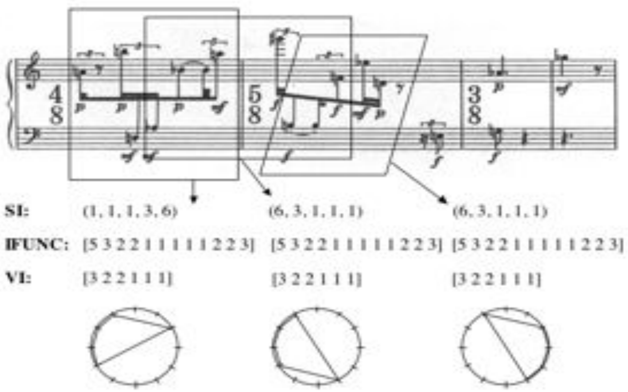


Bamberger, J. (2006). "What develops in musical development?" In G. MacPherson (ed.) *The child as musician: Musical development from conception to adolescence.* Oxford, U.K. Oxford University Press.

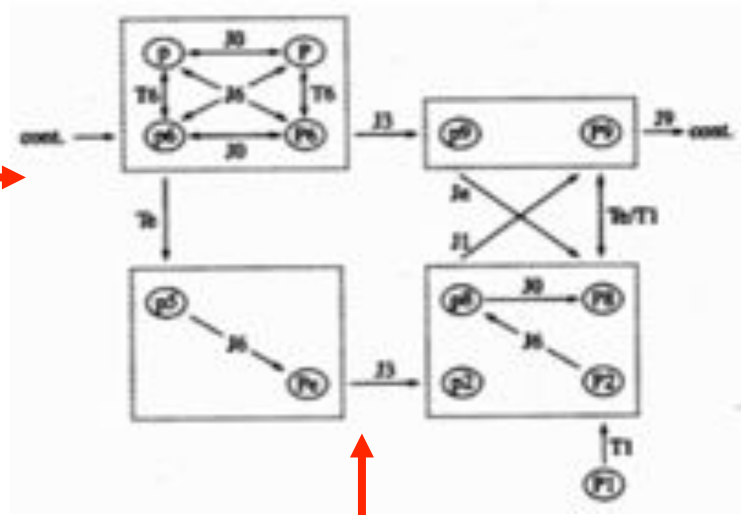
Théorie des groupes et cognition (musicale)

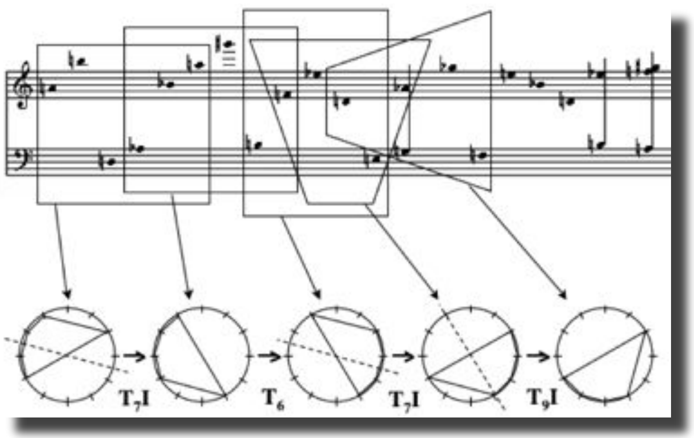
« Group Theory has emerged as a powerful tool for analyzing cognitive structure. The number of cognitive disciplines using group theory is now enormous. The power of group theory lies in its ability to identify organization, and to express organization in terms of **generative actions that structure a space** »

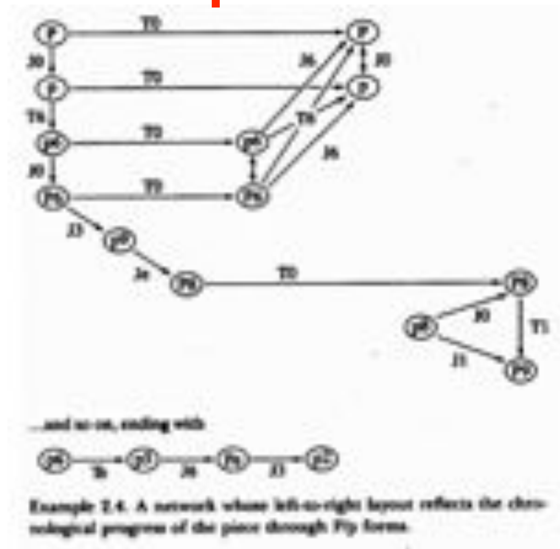
Michael Leyton, The International Society for Group Theory in Cognitive Science



?





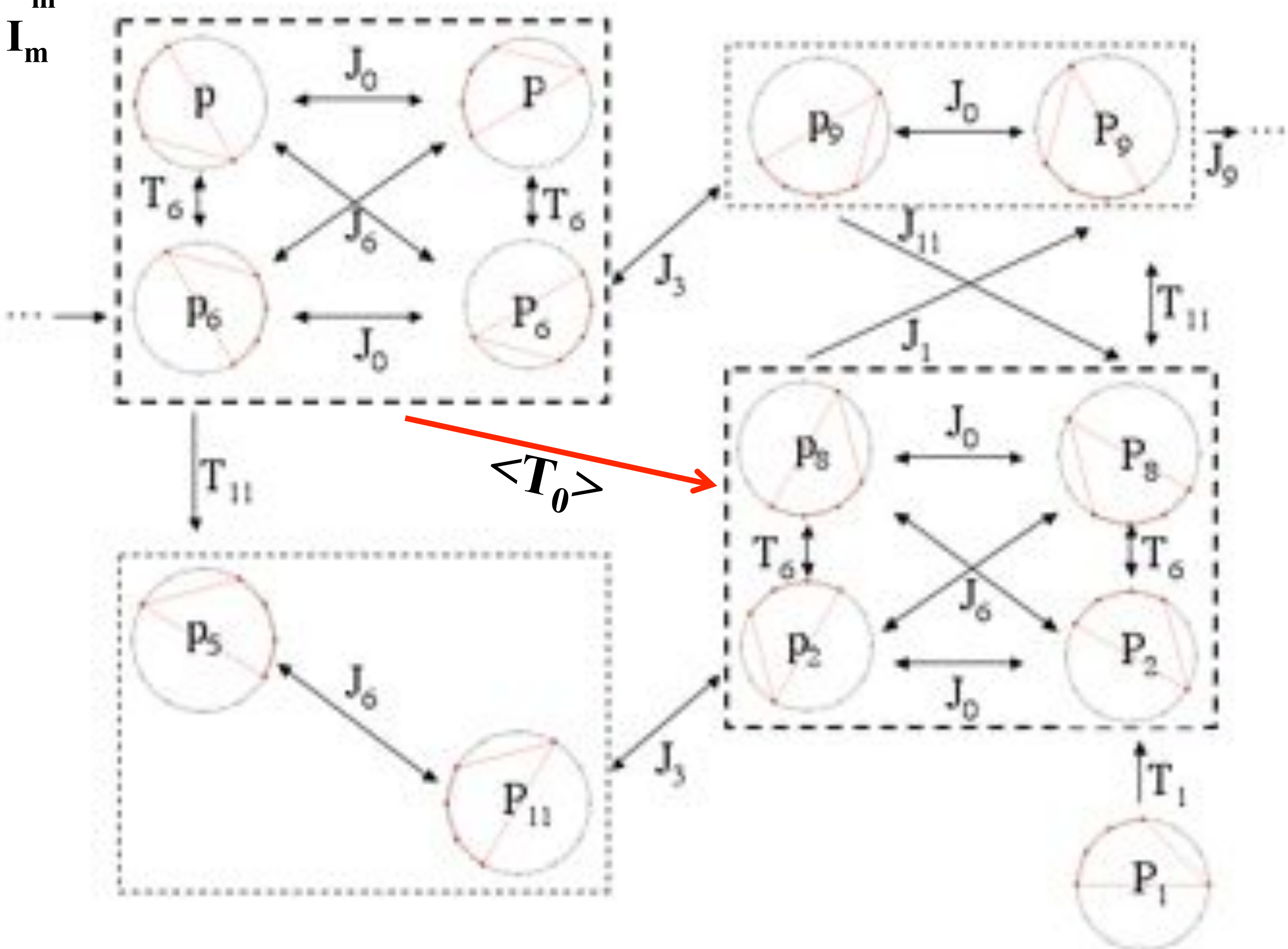


Isographies « fortes » dans le réseau transformationnel

Stockhausen: *Klavierstück III* (Analyse de D. Lewin)

$\langle T_0 \rangle : T_m \rightarrow T_m$

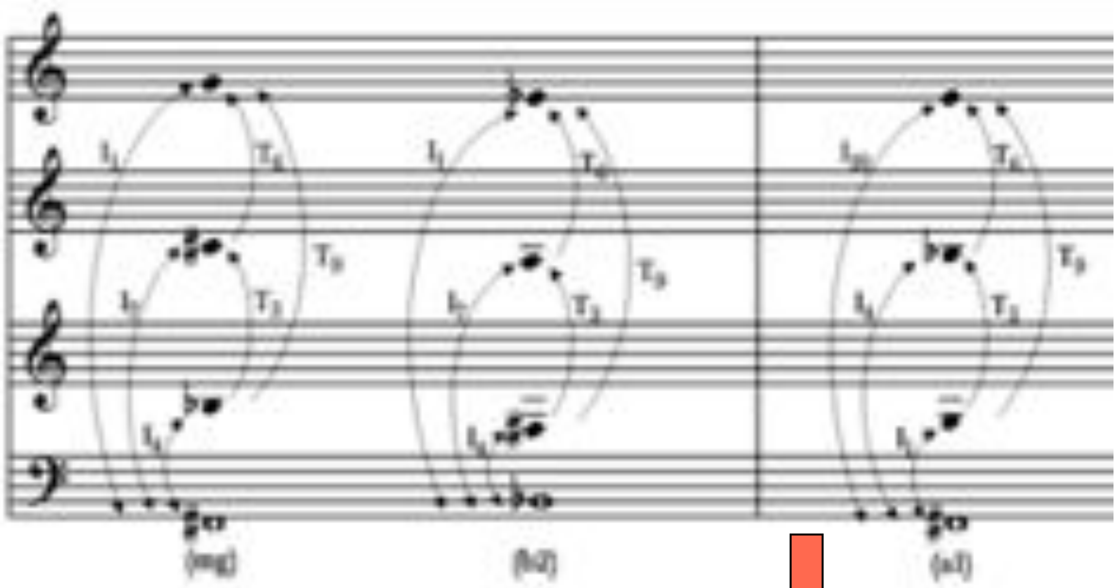
$I_m \rightarrow I_m$

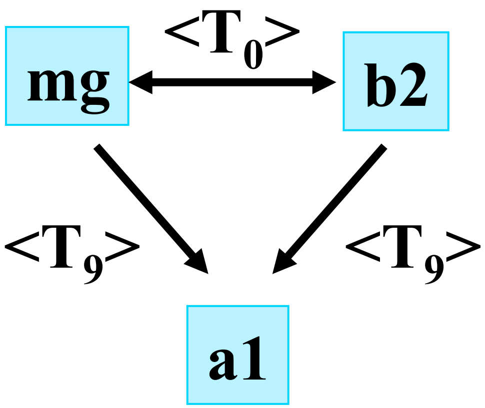


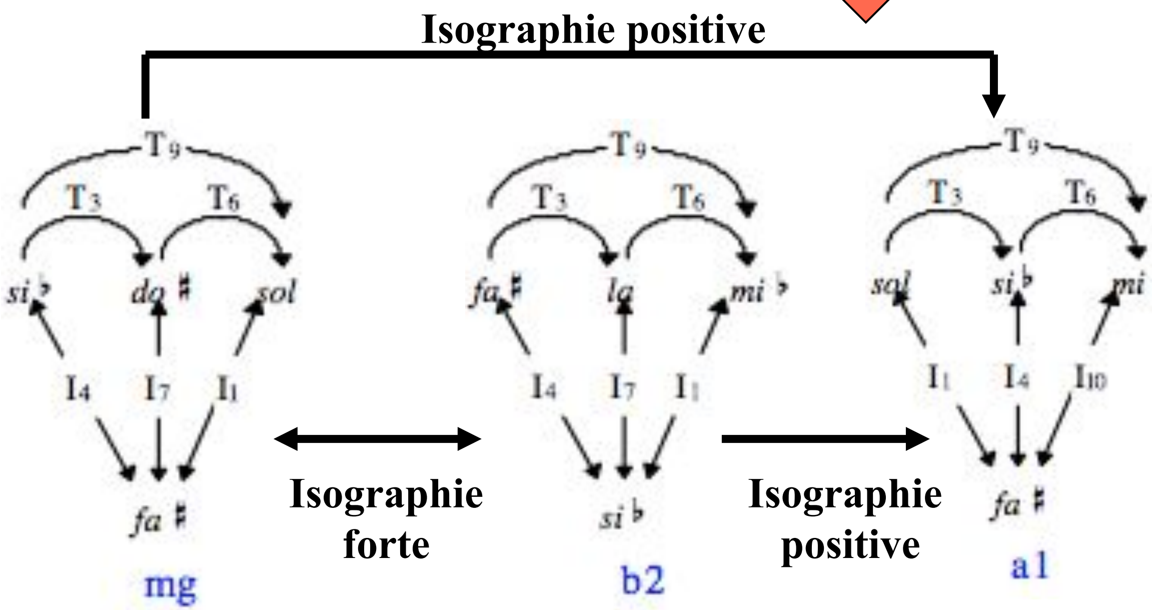
Klumpenhouwer Networks (K-réseaux)

Xavier Hascher: « Liszt et les sources de la notion d'agrégat », Analyse Musicale, 43, 2002

Ex. 1 - « Ladislaus Teleki » (*Historische ungarische Bildnisse* n° 4), mes. 1-7
Les agrégats dans la classification de Forte



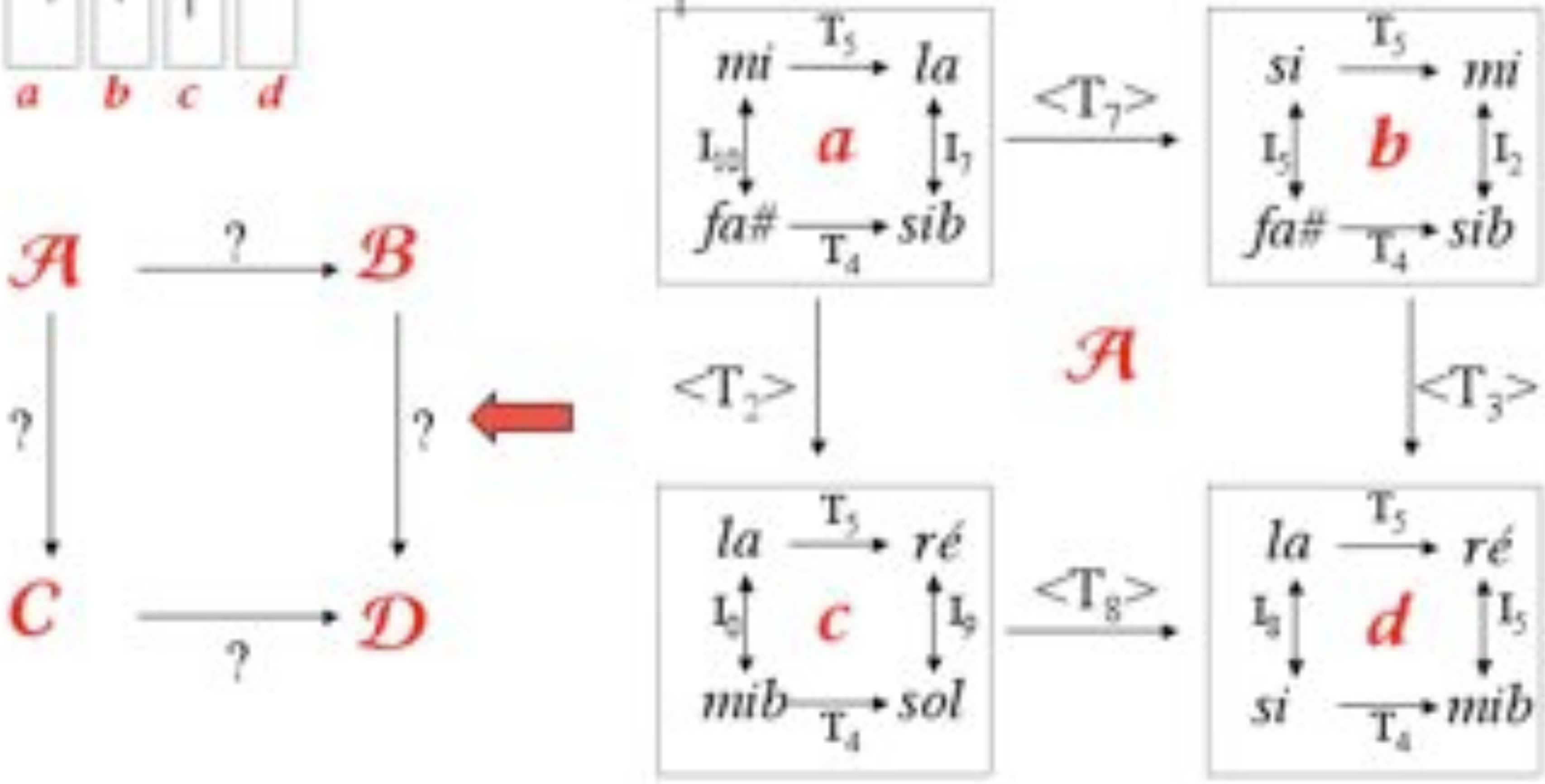




Klumpenhouwer Networks (K-réseaux) : isographies positives et récursivité

David Lewin: «A Tutorial on K-nets using the Chorale in Schoenberg's Op.11, N°2 », JMT, 1994

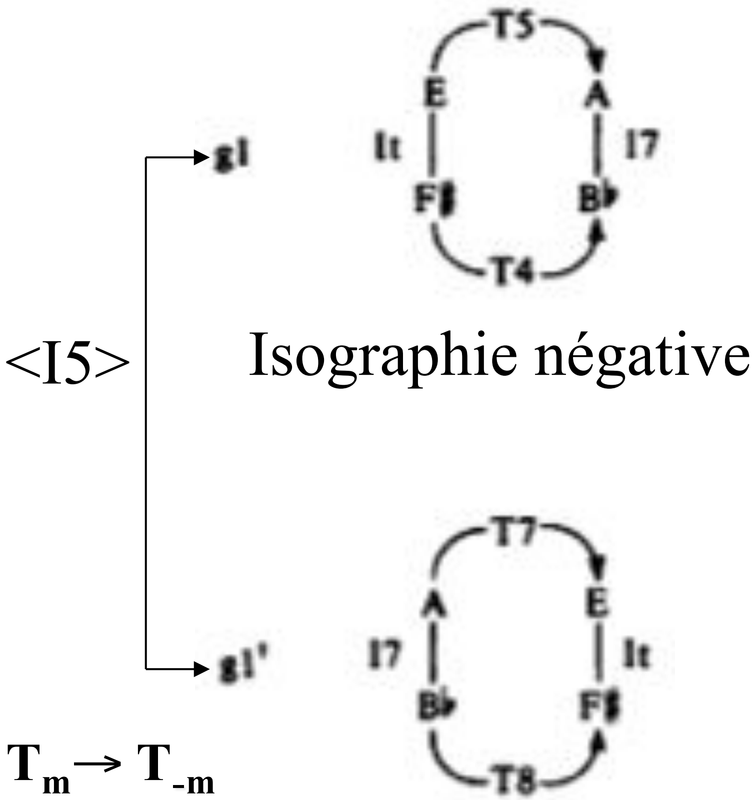
$$<T_k> : T_m \rightarrow T_m$$
$$I_m \rightarrow I_{k+m}$$

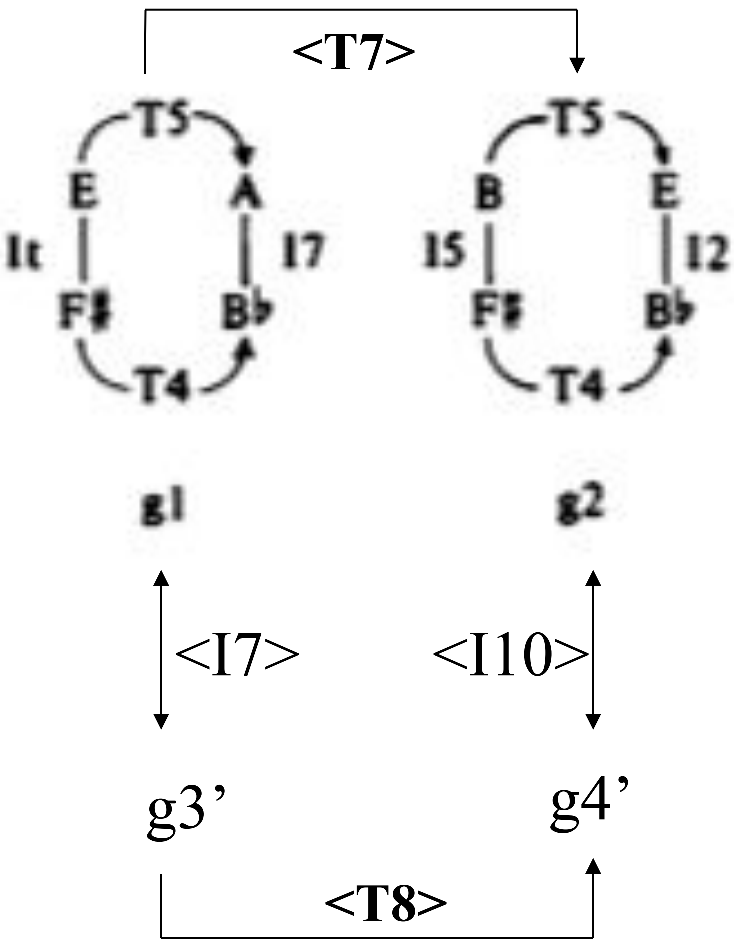


Klumpenhouwer Networks (K-nets) : isographies négatives

David Lewin: «A Tutorial on K-nets using the Chorale in Schoenberg's Op.11, N°2 », JMT, 1994

Example 9

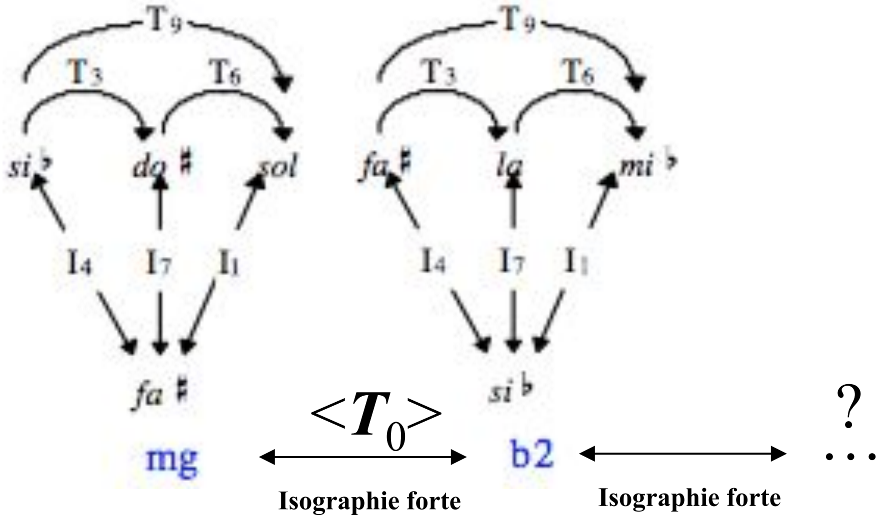




$$\langle I_k \rangle : T_m \rightarrow T_{-m}$$
$$I_m \rightarrow I_{k-m}$$

$$\langle T_k \rangle : T_m \rightarrow T_m$$
$$I_m \rightarrow I_{k+m}$$

Enumération des K-nets en relation d'isographie forte



$$x \xrightarrow{T_3} x+3 \xrightarrow{T_6} x+9$$

$I_4 \quad I_7 \quad I_1$

$$4-x=7-(x+3)=1-(x+9)$$

⇨ 12 solutions

$$\begin{array}{ccc} re & \xrightarrow{T_4} & fa\# \\ M_5 \downarrow & & \downarrow T_2I \\ sib & \xrightarrow{T_6I} & sol\# \end{array} \quad \longleftrightarrow \quad \begin{array}{ccc} x & \xrightarrow{T_4} & x+4 \\ M_5 \downarrow & & \downarrow T_2I \\ 5x & \xrightarrow{T_6I} & 6-5x=2-(x+4) \end{array}$$

Isographie forte

$6-5x=2-(x+4)$ ==> $8=4x$ ==> $x=2, 5, 8, 11$

⇨ 4 solutions

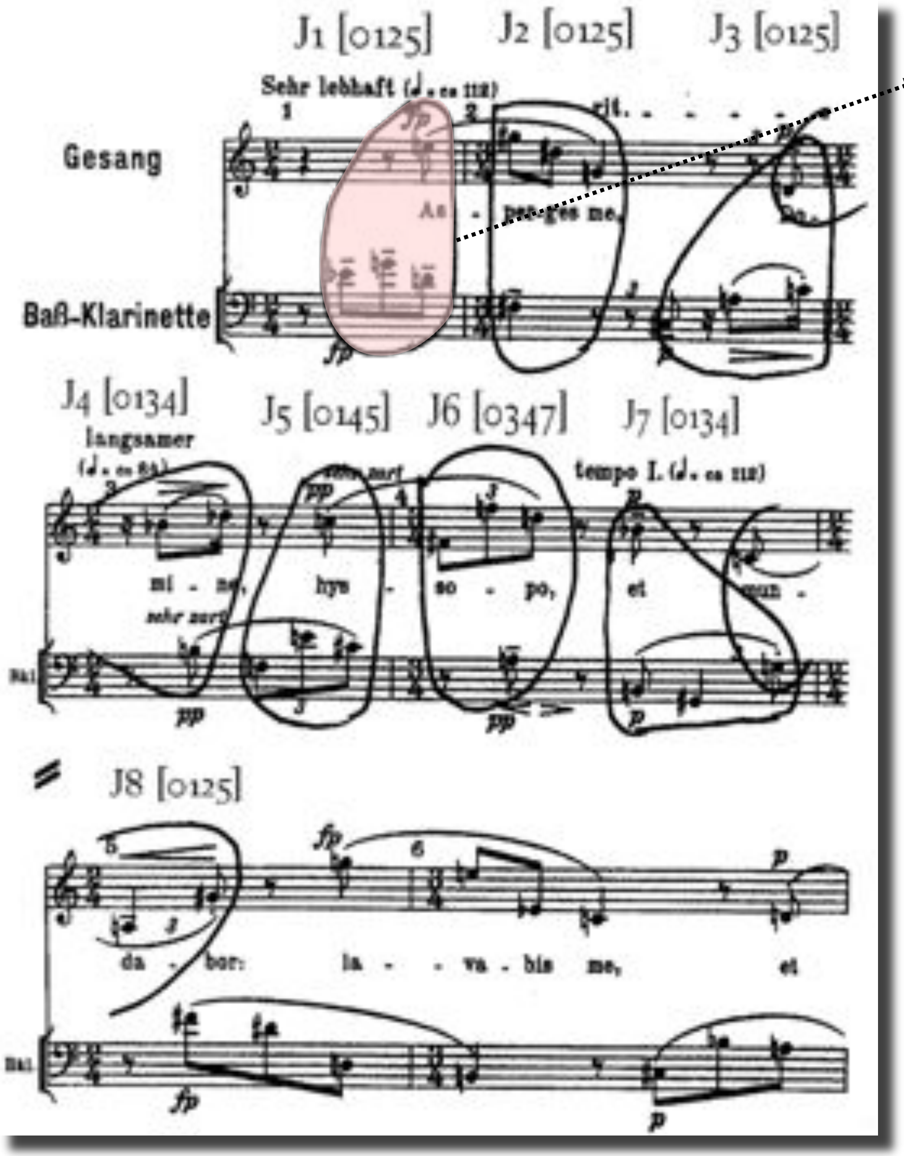
$$\begin{array}{ccc} re\# & \xrightarrow{M_3} & la \\ M_1 \downarrow & & \downarrow M_7 \\ re\# & \xrightarrow{M_{11}} & la \end{array} \quad \longleftrightarrow \quad \begin{array}{ccc} x & \xrightarrow{M_1} & x \\ M_1 \downarrow & & \downarrow M_7 \\ x & \xrightarrow{M_{11}} & 11x=7x \end{array}$$

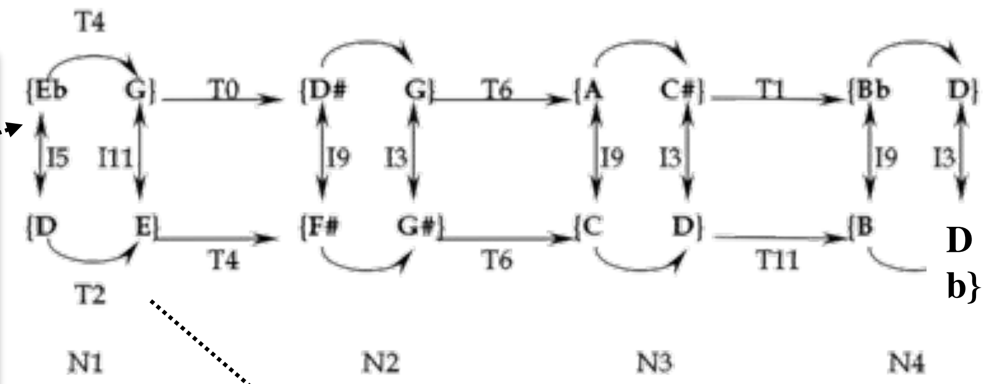
Isographie forte

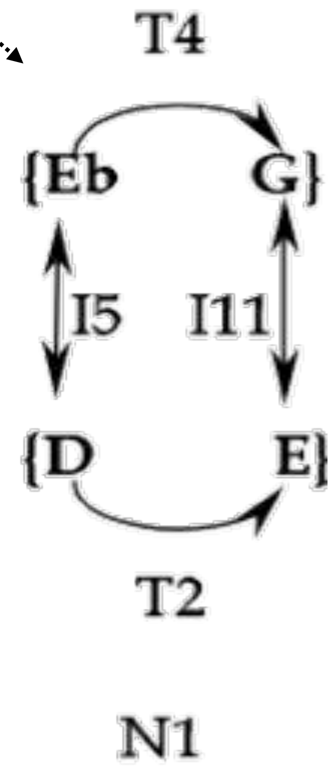
$11x=7x$ ==> $4x=0$ ==> $x=0, 3, 6, 9$

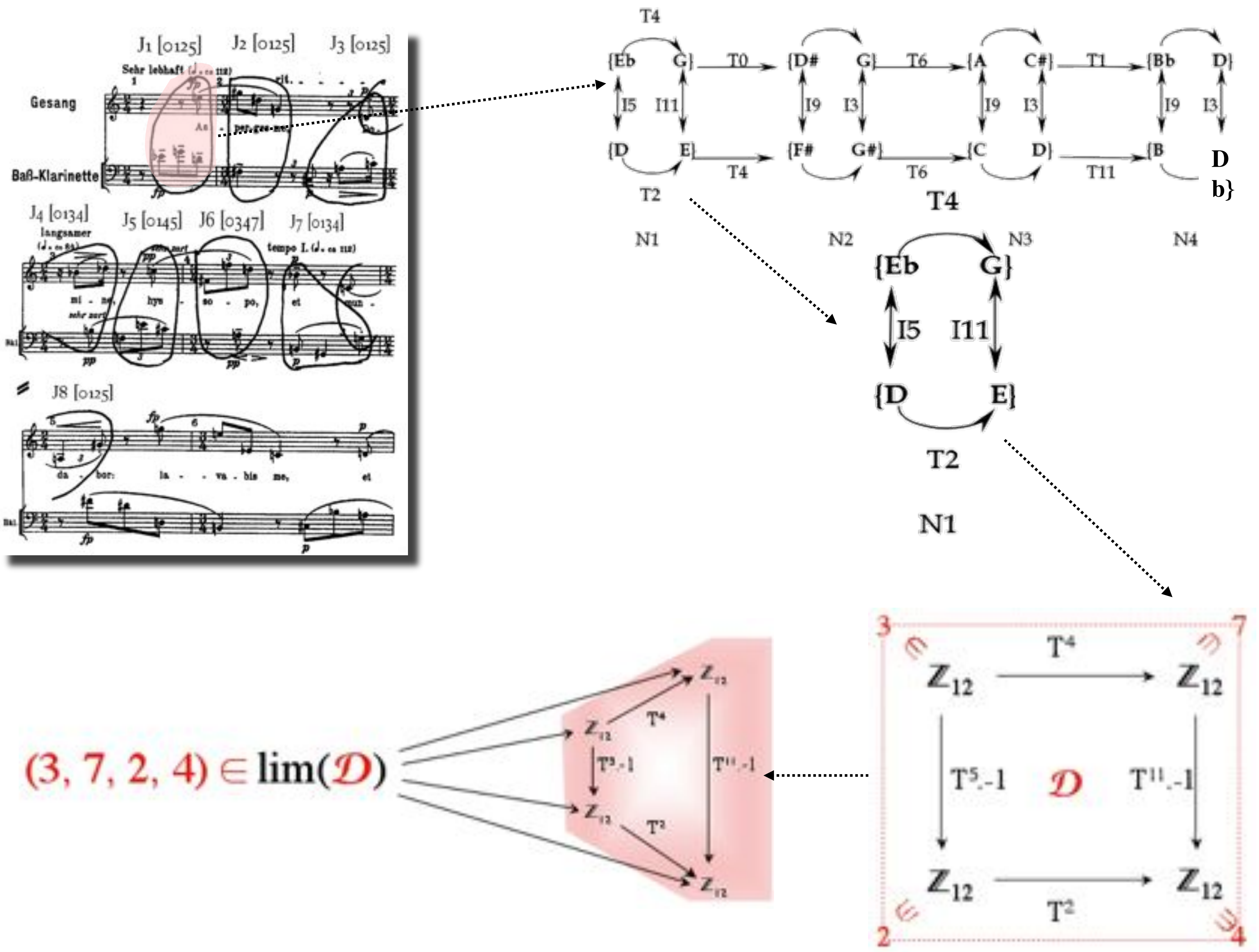
⇨ 4 solutions

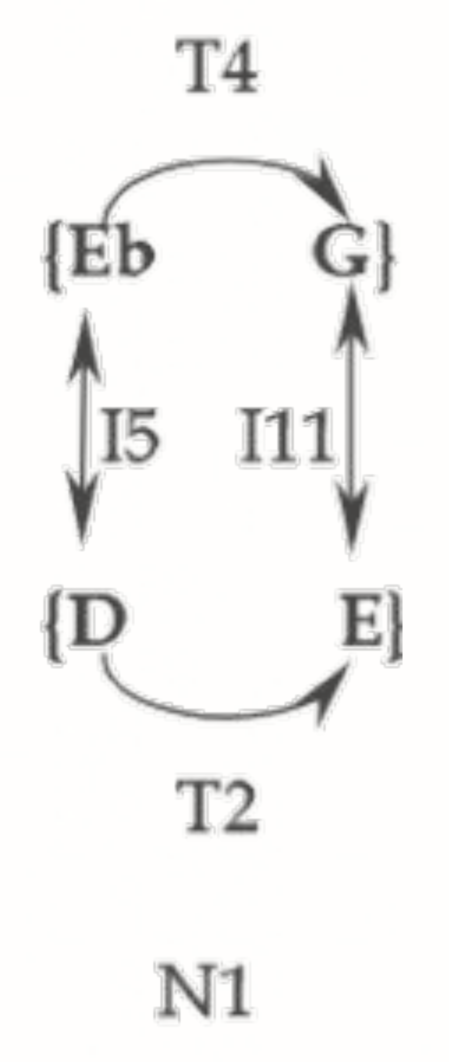
Henry Klumpenhouwer: ***Deep Structure in K-net Analysis with Special Reference to Webern (Opus 16, n°4)***

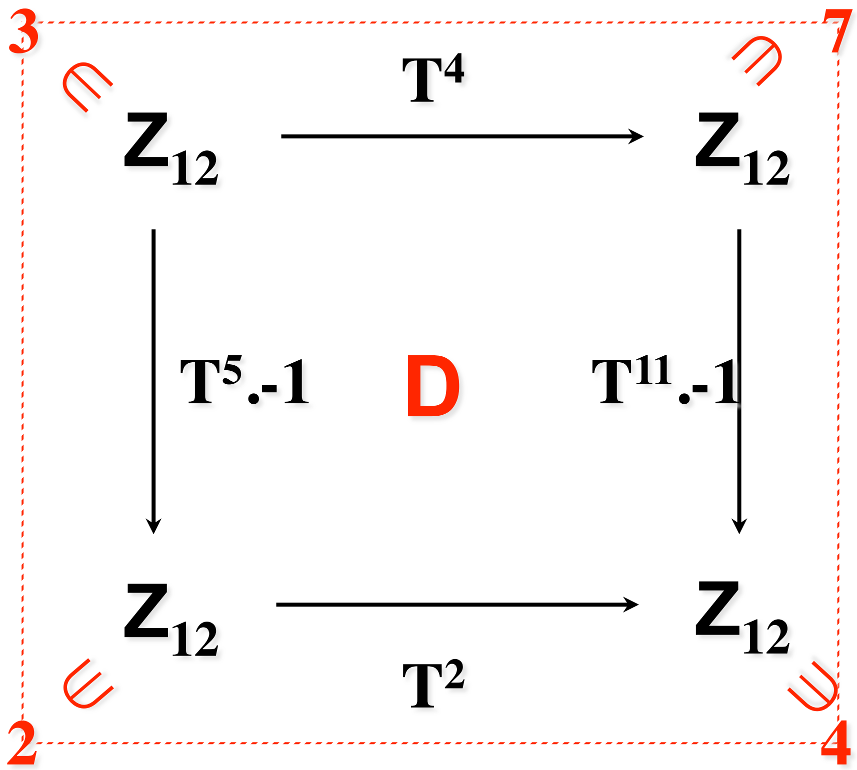




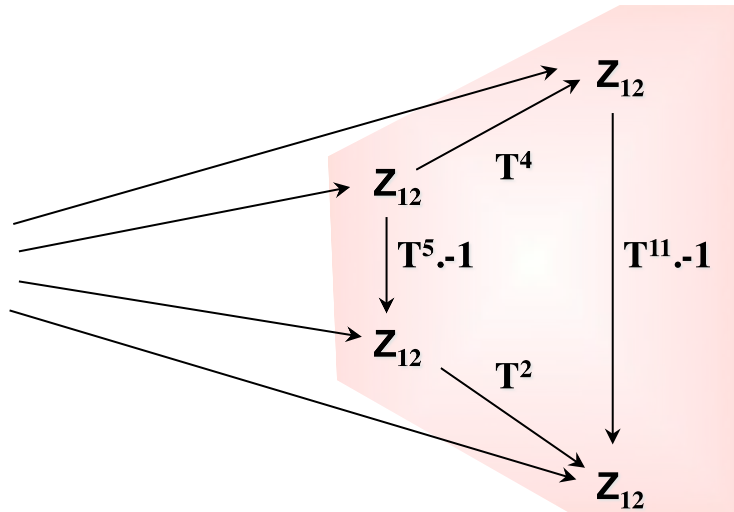



J1 [0125]
J2 [0125]
J3 [0125]
Sehr lebhaft
Gesang
Baß-Klarinette
J4 [0134]
langsamer
J5 [0145]
J6 [0347]
J7 [0134]
tempo I.
mi - ne, hys - so - po, et mun -
J8 [0125]
da - bor: la - va - bis me, et
T4
{Eb G}
T0
{D# G}
T6
{A C#}
T1
{Bb D}
I5 I11
I9 I3
I9 I3
I9 I3
{D E}
T4
{F# G#}
T6
{C D}
T11
{B Db}
T2
T4
N1
N2
N3
N4
{Eb G}
I5 I11
{D E}
T2
N1
3
7
2
4
$\mathbb{Z}_{12}$
T^4
$T^5.-1$
$\mathcal{D}$
$T^{11}.-1$
T^2
$T^3.-1$
$T^{11}.-1$
$(3, 7, 2, 4) \in \lim(\mathcal{D})$



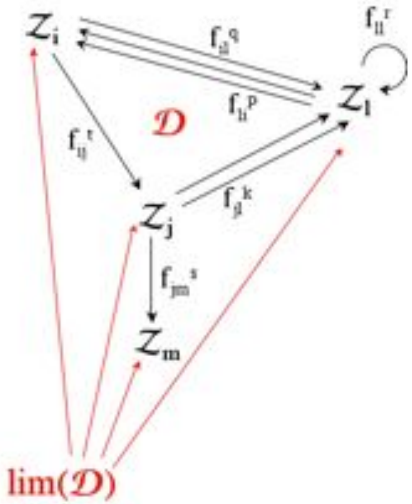


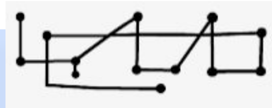
$(3, 7, 2, 4) \in \lim(\mathbf{D})$



$\mathbf{Z}_i = \mathbf{Z}_{12}$

$f_{ij}^t \in \mathbf{Z}_i@\mathbf{Z}_j$

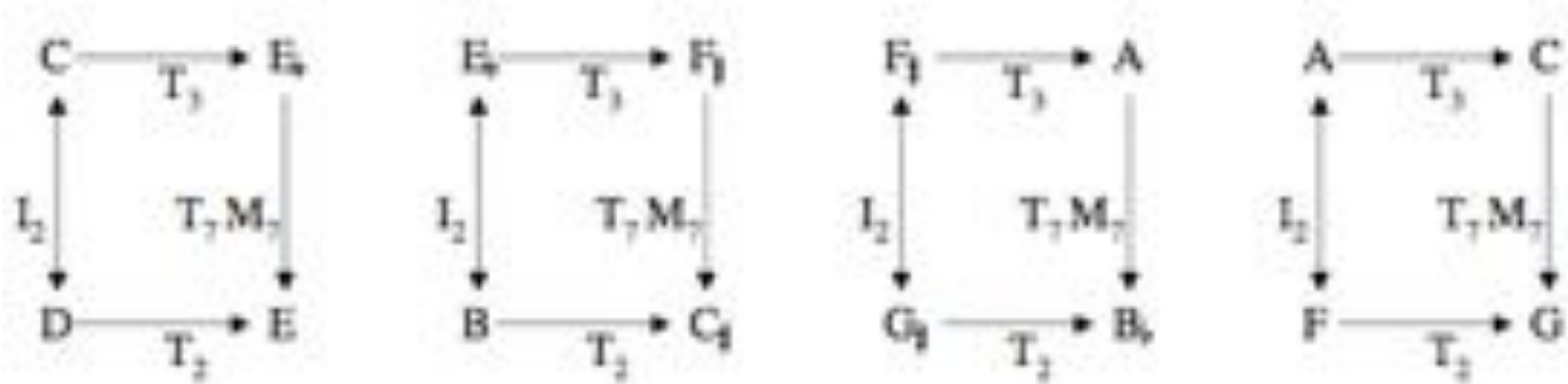




Fact:
lim(D) ≈ U

U = (empty or) subgroup of $(\mathbf{Z}_{12})^n$

If f_{**}^{*} **= isomorphisms** card (U) (= 0 or) divides 12



EXAMPLE 6: THE FOUR SOLUTIONS (STRONGLY ISOGRAPHIC K-NETS) OF THIS DIAGRAM ILLUSTRATE THAT THE CARDINALITY OF THE SOLUTION SET IS A DIVISOR OF 12. HERE, THE OPERATOR M_7 DENOTES THE MULTIPLICATION BY 7

G. Mazzola & M. Andreatta, 2006

Isomorphismes de réseaux de Klumpenhouwer

Isographie positive

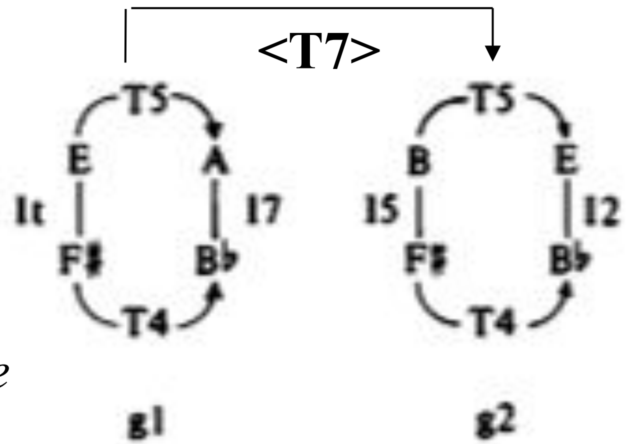
$<T_k> : T_m \rightarrow T_m$

$\qquad\qquad I_m \rightarrow I_{k+m}$

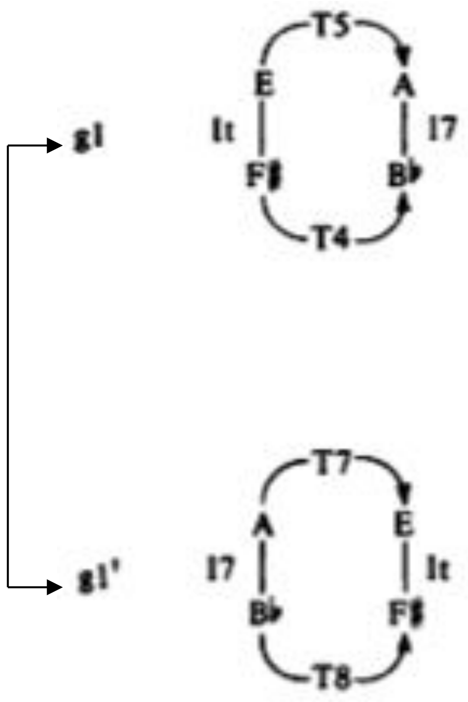
Isographie négative

$<I_k> : T_m \rightarrow T_{-m}$

$\qquad\qquad I_m \rightarrow I_{k-m}$



<I5 >



$$\begin{array}{ccc} x & \xrightarrow{\varphi} & y \\ \gamma\downarrow & & \downarrow\delta \\ w & \xrightarrow{\psi} & z \end{array}$$

$$\Big\downarrow F$$

$$\begin{array}{ccc} F(x) & \xrightarrow{F\varphi F^{-1}} & F(y) \\ F\gamma F^{-1}\downarrow & & \downarrow F\delta F^{-1} \\ F(w) & \xrightarrow{F\psi F^{-1}} & F(z) \end{array}$$

Isomorphismes de réseaux de Klumpenhouwer

Isographie positive

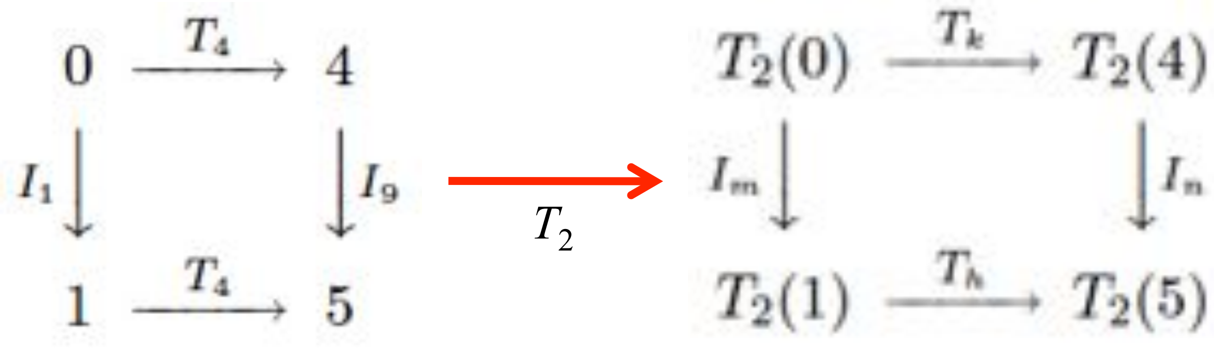
$\langle T_k \rangle : T_m \to T_m$

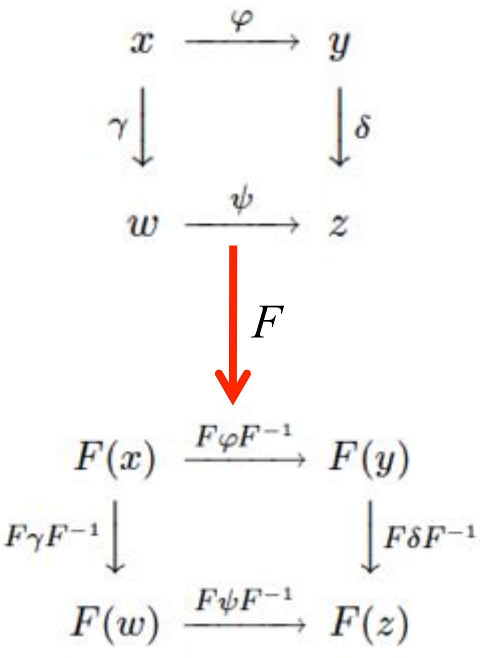
$I_m \to I_{k+m}$

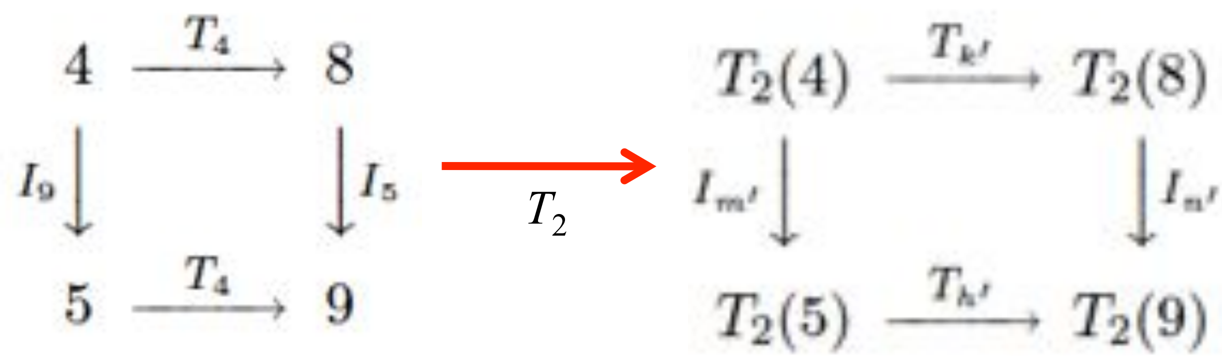
Isographie négative

$\langle I_k \rangle : T_m \to T_{-m}$

$I_m \to I_{k-m}$







Isomorphismes de réseaux de Klumpenhouwer

Isographie positive

Examen 7/3/2006

$\langle T_k \rangle : T_m \rightarrow T_m$

$I_m \rightarrow I_{k+m}$

$$\begin{array}{ccc} 2 & \xrightarrow{T_1} & 3 \\ I_2 \downarrow & & \downarrow I_8 \\ 0 & \xrightarrow{T_5} & 5 \end{array} \quad \xrightarrow{I_5} \quad \begin{array}{ccc} I_5(2) & \xrightarrow{T_k} & I_5(3) \\ I_a \downarrow & & \downarrow I_b \\ I_5(0) & \xrightarrow{T_h} & I_5(5) \end{array}$$

Isographie négative

$\langle I_k \rangle : T_m \rightarrow T_{-m}$

$I_m \rightarrow I_{k-m}$

$$\begin{array}{ccc} x & \xrightarrow{\varphi} & y \\ \gamma \downarrow & & \downarrow \delta \\ w & \xrightarrow{\psi} & z \end{array}$$

$$\downarrow F$$

$$\begin{array}{ccc} F(x) & \xrightarrow{F\varphi F^{-1}} & F(y) \\ F\gamma F^{-1} \downarrow & & \downarrow F\delta F^{-1} \\ F(w) & \xrightarrow{F\psi F^{-1}} & F(z) \end{array}$$

$$\begin{array}{ccc} A & \xrightarrow{\varphi} & B \\ \gamma \downarrow & & \downarrow \delta \\ C & \xrightarrow{\psi} & D \end{array} \quad \longrightarrow \quad \begin{array}{ccc} A^C & \xrightarrow{\varphi'} & B^C \\ \gamma' \downarrow & & \downarrow \delta' \\ C^C & \xrightarrow{\psi'} & D^C \end{array}$$

Passage au complémentaire

$A=\{1,2,3,4,5,7\}$

$B=\{0,6,8,9,10,11\}$

$C=\{1,2,4,7,9,11\}$

$D=\{0,2,4,5,7,10\}$

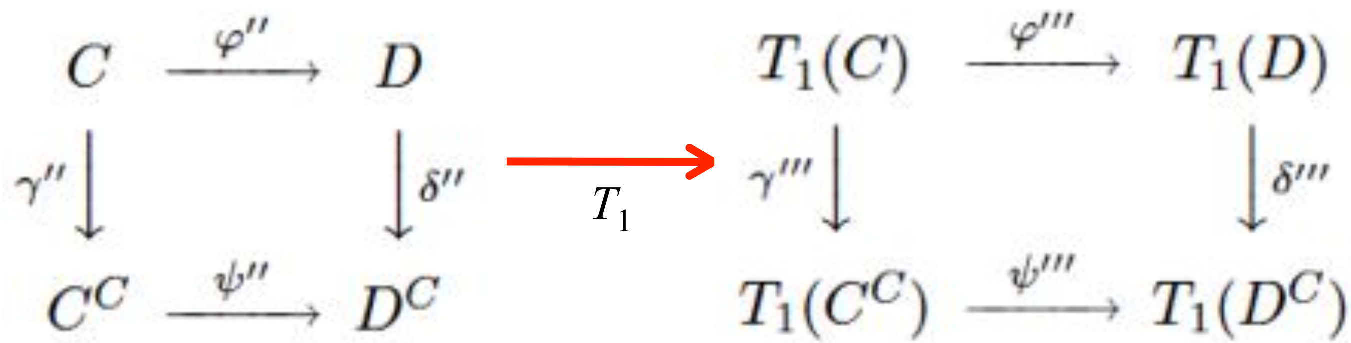
Examen 25/2/2009

Isomorphismes de réseaux de Klumpenhouwer

Isographie positive

$<T_k> : T_m \rightarrow T_m$

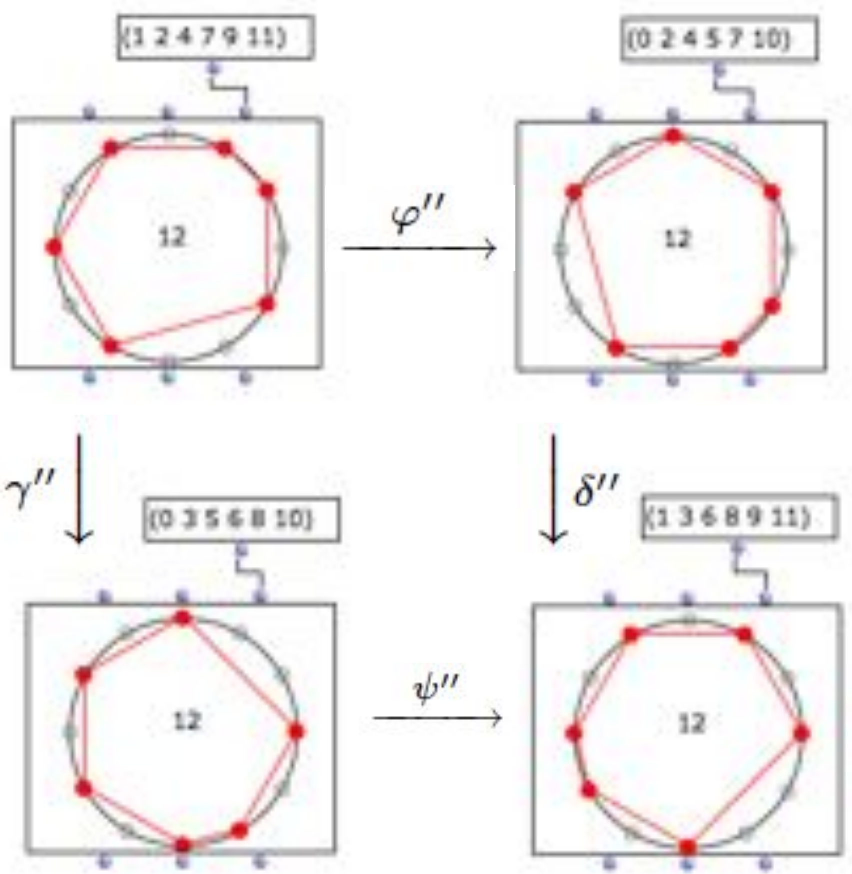
$I_m \rightarrow I_{k+m}$

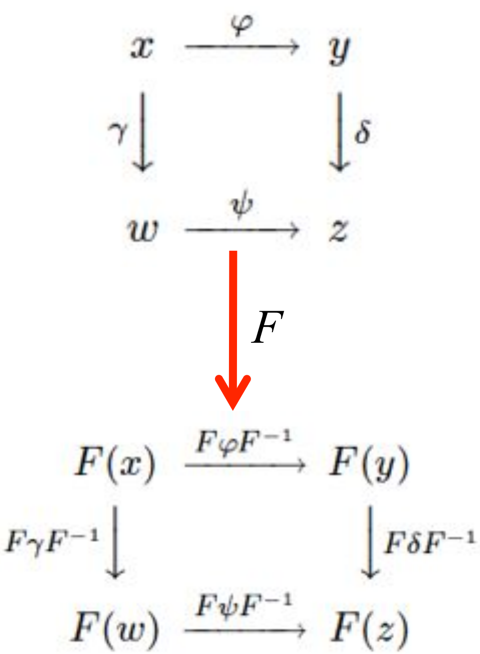


Isographie négative

$<I_k> : T_m \rightarrow T_{-m}$

$I_m \rightarrow I_{k-m}$





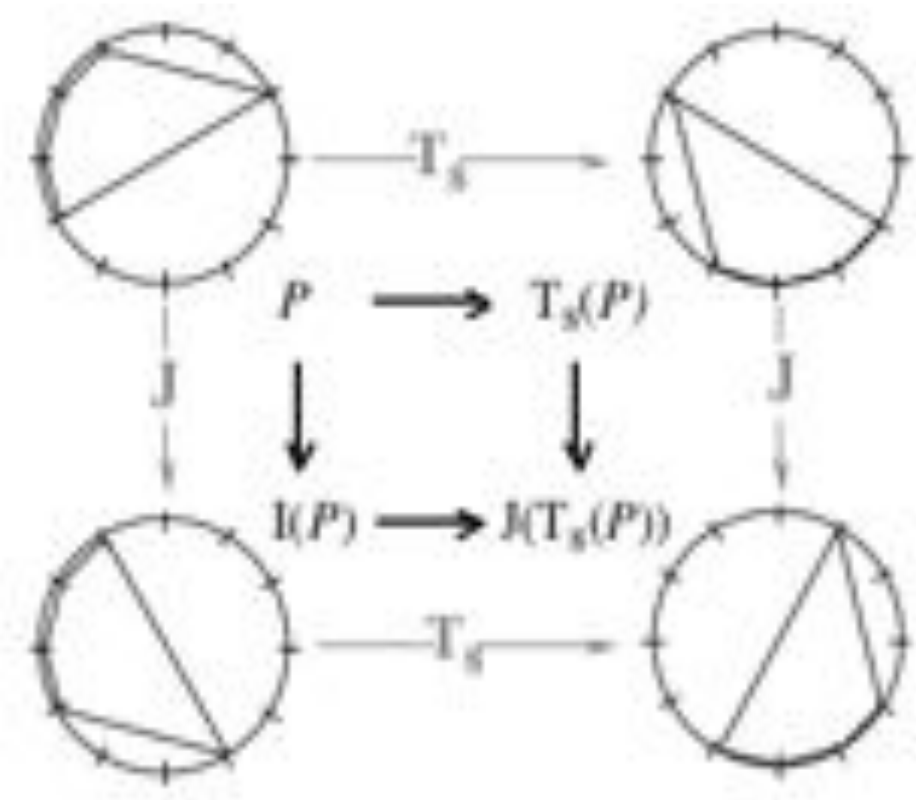
$A=\{1,2,3,4,5,7\}$

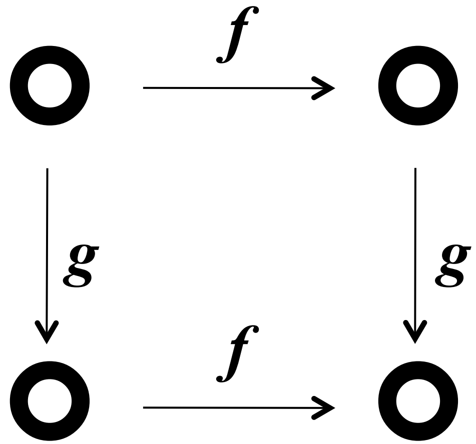
$B=\{0,6,8,9,10,11\}$

$C=\{1,2,4,7,9,11\}$

$D=\{0,2,4,5,7,10\}$

Action de groupe et commutativité des diagrammes



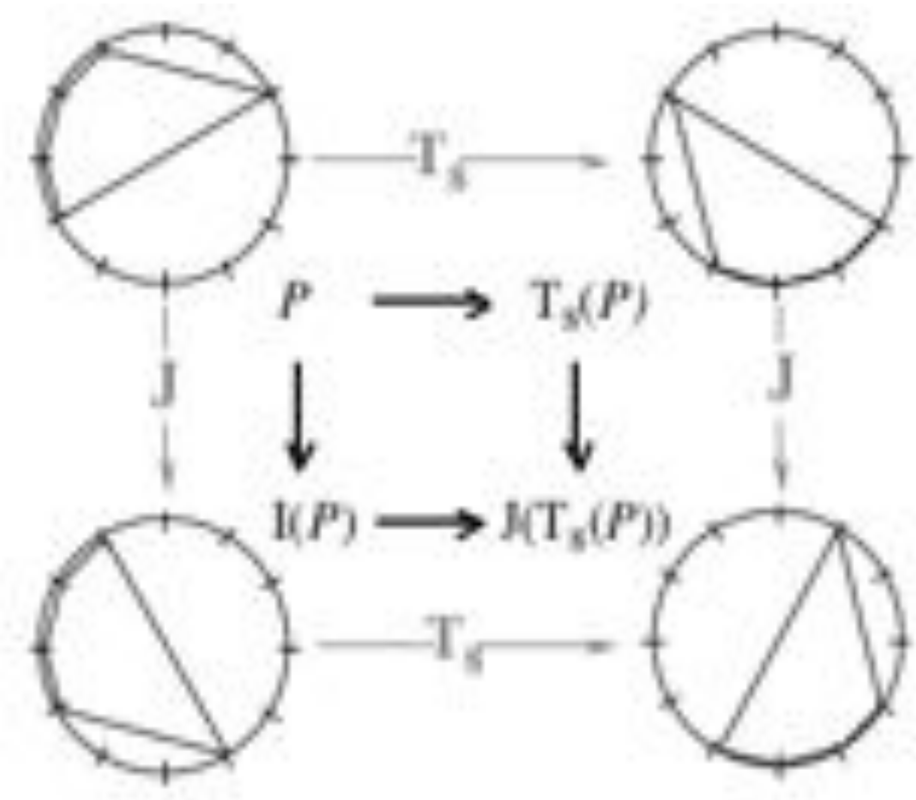


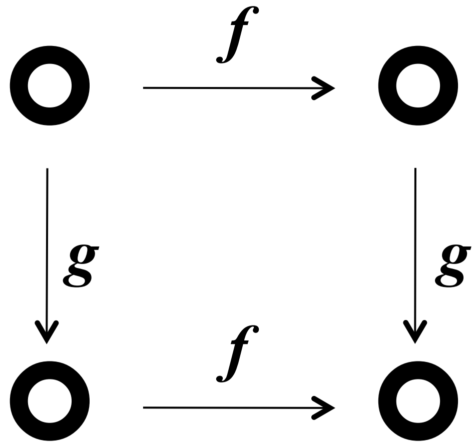
Tout diagramme commute

$$\forall f , g \in < \mathrm{T}, \mathrm{J} >$$

Le groupe des 24 transformations $\sigma = \{T_0, T_1, \ldots, T_{11}, T_0J, T_1J, \ldots, T_{11}J\}$ est commutatif et opère de manière simplement transitive sur l'espace *S* des 24 formes du pentacorde de base (i.e. l'ensemble de ses 12 transpositions et de ses 12 inversions)

Inversions « contextuelles » et commutativité des diagrammes





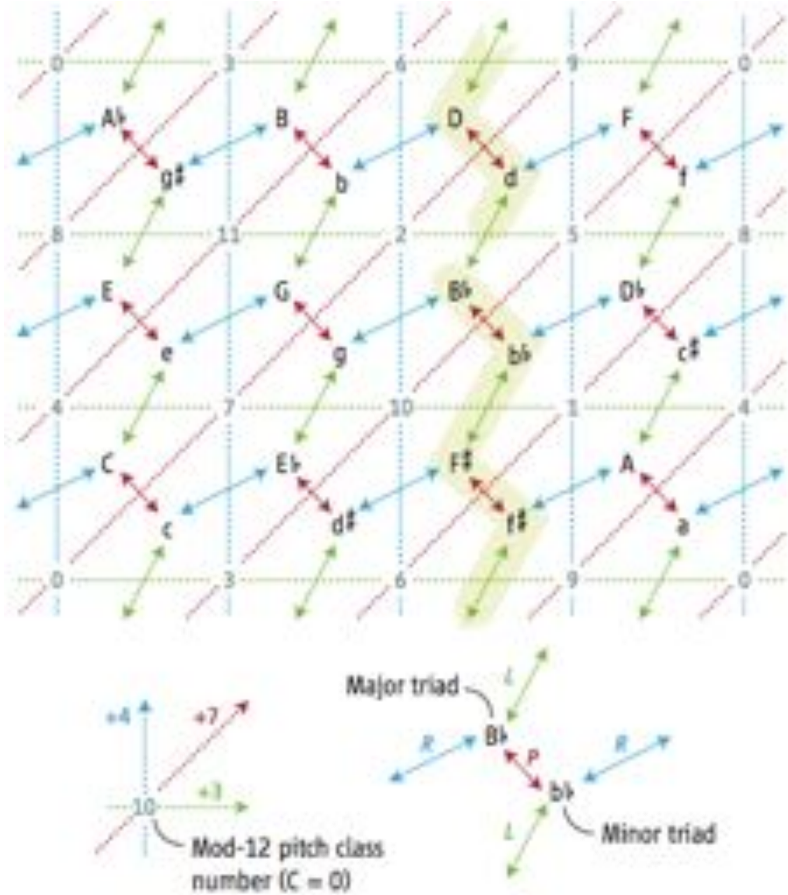
Tout diagramme commute

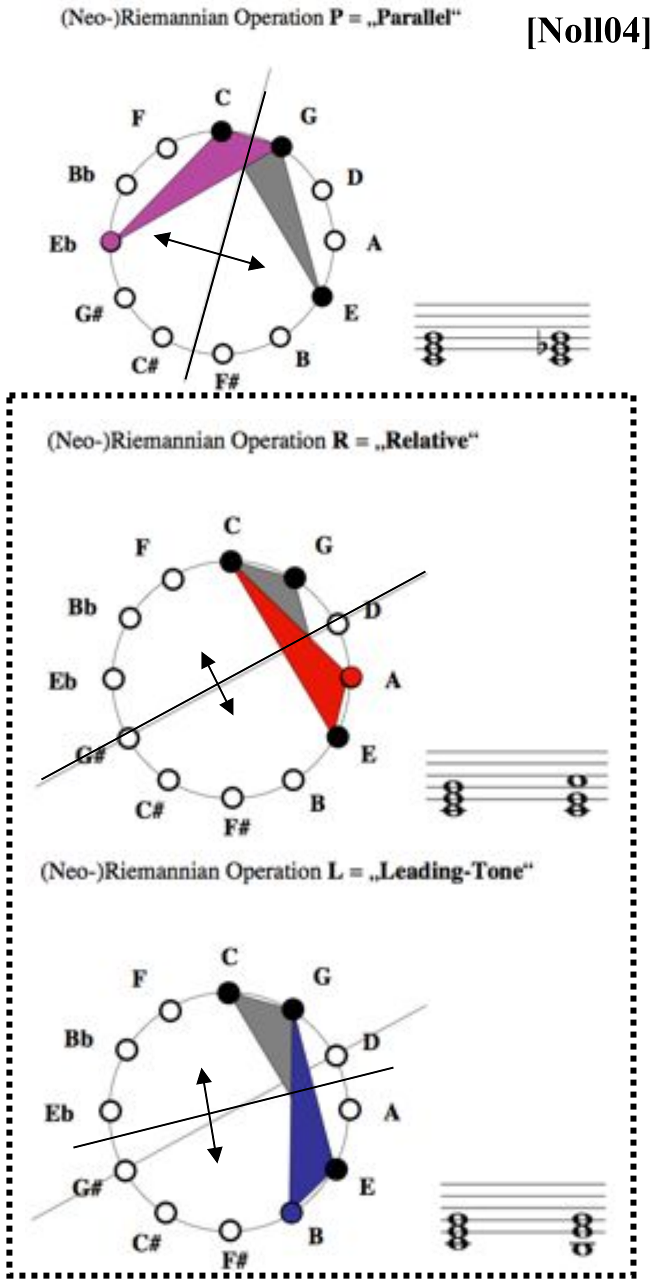
$\forall f, g \in \langle T, J \rangle$

Le groupe des 24 transformations σ = {T_0, T_1,…, T_{11}, T_0J, T_1J, …, T_{11}J} est commutatif et opère de manière simplement transitive sur l'espace *S* des 24 formes du pentacorde de base (i.e. l'ensemble de ses 12 transpositions et de ses 12 inversions)

⇒ (*S*, σ, int) est un GIS

Le *Tonnetz* en tant que GIS





$$\rho = \langle L, R \mid L^2 = (LR)^{12} = 1 \; ; \; LRL = L(LR)^{-1} \rangle$$

- **ρ opère de façon simplement transitive sur l'ensemble *S* des 24 triades consonantes**

⇒ (S, ρ, int) est un GIS

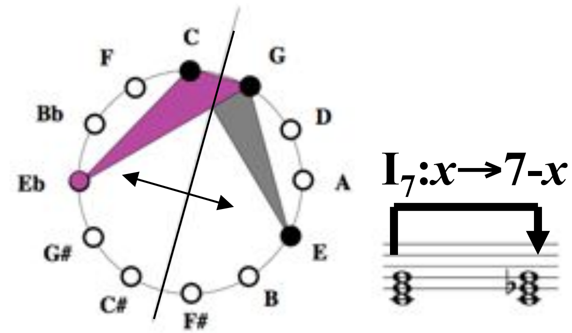
Une autre structure de GIS sur l’espace *S*

L	R	RL	I_{11}
C → e	C → a	C → G	C → e
c → A♭	c → E♭	c → f	c → E
D♭ → f	D♭ → b♭	D♭ → A♭	D♭ → e♭
c♯ → A	c♯ → E	c♯ → f♯	c♯ → E♭
D → f♯	D → b	D → A	D → d
d → B♭	d → F	d → g	d → D
E♭ → g	E♭ → c	E♭ → B♭	E♭ → c♯
d♯ → B	d♯ → F♯	d♯ → g♯	d♯ → C♯
E → g♯	E → c♯	E → B	E → c
e → C	e → G	e → a	e → C
F → a	F → d	F → C	F → b
f → D♭	f → A♭	f → b♭	f → B
F♯ → a♯	F♯ → d♯	F♯ → C♯	F♯ → b♭
f♯ → D	f♯ → A	f♯ → b	f♯ → B♭
G → b	G → e	G → D	G → a
g → E♭	g → B♭	g → c	g → A
A♭ → c	A♭ → f	A♭ → E♭	A♭ → a♭
g♯ → E	g♯ → B	g♯ → c♯	g♯ → G♯
A → c♯	A → f♯	A → E	A → g
a → F	a → C	a → d	a → G
B♭ → d	B♭ → g	B♭ → F	B♭ → f♯
a♯ → F♯	a♯ → C♯	a♯ → d♯	a♯ → F♯
B → e♭	B → g♯	B → F♯	B → f
b → G	b → D	b → e	b → F

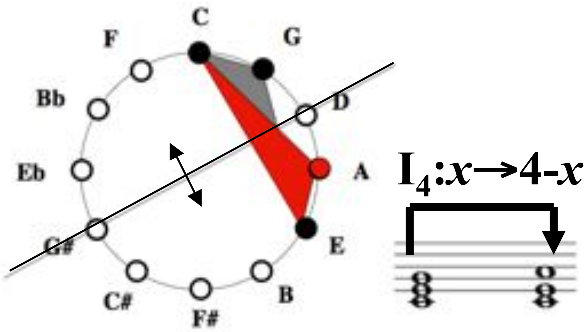
[Satyendra 2004]

(Neo-)Riemannian Operation **P** = „**Parallel**“

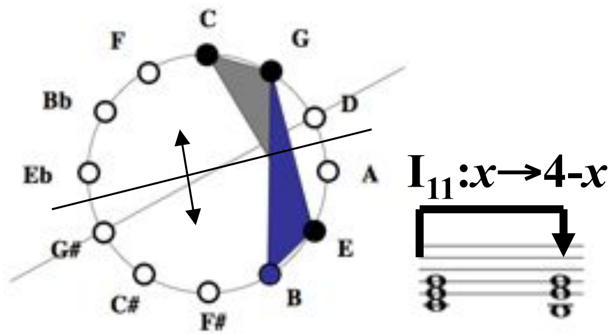
[Noll04]



(Neo-)Riemannian Operation **R** = „**Relative**“



(Neo-)Riemannian Operation **L** = „**Leading-Tone**“



D_{12} = <I, T | $I^2 = T^{12} = 1$; ITI=I$(IT)^{-1}$ >

- **D_{12} opère de façon simplement transitive sur l’ensemble *S* des 24 triades consonantes**

⇒ (*S*, D_{12}, int) est un GIS