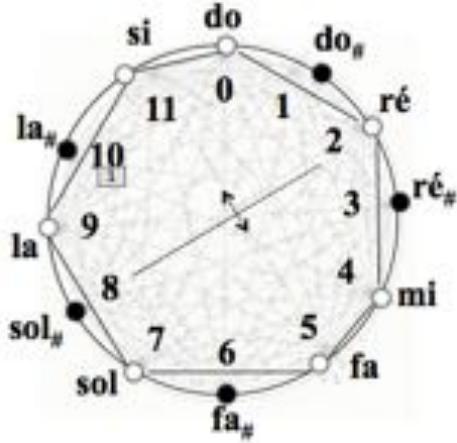


Algebra and Geometry in Music and Musicology



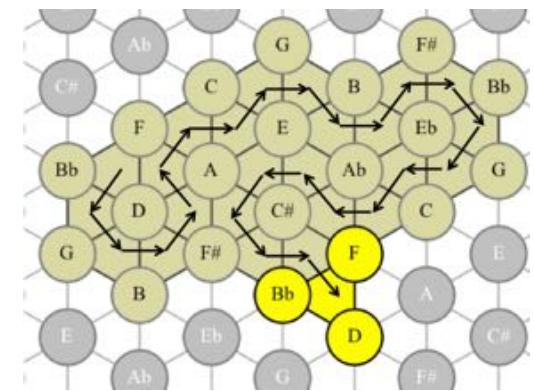
Séminaire PMMH / ESPCI

9 Juin 2017

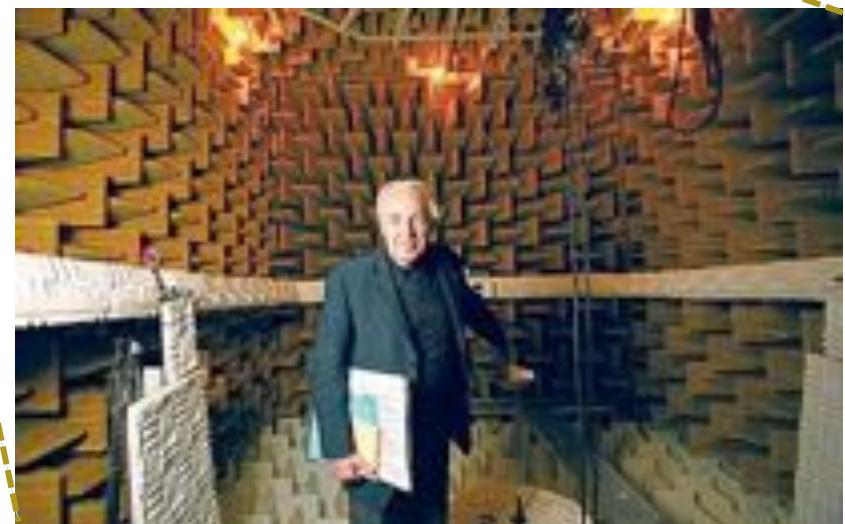
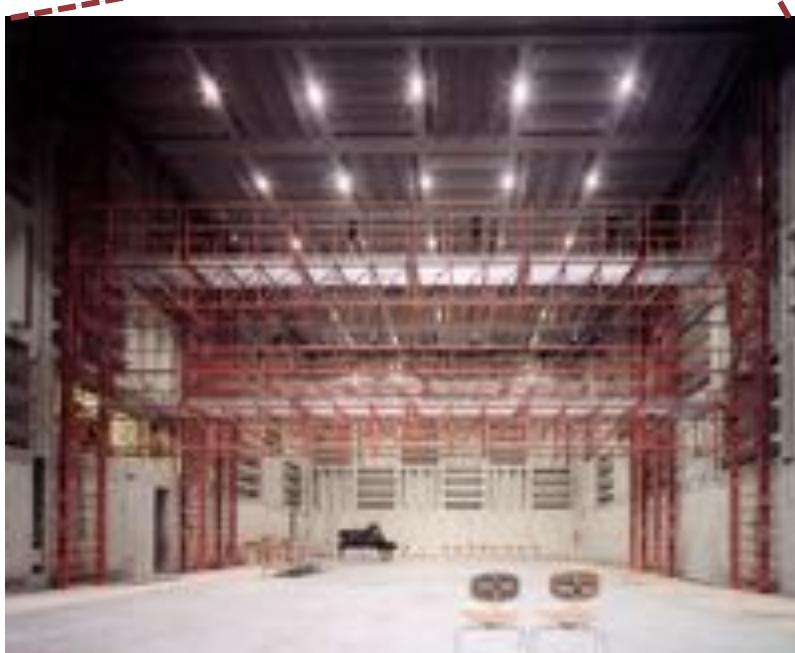
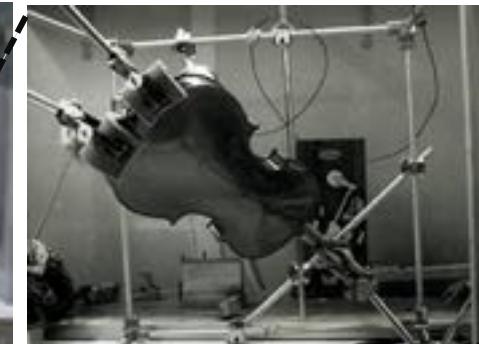
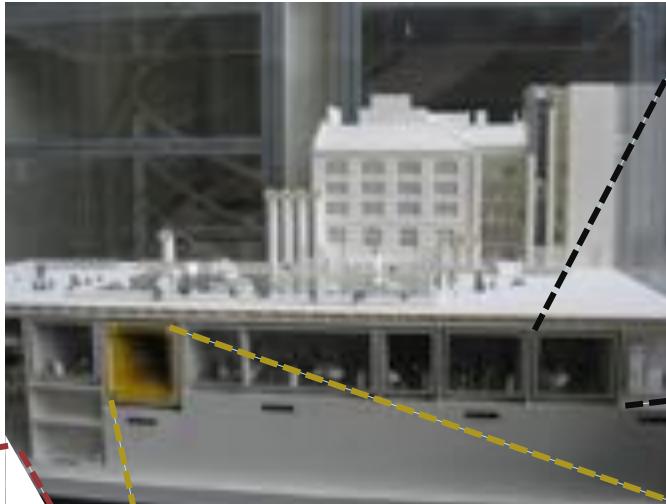
Moreno Andreatta

Music Representations Team

IRCAM / CNRS UMR 9912 / UPMC, Paris
IRMA & GREAM, University of Strasbourg



The musical and scientific research at IRCAM...



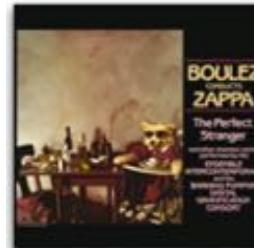
... at the interface between contemporary and pop music



MusiqueLab 2

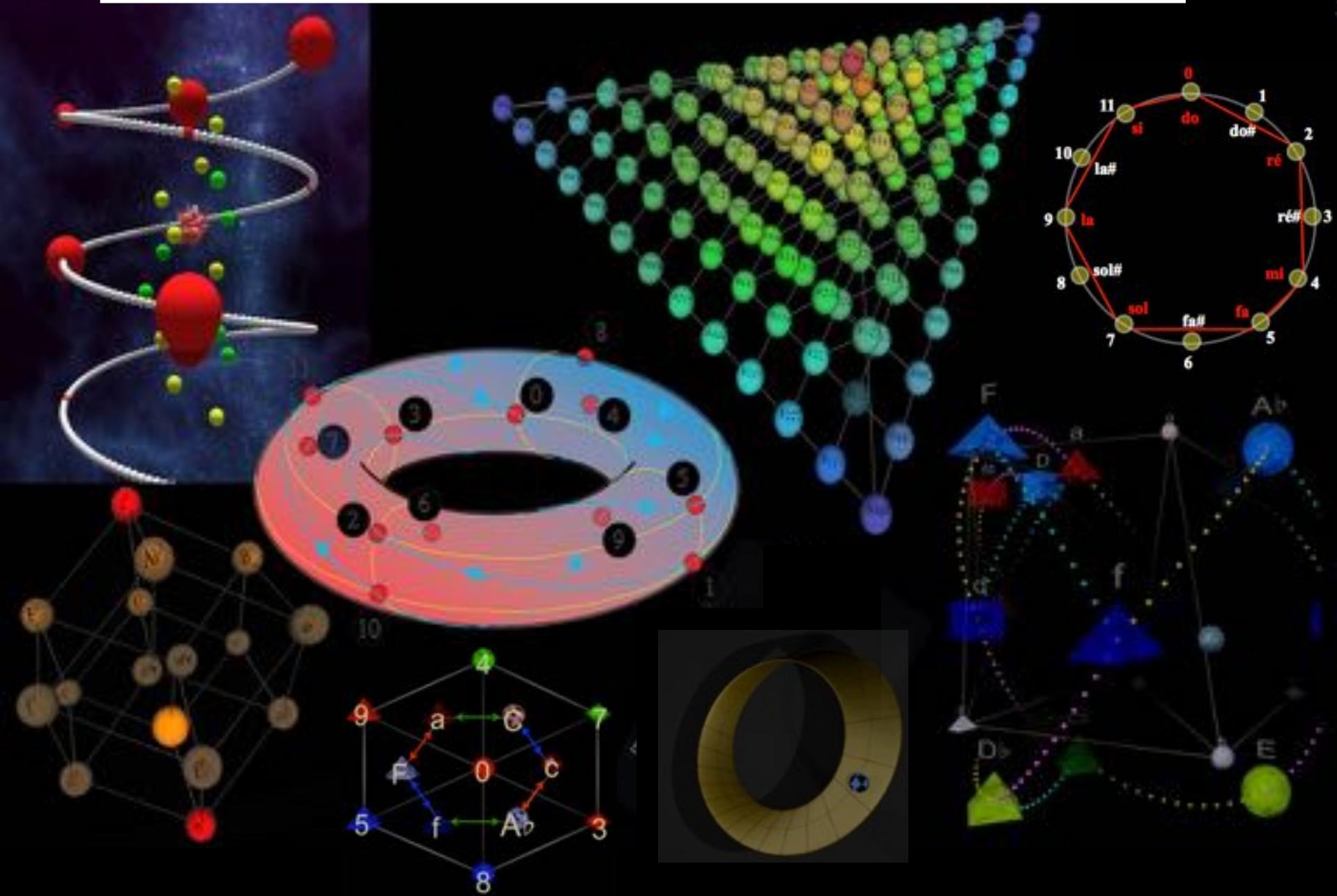


OMAX (computer-aided impro)

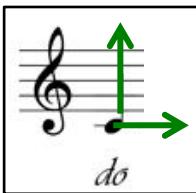
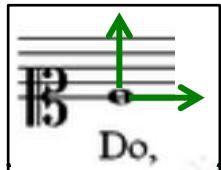


www.ircam.fr

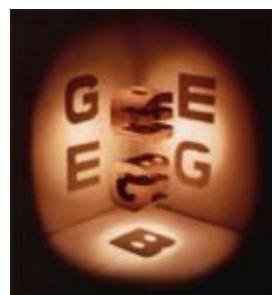
The galaxy of geometrical models at the service of music



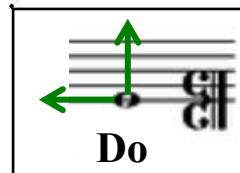
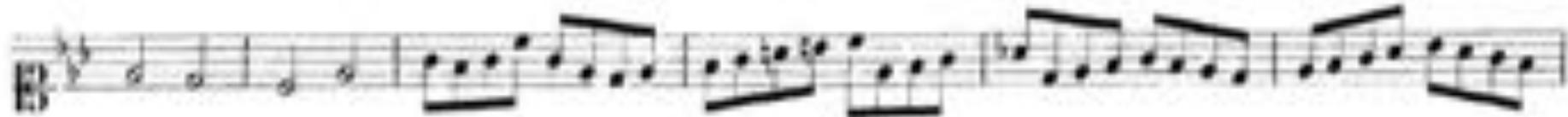
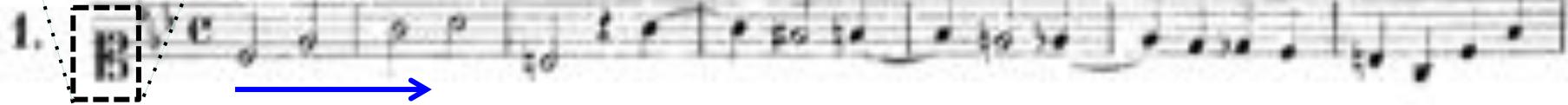
Bach's enigmatic canons and geometry



Canones diversi
super thema regium

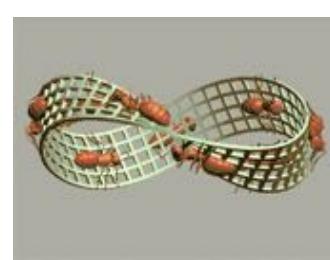


Canon a 2.





My end is my beginning (but twisted!)



Canones diversi

super thema regium

1.

Canon n. 2

1. Canon n. 2

Canones diversi
super thema regium

...

1.

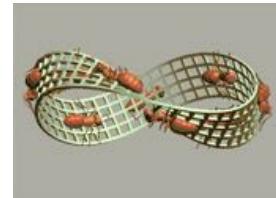
Canon n. 2

1. Canon n. 2

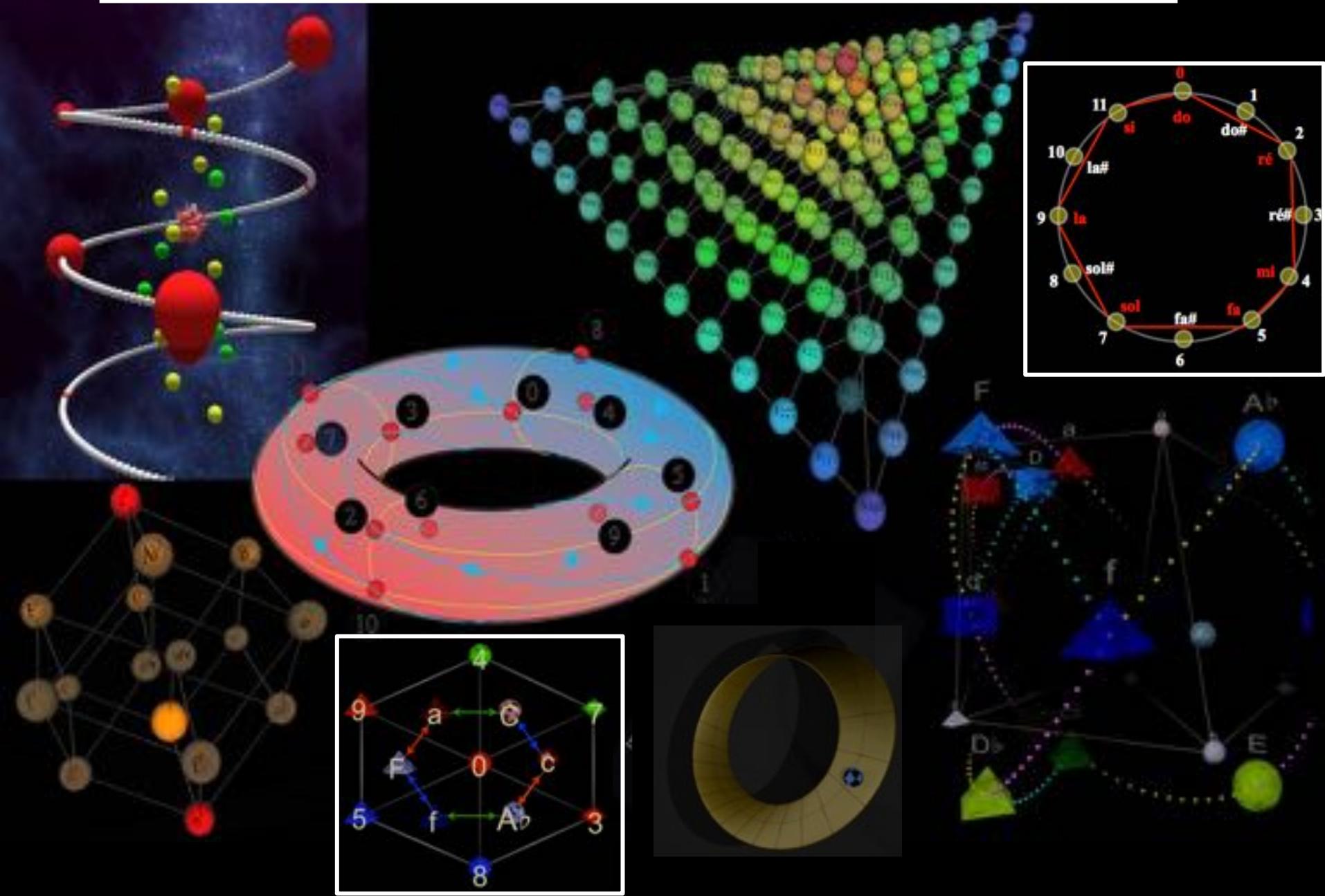


<http://www.josleys.com/Canon/Canon.html>

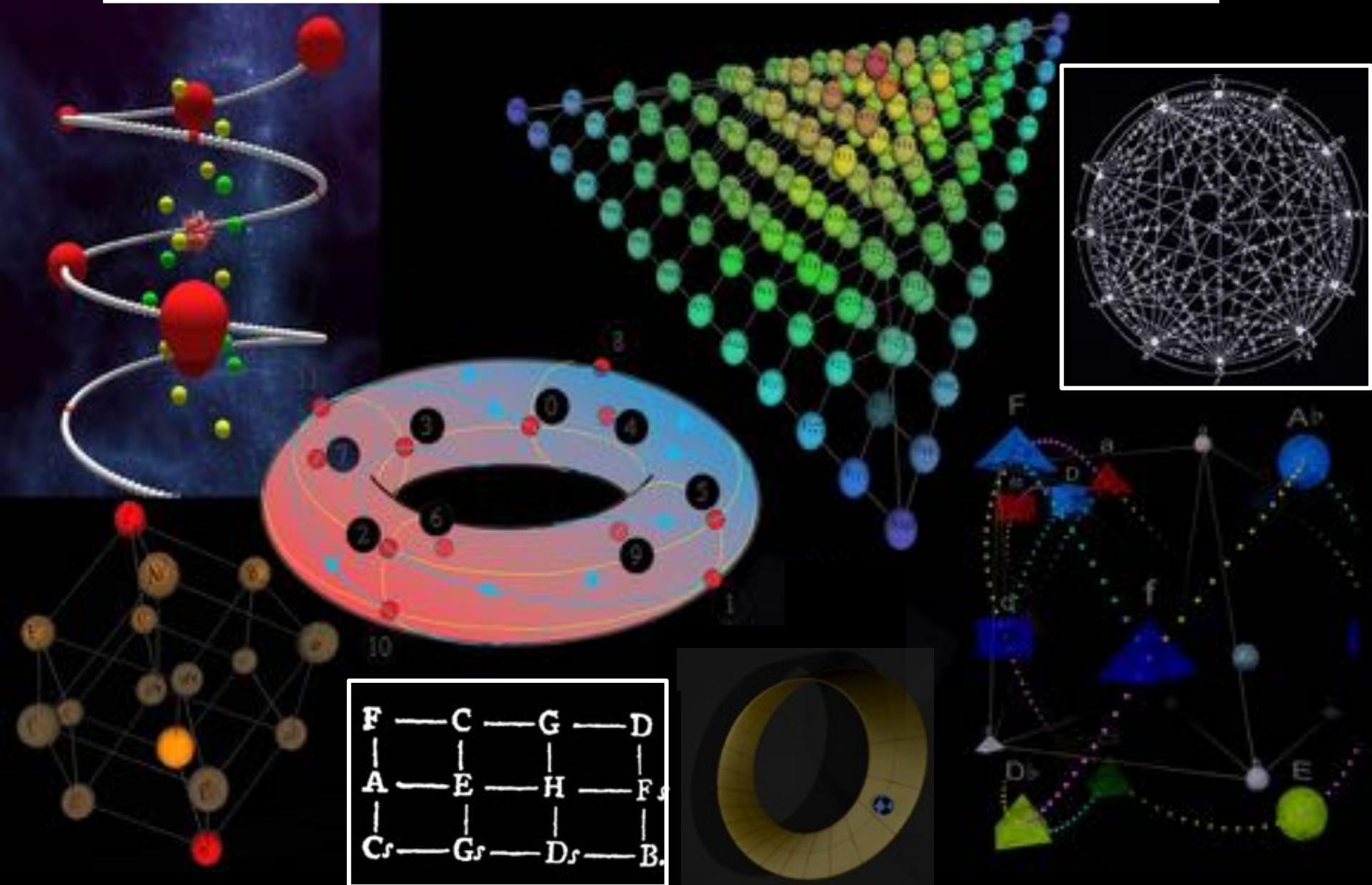
[min. 1'14"]



The galaxy of geometrical models at the service of music



The galaxy of geometrical models at the service of music



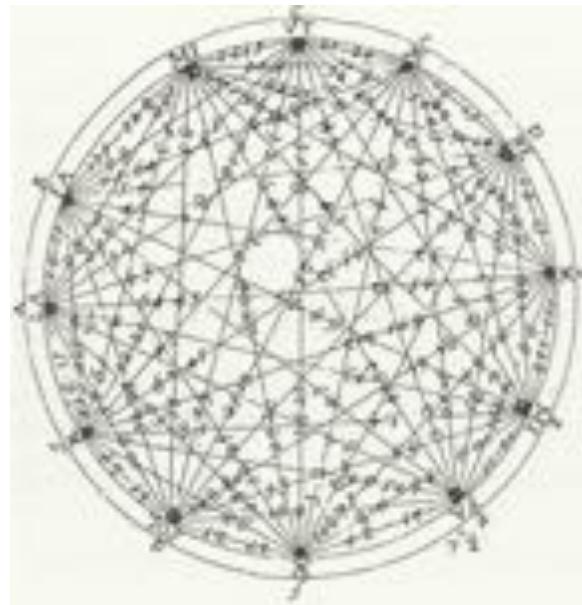
Mersenne and the birth of (musical) combinatorics

114 Marin Mersenne, *Harmonicorum Libri XII*, 1648

LIBER SEPTIMVS. DE CANTIBVS, SEV CANTILENIS, EARVMQ; NVMERO, PARTIBVS, ET SPECIEBVS.

Tabela Combinationis ab I ad XXII.

I	1
II	2
III	6
IV	24
V	120
VI	720
VII	5040
VIII	40320
IX	362880
X	3628800
XI	39916800
XII	479001600
XIII	6117020800
XIV	8717819200
XV	1107674368000
XVI	10933789888000
XVII	311687418296000
XVIII	6401173705718000
XIX	1116410040818000
XX	1433904008176640000
XXI	31090941171709440000
XXII.	384000737777607680000



Marin Mersenne

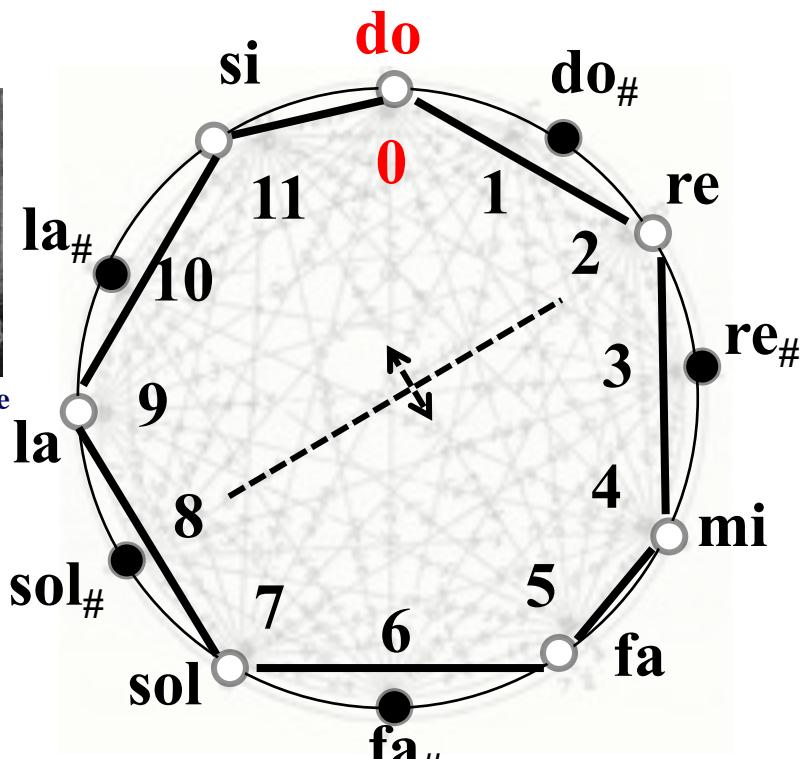
Varietas, seu Combinatio quatuor notarum.



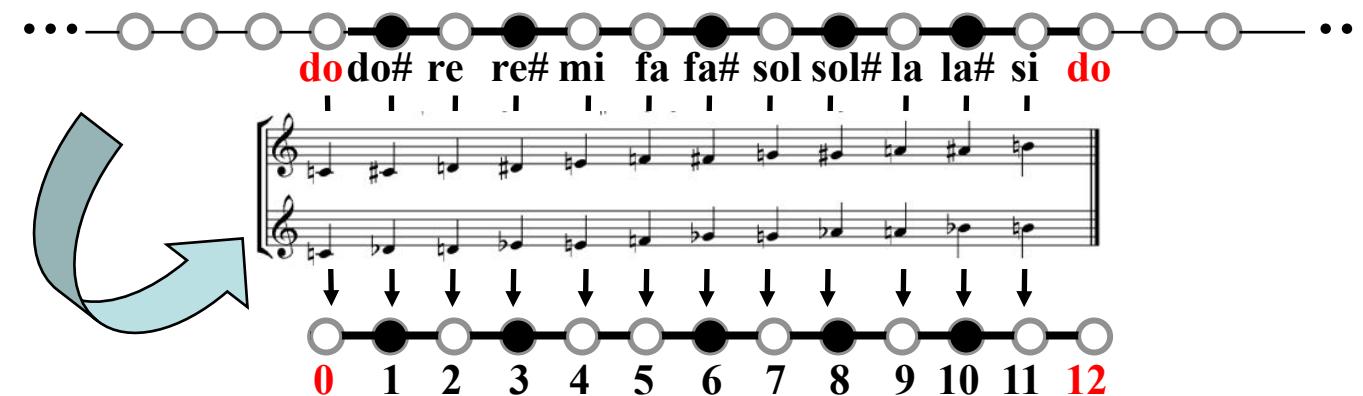
The circular representation of the pitch space



Marin Mersenne



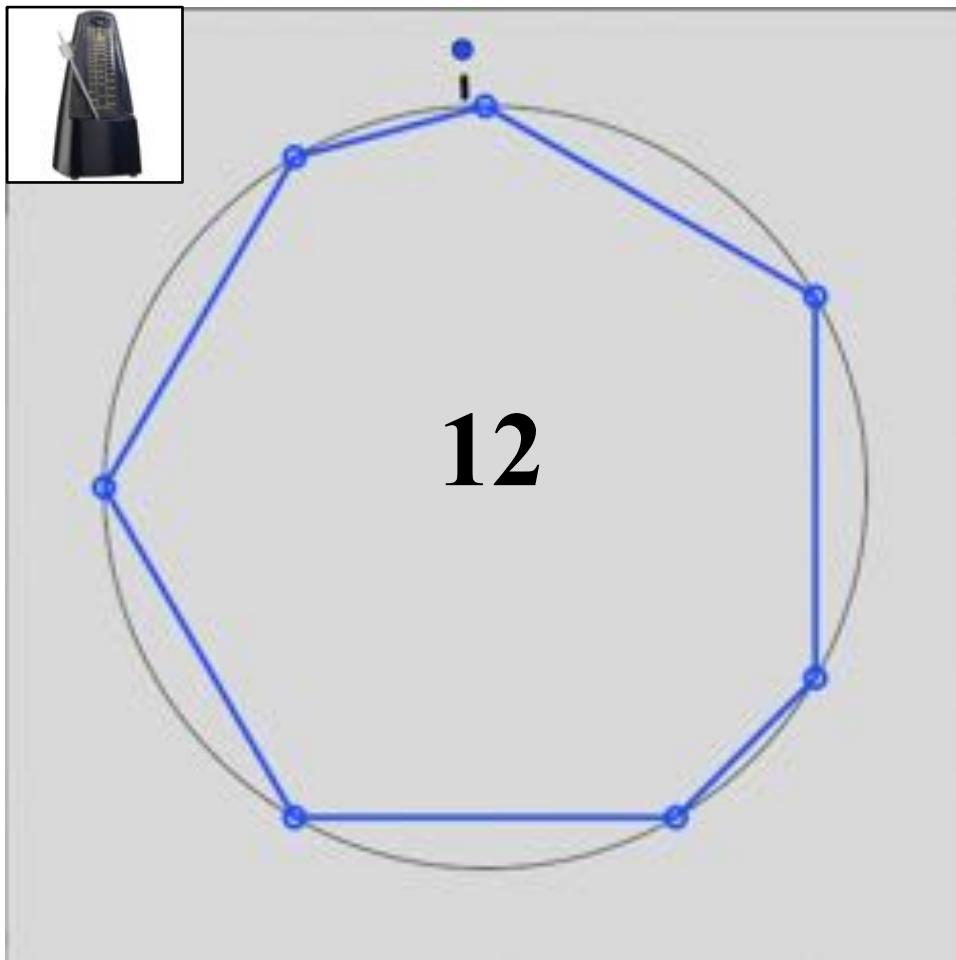
Harmonicorum Libri XII, 1648



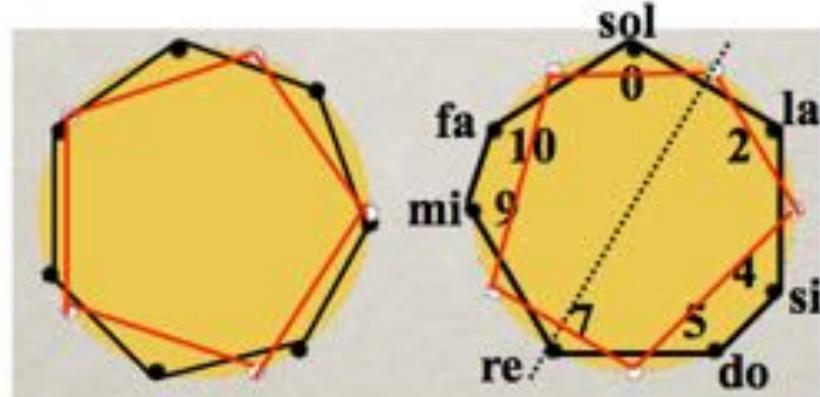
Tabelă Combinatorie ab 1 ad 22.	
I	1
II	2
III	6
IV	24
V	110
VI	720
VII	5040
VIII	40320
IX	362880
X	3628800
XI	3628800
XII	479001600
XIII	6117016000
XIV	877399600
XV	11076496000
XVI	130320880000
XVII	1516874180000
XVIII	640473705718000
XIX	1316450004513000
XX	1451900004513000
XXI	5109204181709440000
XXII	1884000727777607430000

→ DEMO

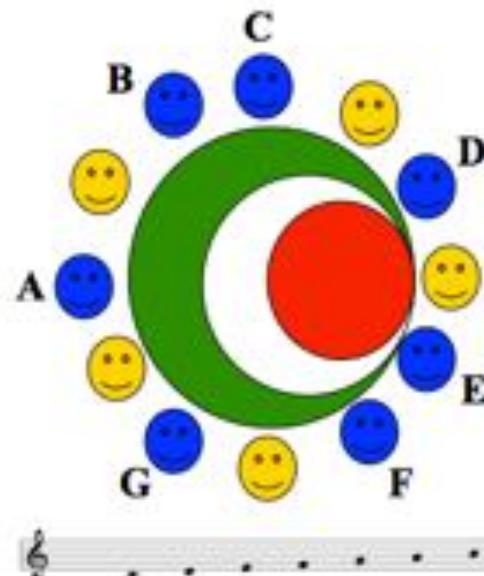
The Pitch/Rhythm Isomorphism



Abadja or Bembé



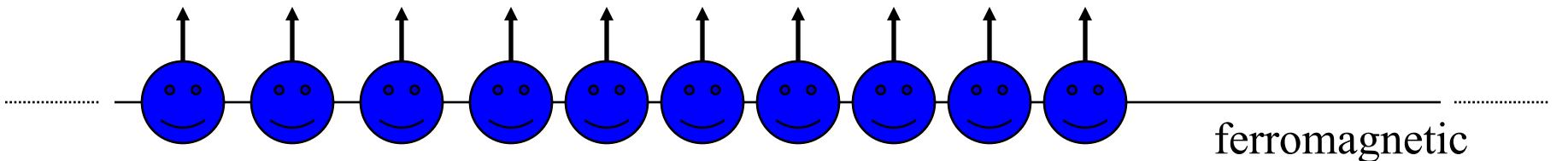
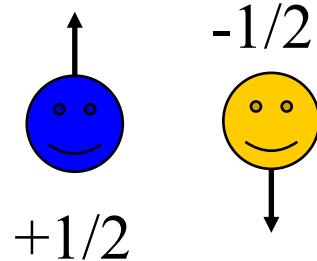
The problem of concentric circles



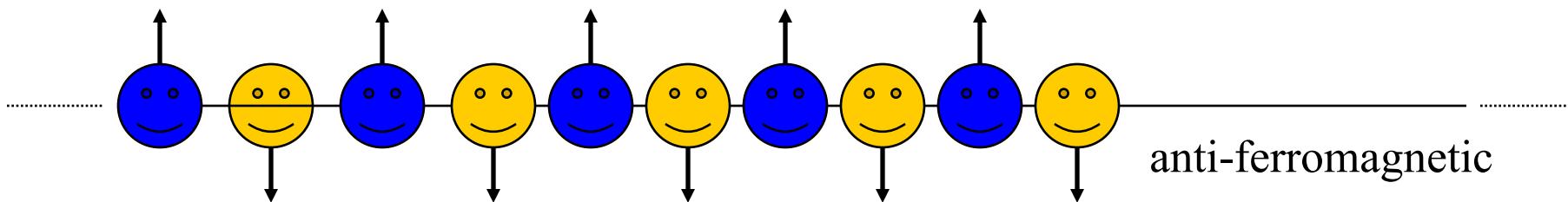
Dinner Table Problem

ME-sets and the Ising antiferromagnetic spin-1/2 model

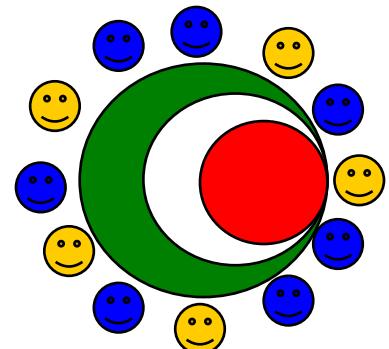
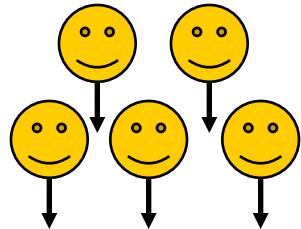
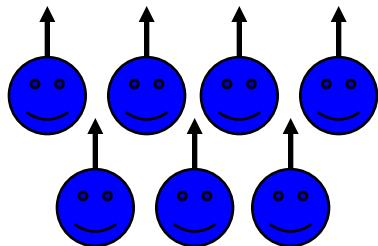
- Jack Douthett & Richard Krantz, “**Energy extremes and spin configurations for the one-dimensional antiferromagnetic Ising model with arbitrary-range interaction**”, *J. Math. Phys.* 37 (7), July 1996
- Jack Douthett & Richard Krantz, “Maximally even sets and configurations: common threads in mathematics, physics, and music”, *J. Comb. Optim.* 14, 2007



ferromagnetic

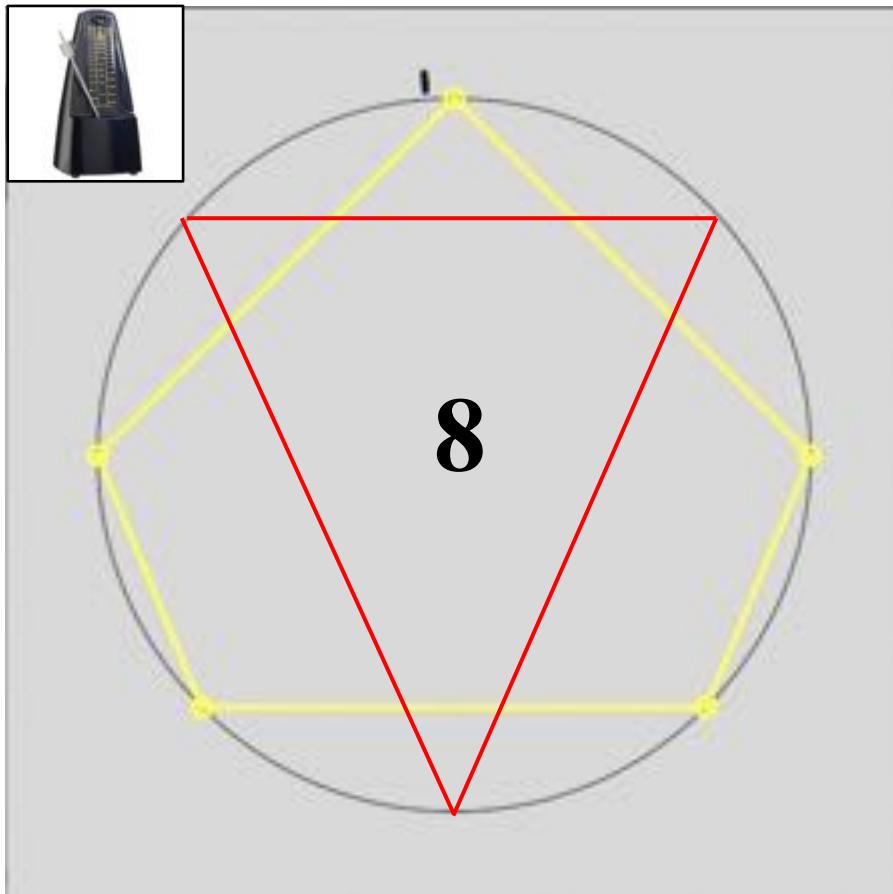


anti-ferromagnetic

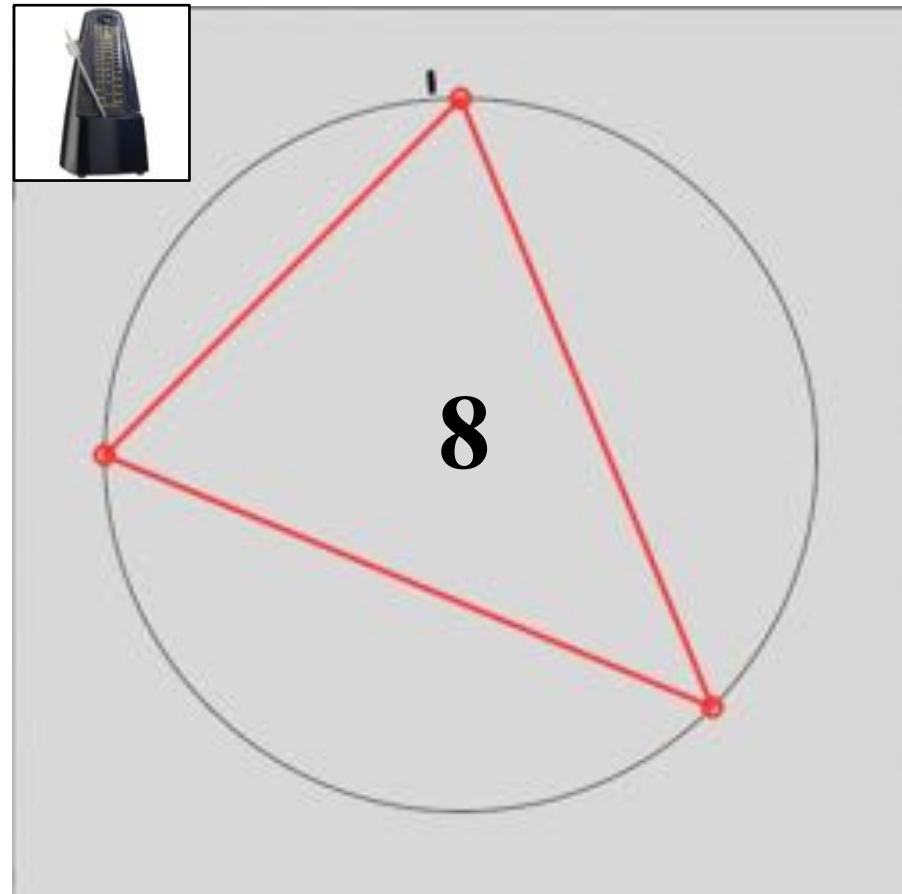


African-cuban ME-rhythms

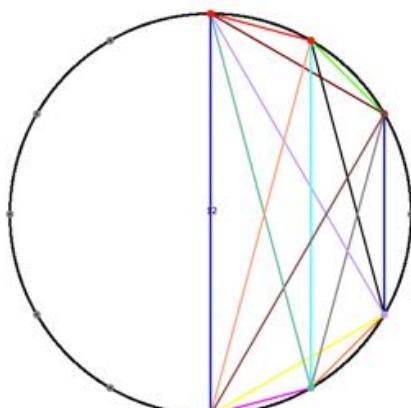
El cinquillo



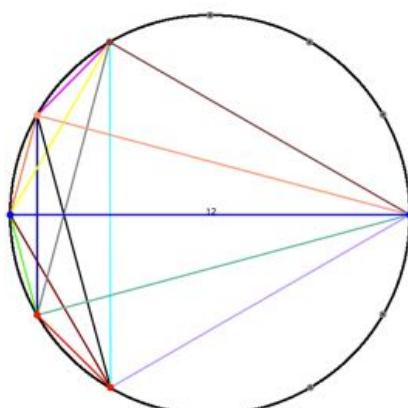
El trecillo



The Discrete Fourier Transform in symbolic MIR



\approx
Hexachord
Theorem



\approx
Homometric
structures

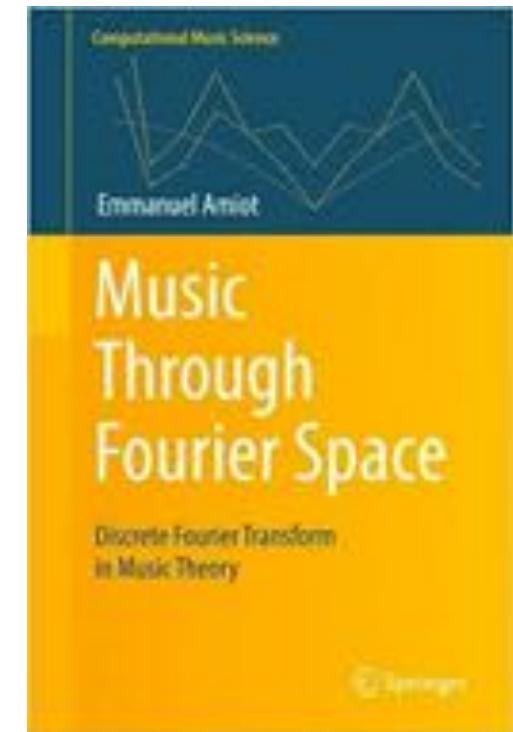
$$IC_A = [4, 3, 2, 3, 2, 1] = [4, 3, 2, 3, 2, 1] = IC_{A'}$$

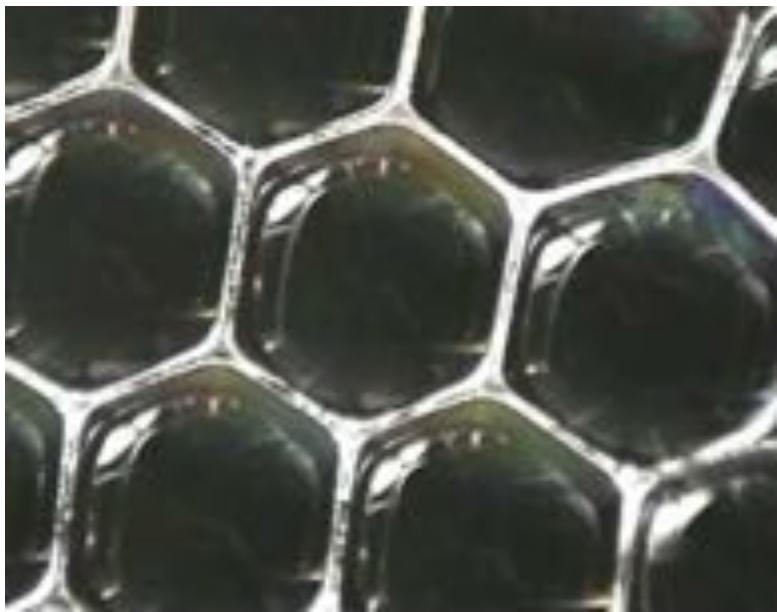
$$IC_A(k) = (1_A \star 1_{-A})(k)$$

$$\mathcal{F}_A : t \mapsto \sum_{k \in A} e^{-2i\pi kt/c}$$

$$\forall k \quad \mathcal{F}(IC_{\mathbb{Z}_c \setminus A})(k) = \mathcal{F}(IC_A)(k)$$

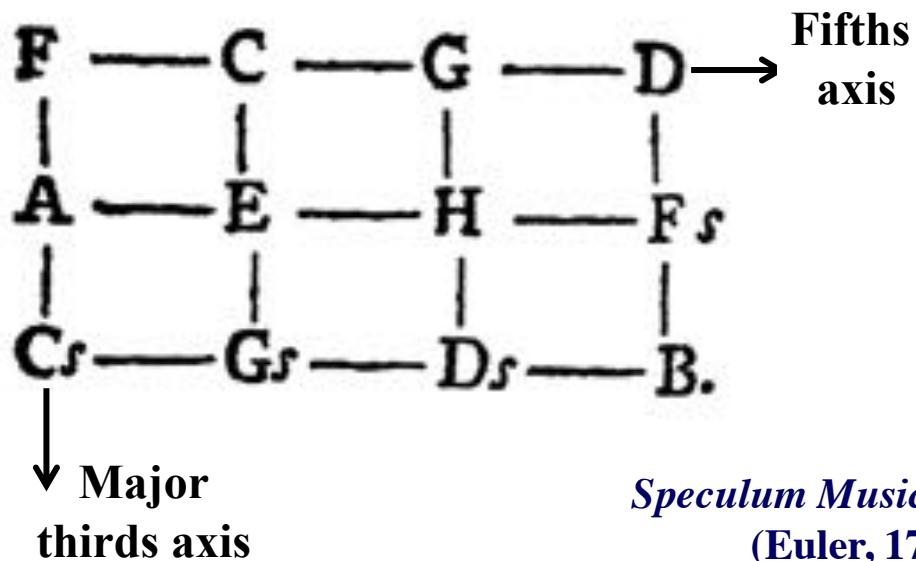
E. Amiot : « Une preuve élégante du théorème de Babbitt par transformée de Fourier discrète », Quadrature, 61, 2006.





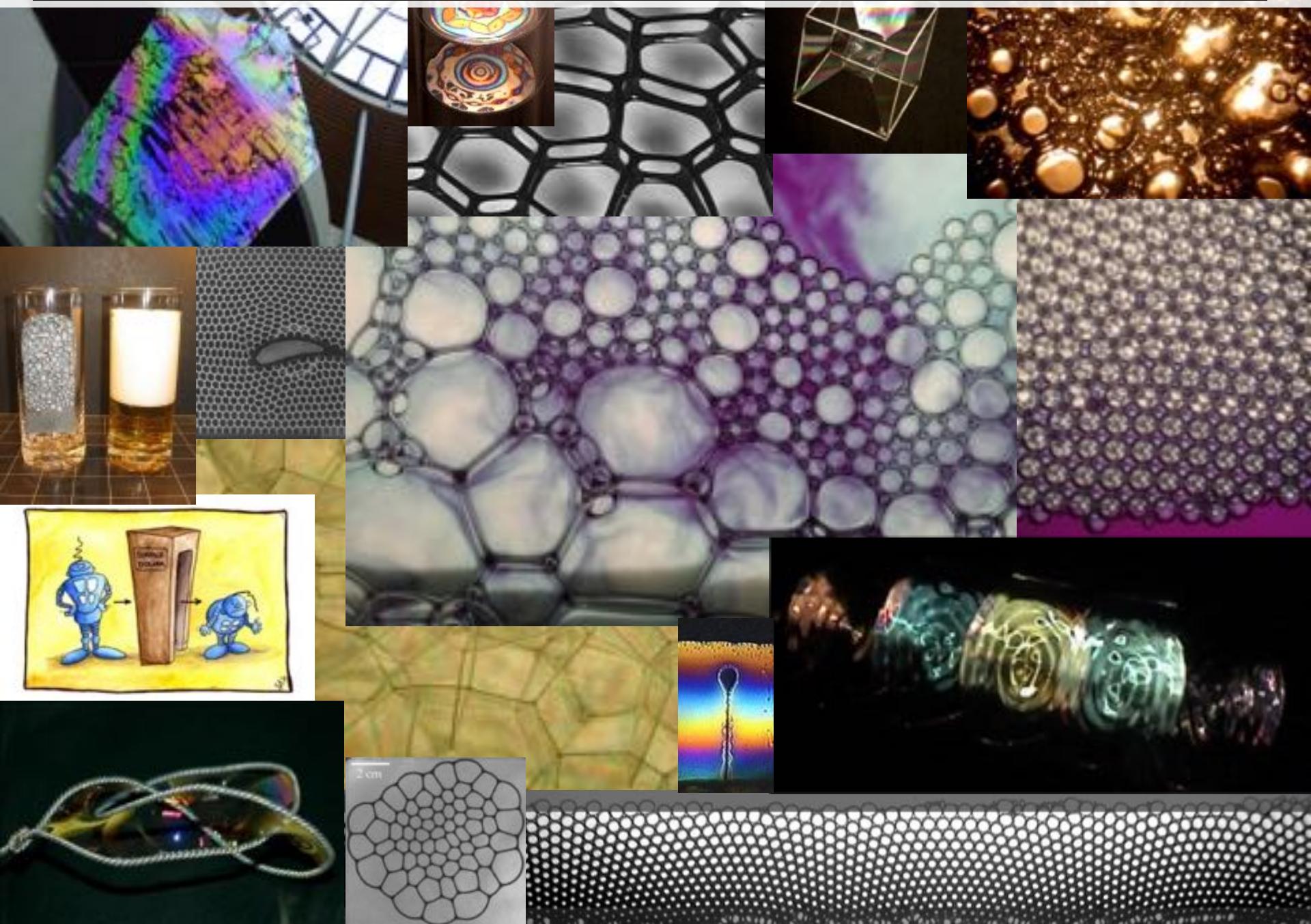
The Tonnetz

(or ‘honeycomb’ hexagonal tiling)

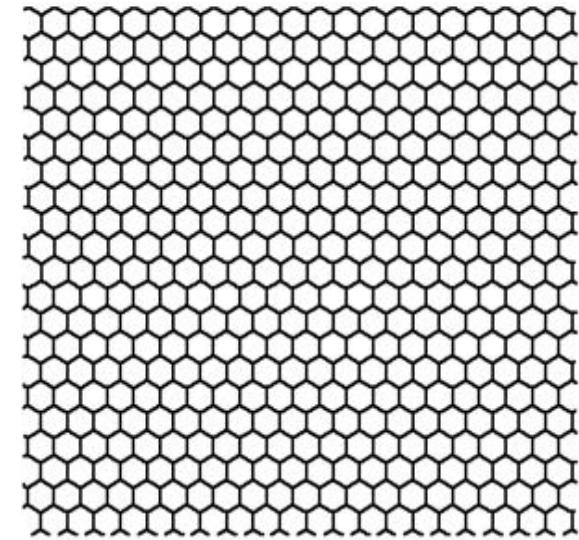


Speculum Musicum
(Euler, 1773)

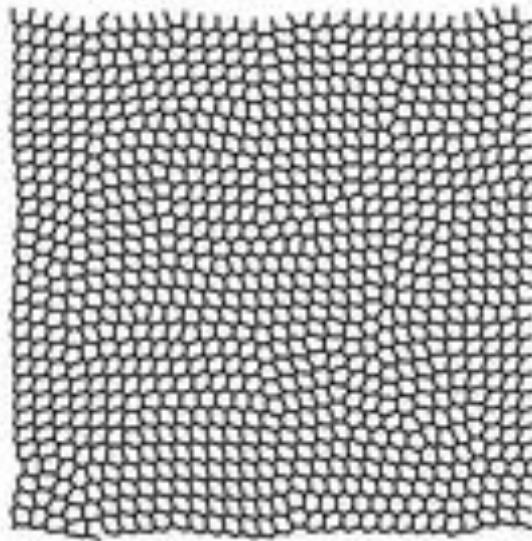
From music analysis to the sonifications of complex systems



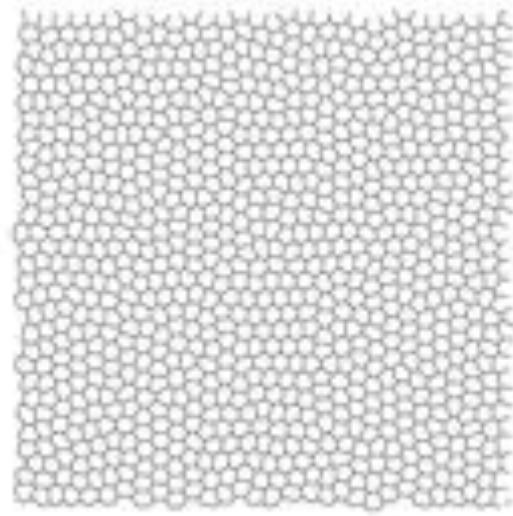
Can you hear the shape of a tiling?



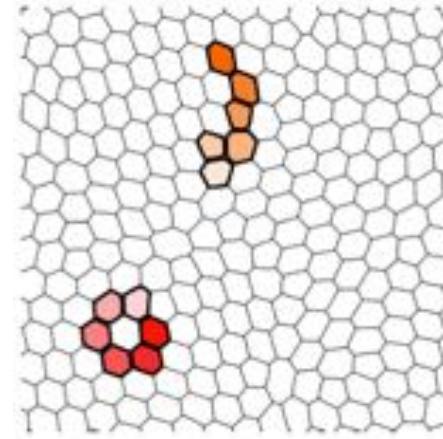
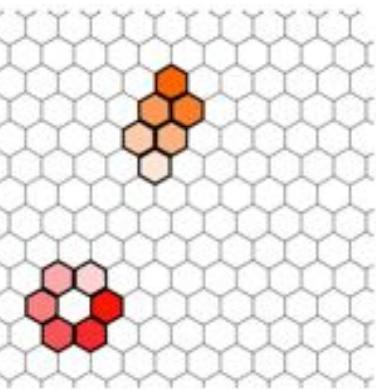
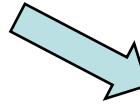
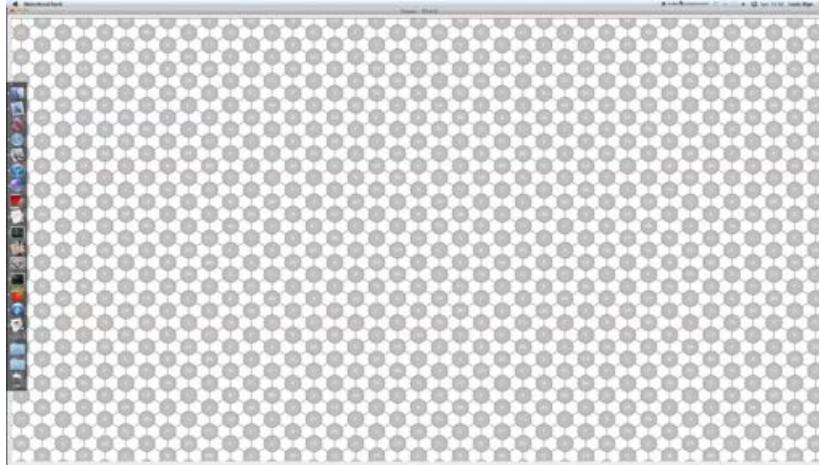
Honeycomb Tonnetz



Grain boundaries

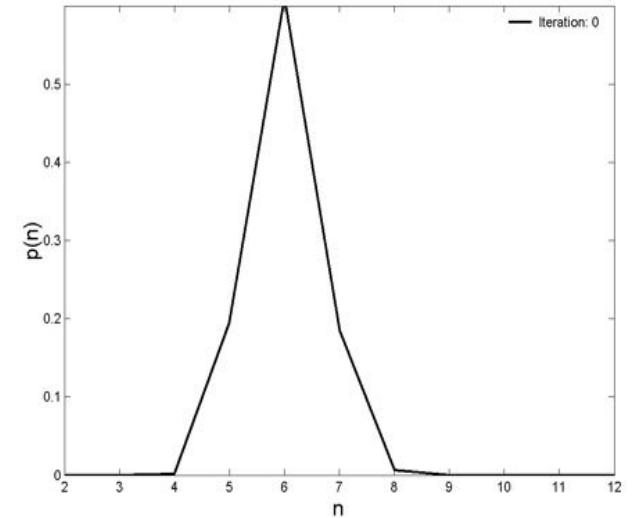
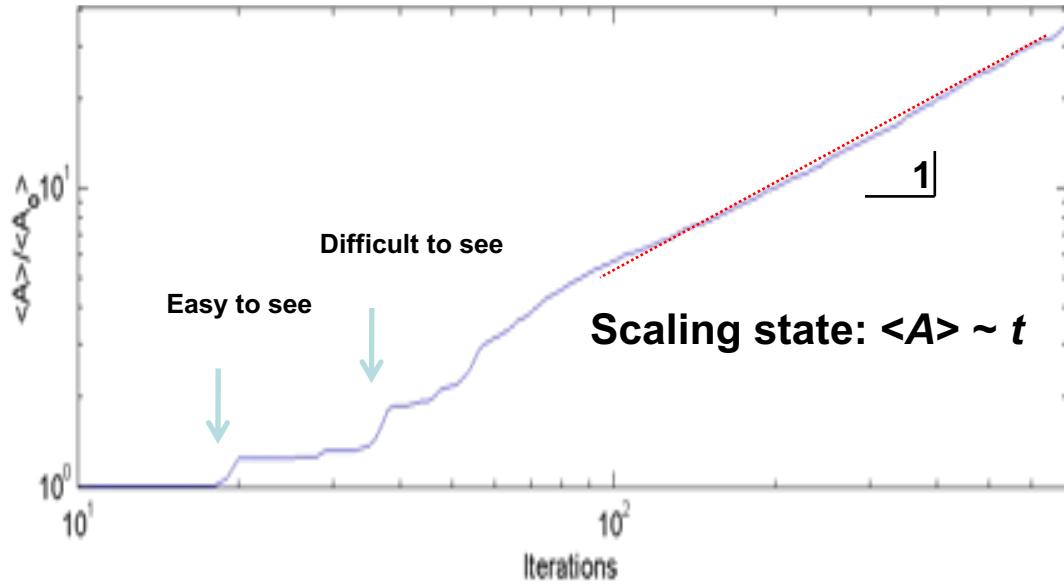


Disordered

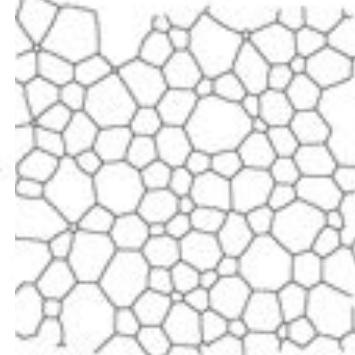
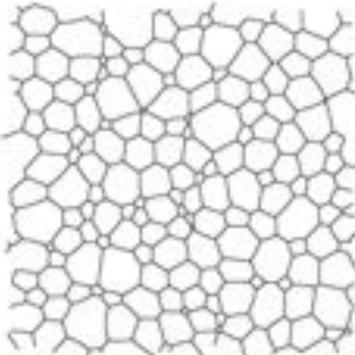
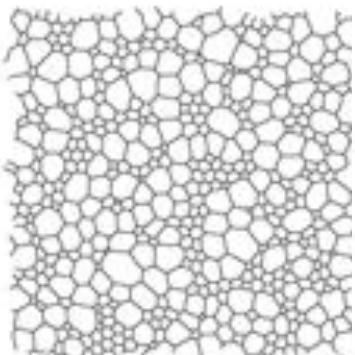
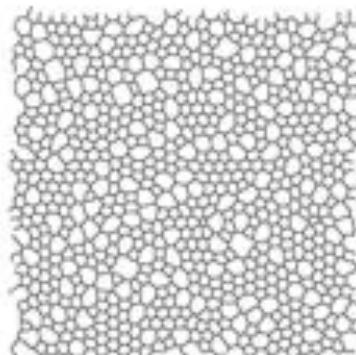
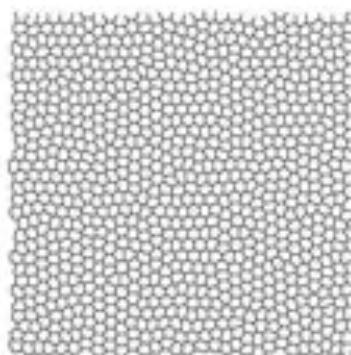


Can you hear the transition towards the ‘scaling state’?

Average cell area



Number n of neighbours of cell



Iteration: 0

10

20

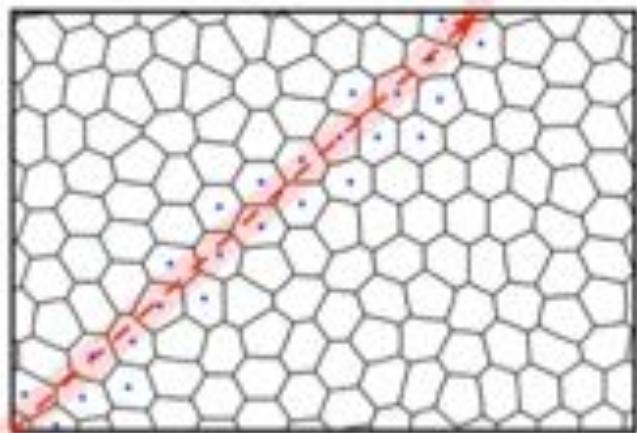
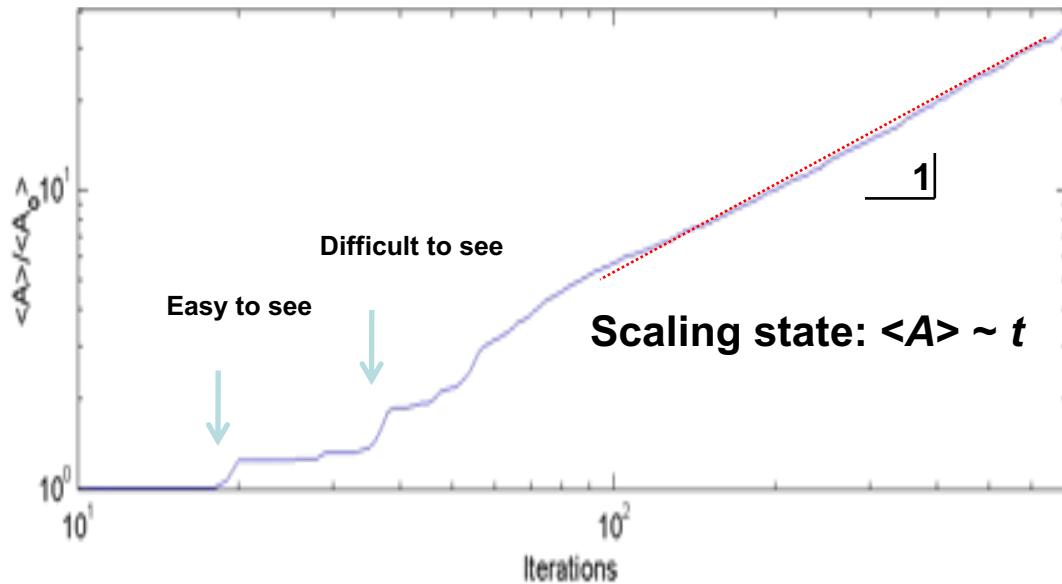
40

80

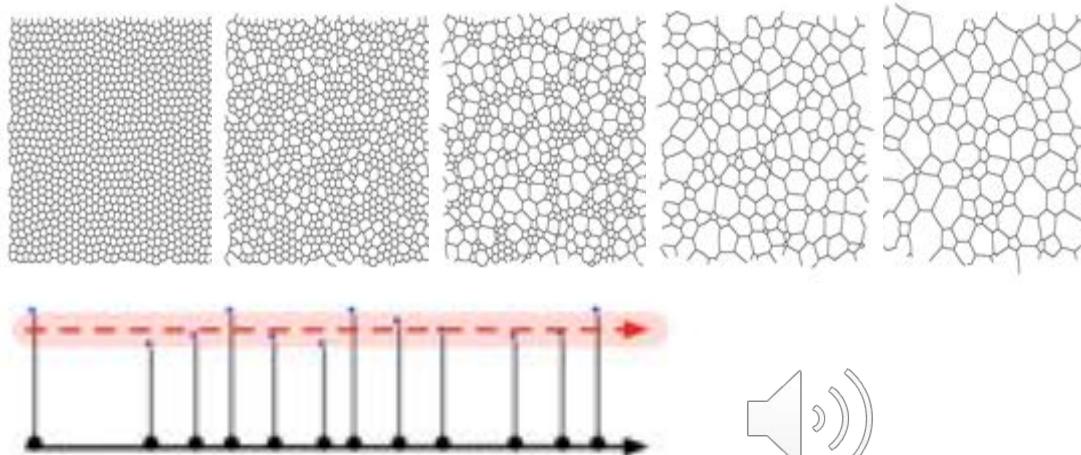
Iteration (time)

Can you hear the transition towards the ‘scaling state’?

Average cell area

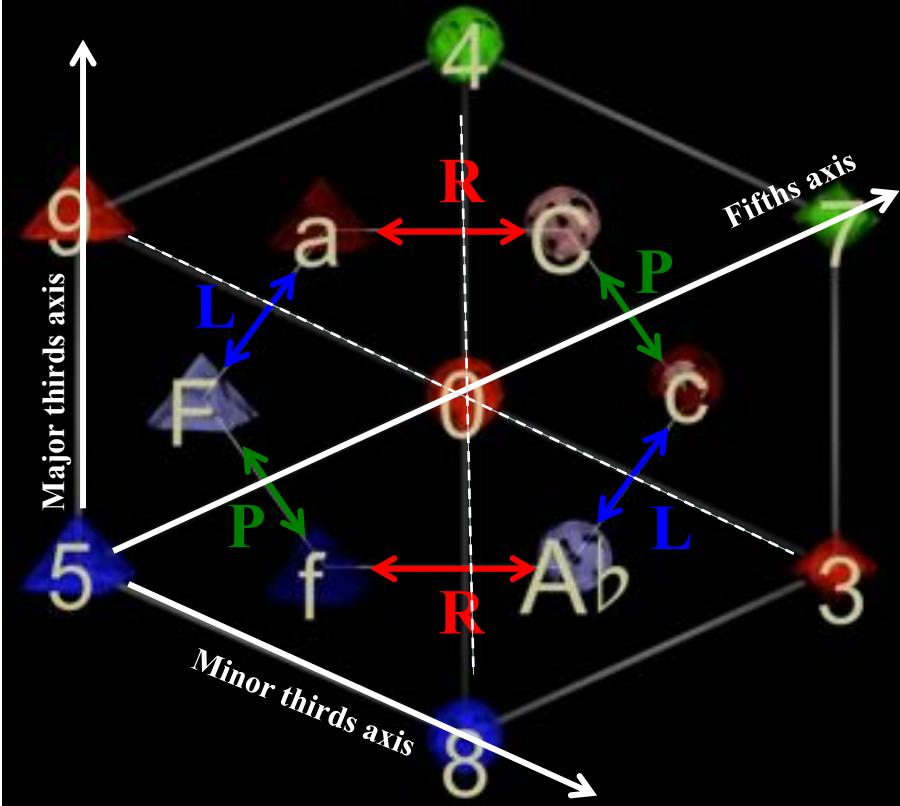


(a) En pointillés, (Δ) traverse la mousse. Les centres des bulles proches sont sélectionnés



(b) Projection orthogonale des centres de bulles sur (Δ) pour obtenir une phrase rythmique



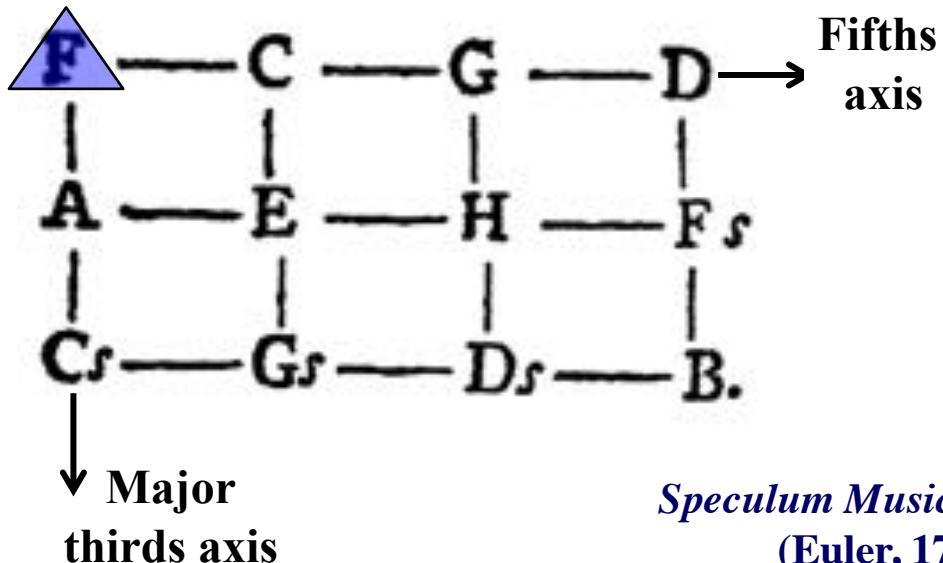


The Tonnetz

(or ‘honeycomb’ hexagonal tiling)

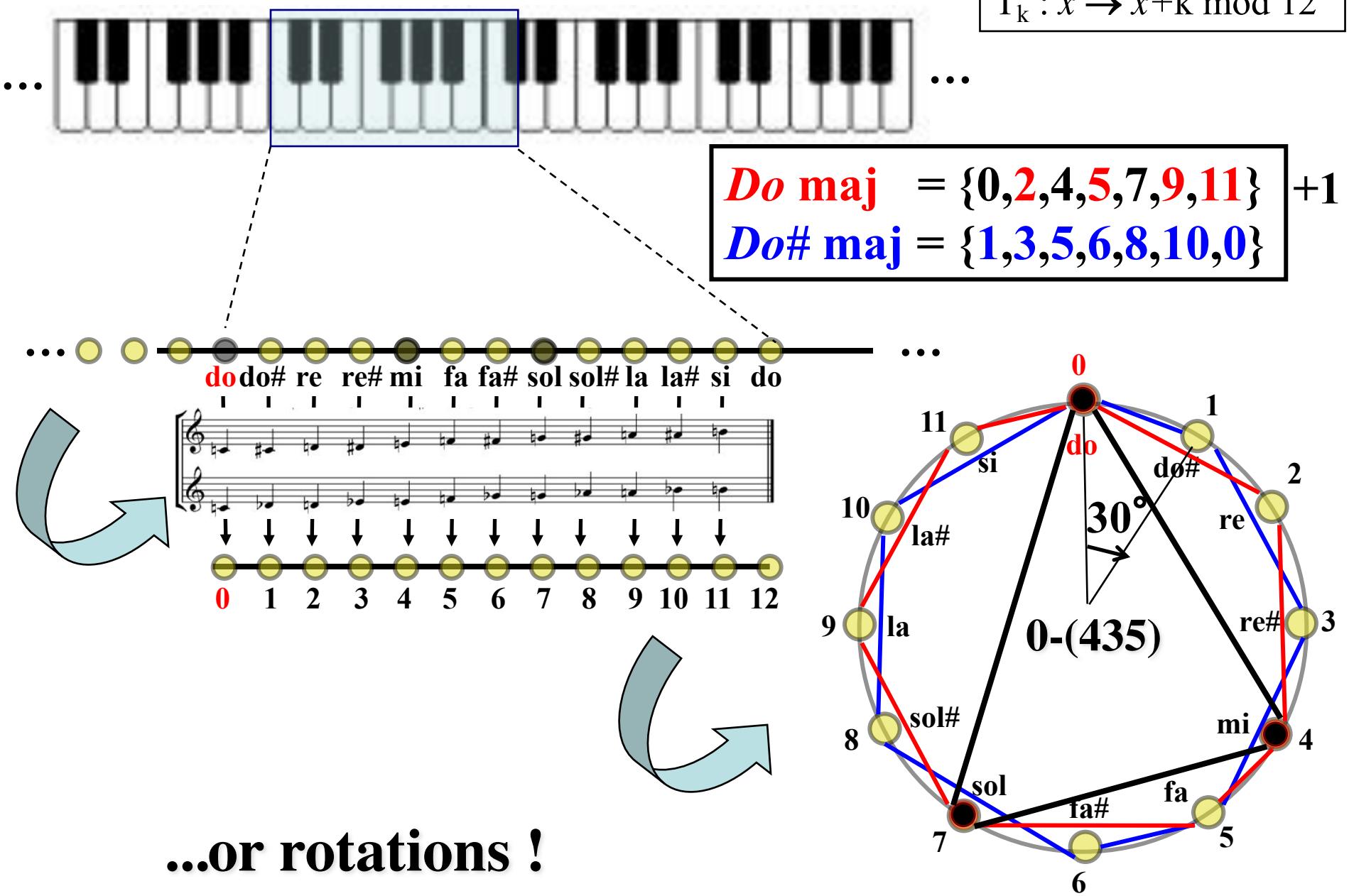


Gilles Baroin

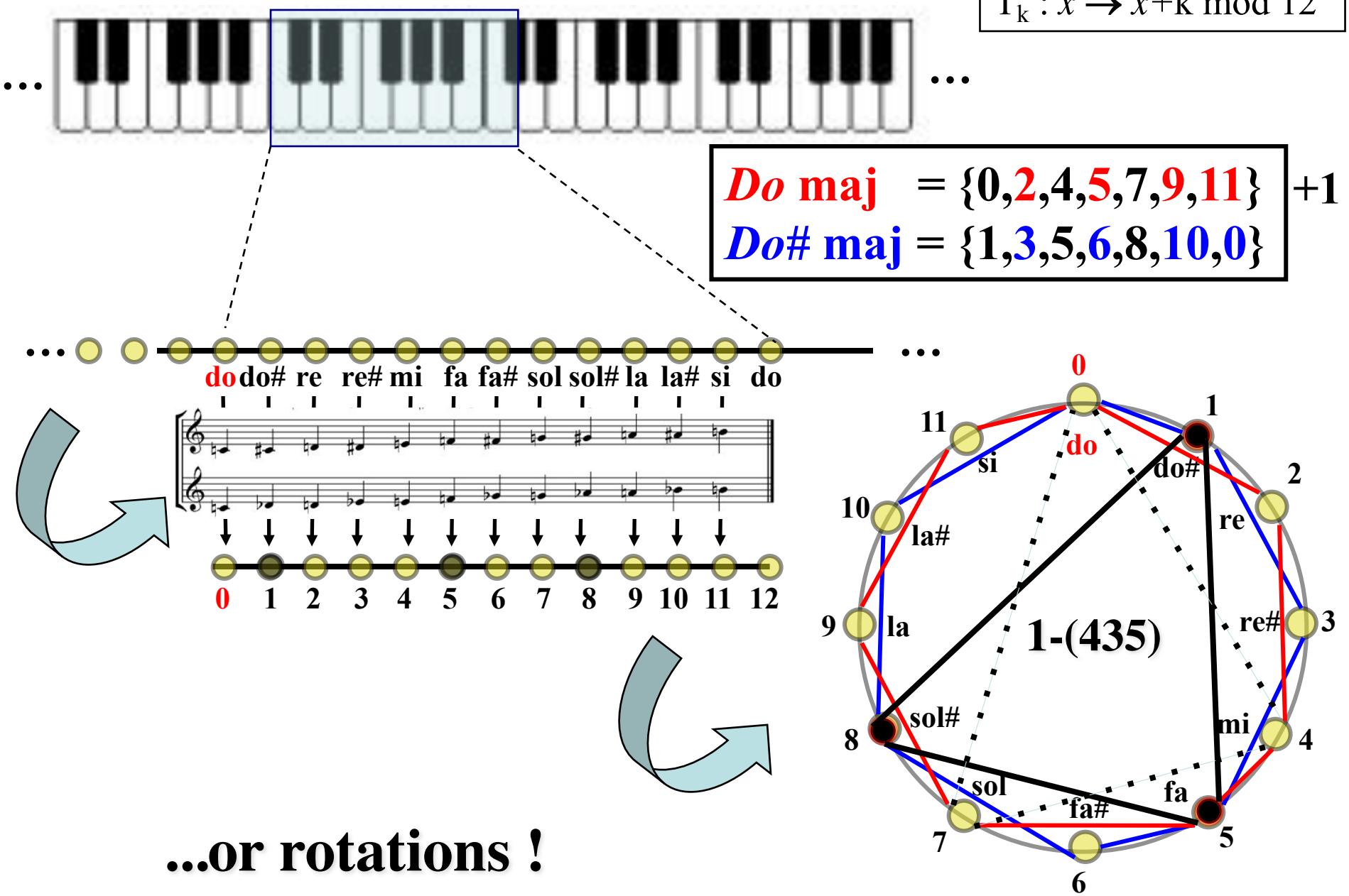


Speculum Musicum
(Euler, 1773)

Musical transpositions are additions...



Musical transpositions are additions...



Musical inversions are differences...

... or axial symmetries!

The diagram illustrates musical inversions and axial symmetries in a 12-note system, mapping notes to integers from 0 to 11. A blue box highlights a segment of the piano keyboard, and a red box shows the sets for **Do maj** and **La min**. A blue arrow indicates a clockwise cycle, and a red arrow indicates an inversion operation I_4 .

Piano Keyboard: Shows a segment of a piano keyboard with black and white keys. A blue dashed box highlights a segment of five keys.

Notes and Integers: A horizontal line with 12 yellow circles numbered 0 to 11. Below it, a musical staff shows notes corresponding to these numbers: 0=do, 1=do#, 2=re, 3=re#, 4=mi, 5=fa, 6=fa#, 7=sol, 8=sol#, 9=la, 10=la#, 11=si, 12=do. Arrows point from each note name to its corresponding integer.

Set Definitions:

- Do maj** = {0, 4, 7}
- La min** = {0, 4, 9}

Mathematical Transformation: $I_4(x) = 4 - x$

Graph: A circular graph with 12 nodes labeled 0 to 11. Nodes 0, 4, and 7 are highlighted in red. Nodes 1, 2, 3, 5, 6, 8, 9, 10, 11, and 12 are highlighted in grey. Edges connect adjacent nodes. A red line connects nodes 0, 4, and 7. A blue line connects nodes 0, 4, and 9. A dashed black line connects node 0 to node 12. A blue curved arrow at the bottom left indicates a clockwise cycle, and a red curved arrow at the bottom right indicates the I_4 operation.

Equation: $I : x \rightarrow -x \bmod 12$

Musical inversions are differences...

... or axial symmetries!

The diagram illustrates musical inversions and axial symmetries using a piano keyboard, a circle of fifths, and a musical staff.

Piano Keyboard: A horizontal piano keyboard is shown with a blue box highlighting a segment of keys. Dashed arrows point from this segment to a circle of fifths and a musical staff.

Circle of Fifths: A circular diagram showing the 12 notes of the chromatic scale. The notes are labeled: do, do#, re, re#, mi, fa, fa#, sol, sol#, la, la#, si, do. The circle is divided into 12 equal segments, each representing an interval of a fifth. A red arrow labeled $I_7(x) = 7 - x$ points from the top note "do" to the note at position 7, "sol". A blue arrow labeled I_7 points from the note at position 7, "sol", back to the top note "do".

Musical Staff: A musical staff with two staves is shown. The notes are labeled: do, do#, re, re#, mi, fa, fa#, sol, sol#, la, la#, si, do. Arrows point from the notes on the staff down to the corresponding positions on the circle of fifths.

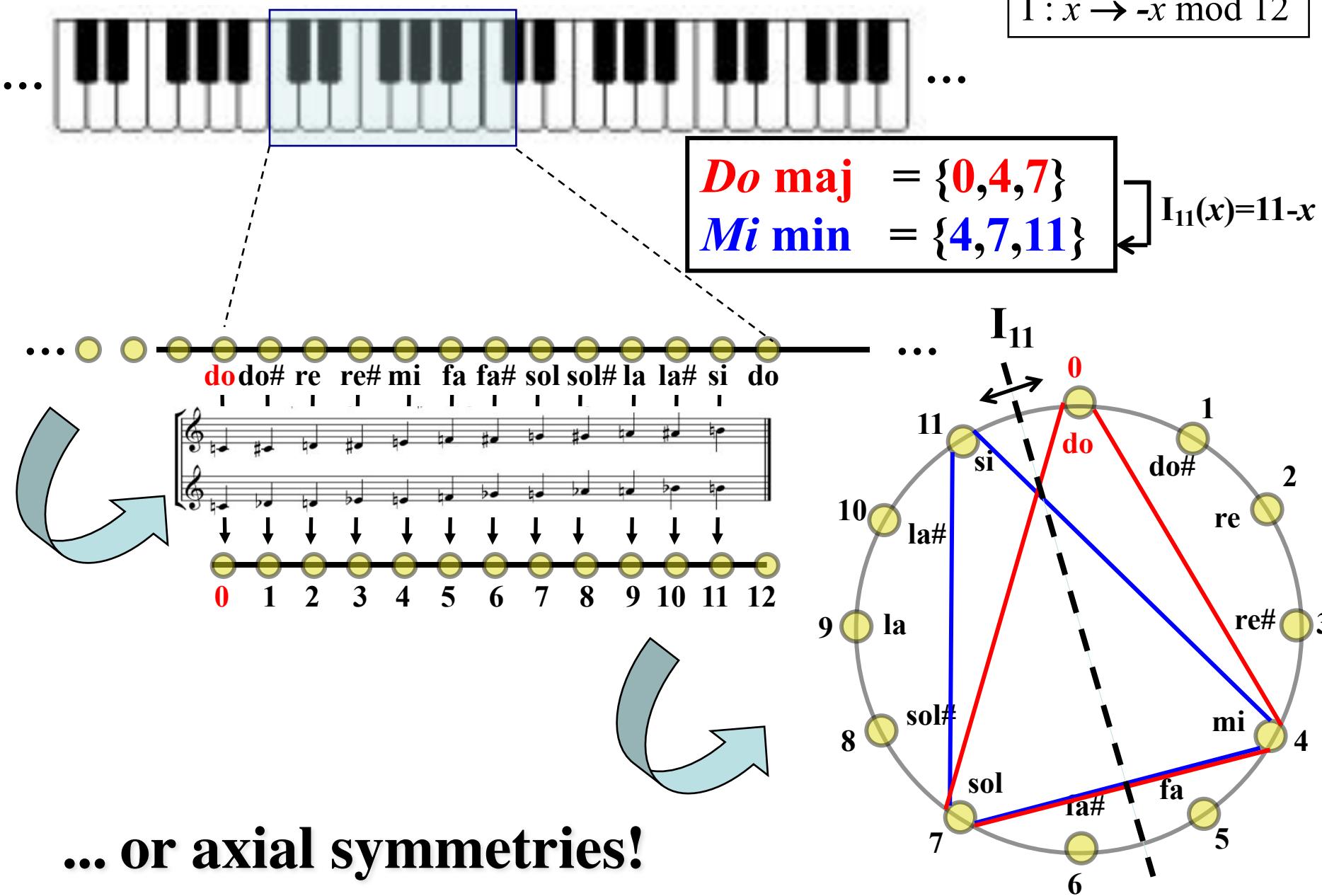
Equation: $I : x \rightarrow -x \bmod 12$

Set Definitions:

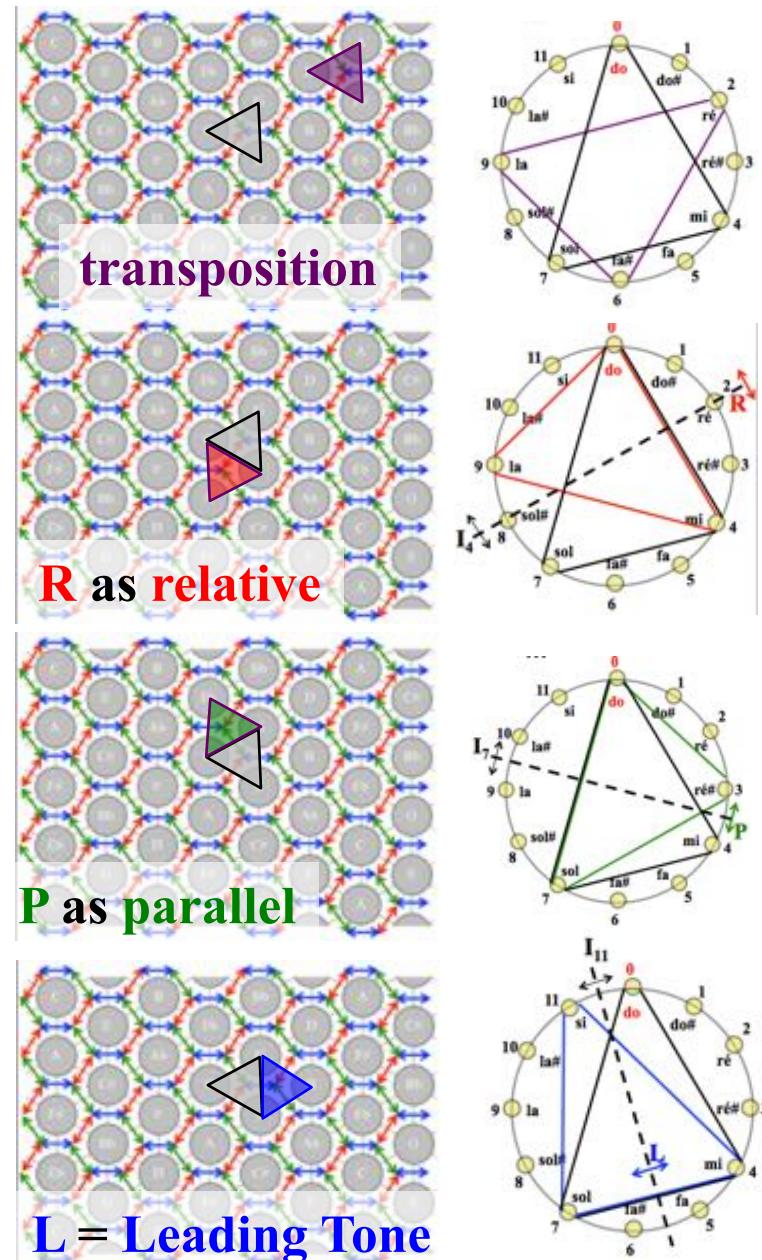
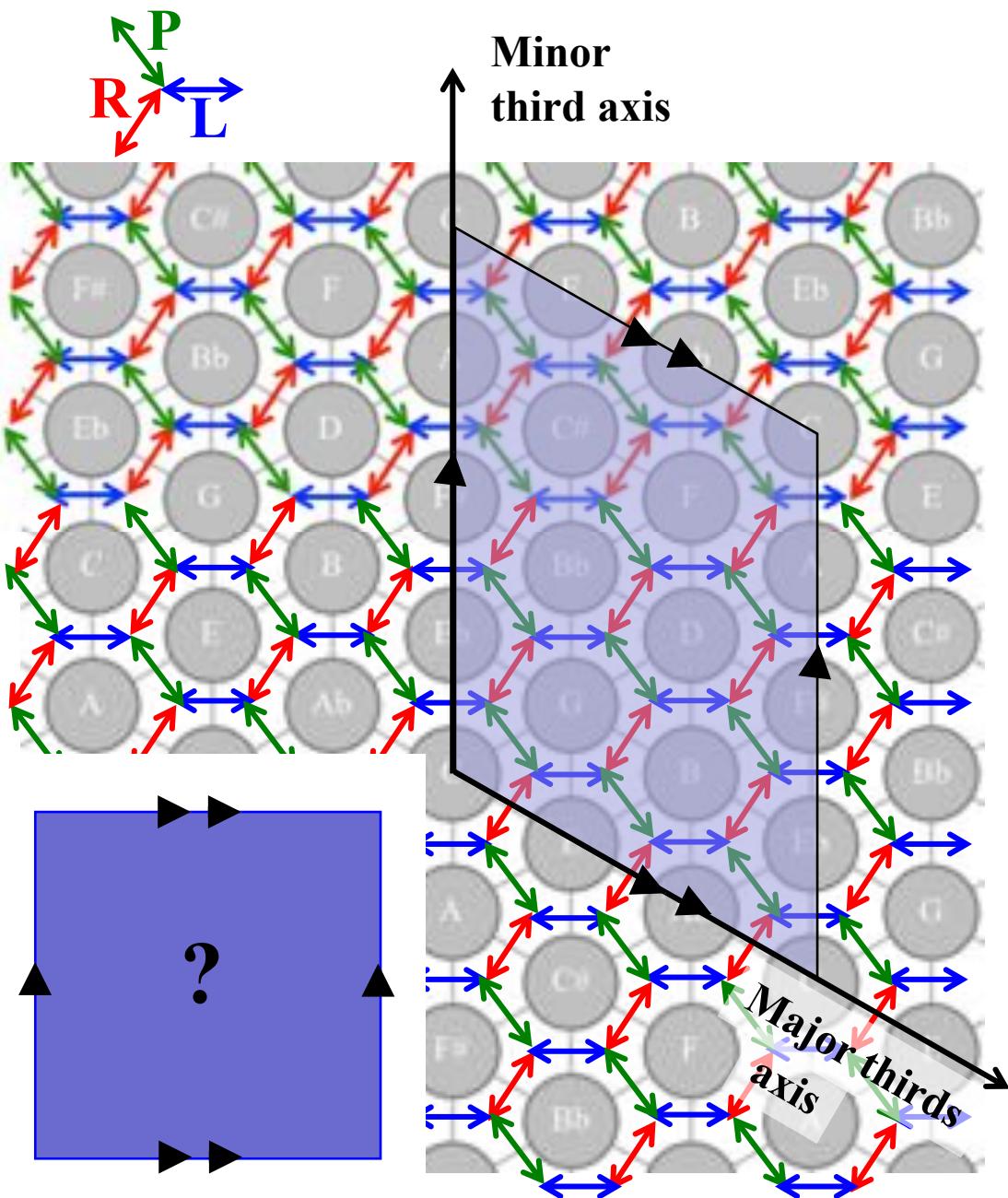
- Do maj** = {0, 4, 7}
- Do min** = {0, 3, 7}

$I_7(x) = 7 - x$

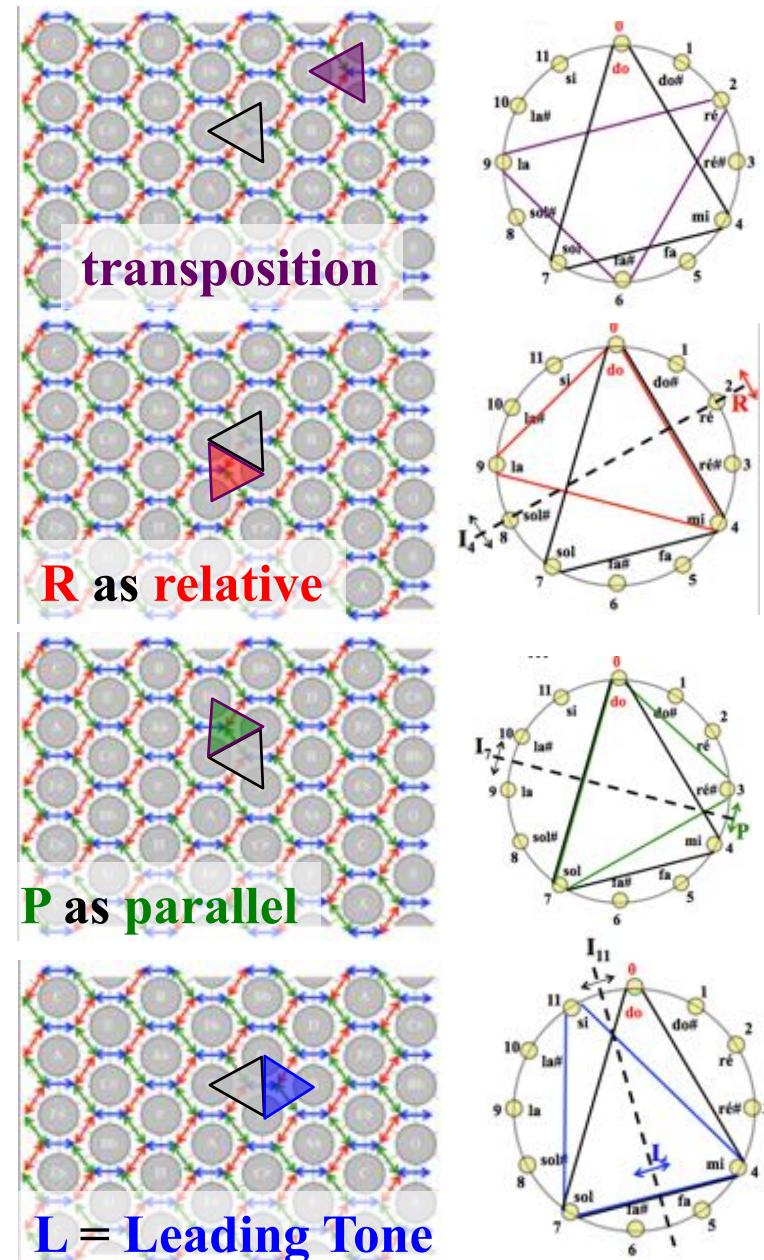
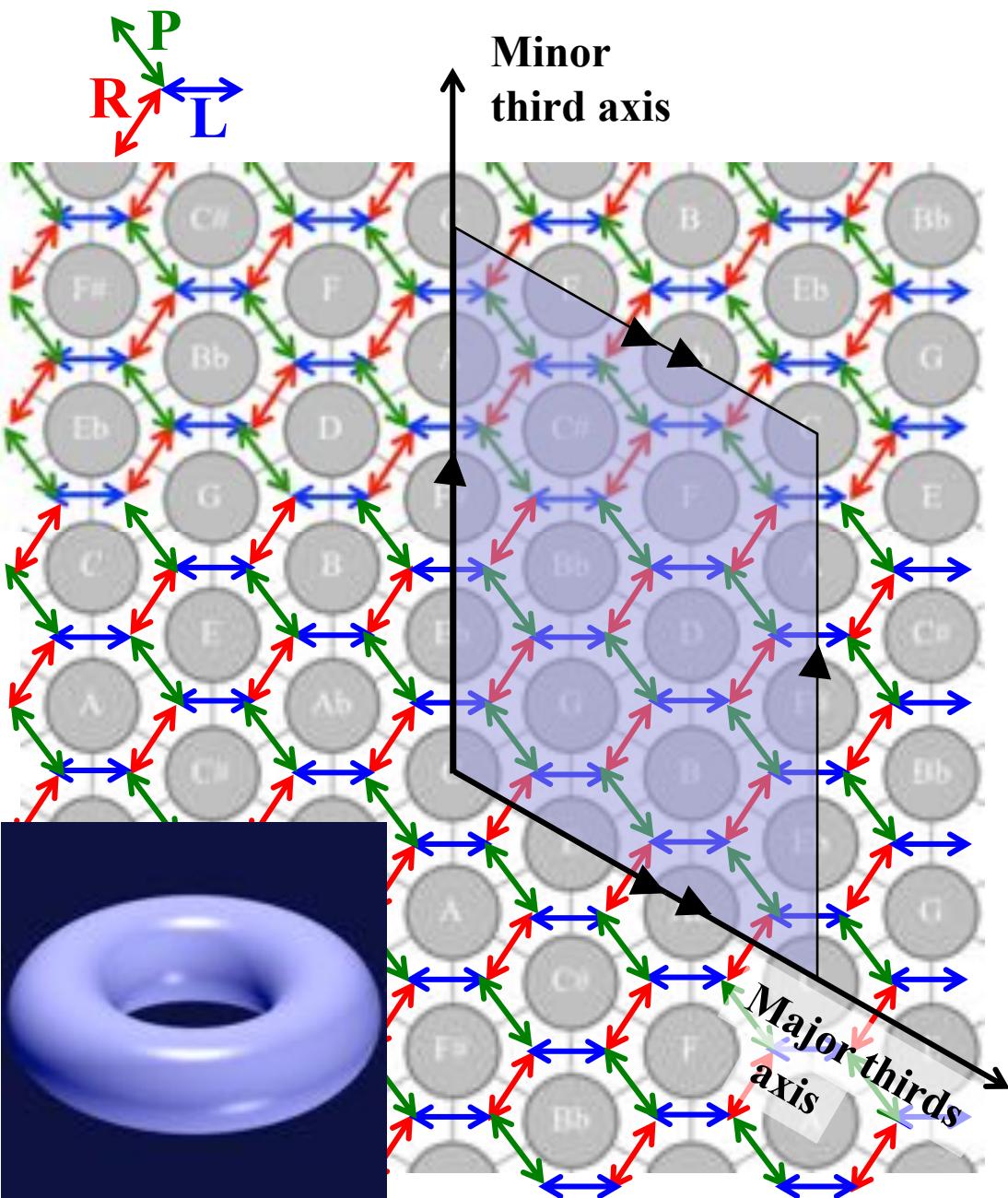
Musical inversions are differences...



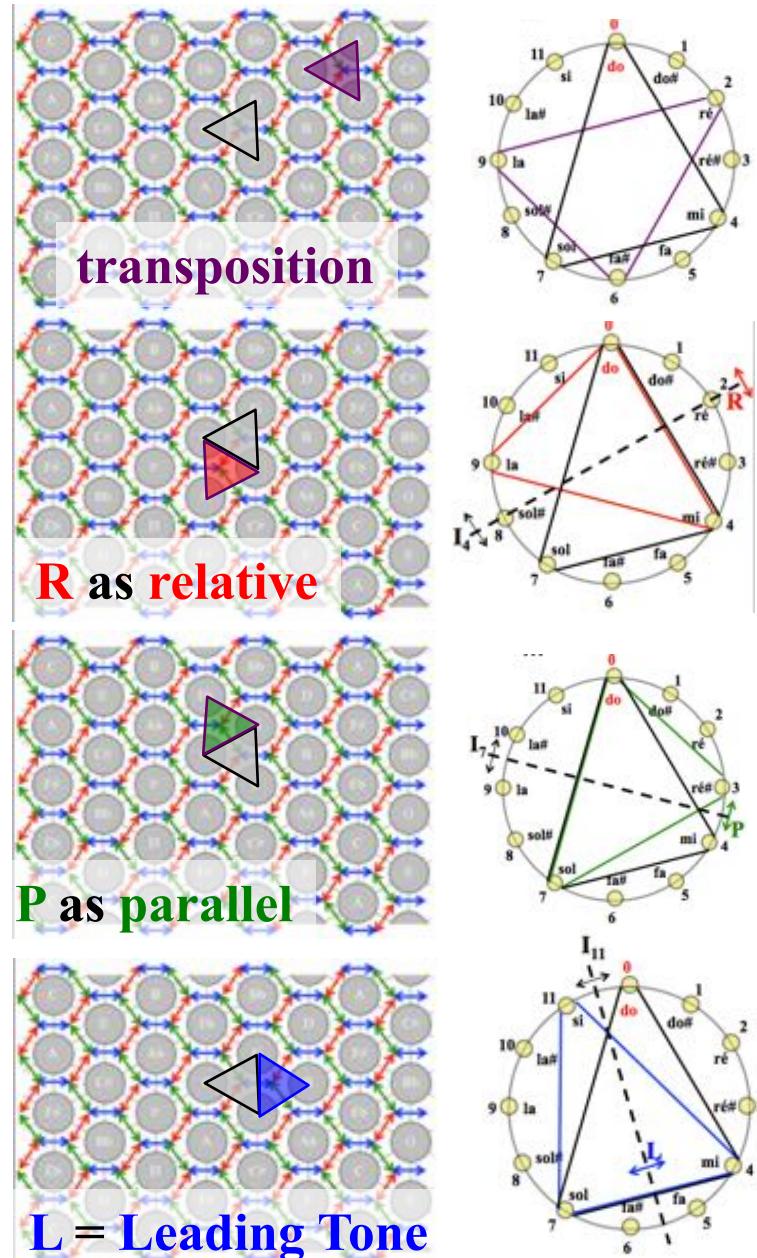
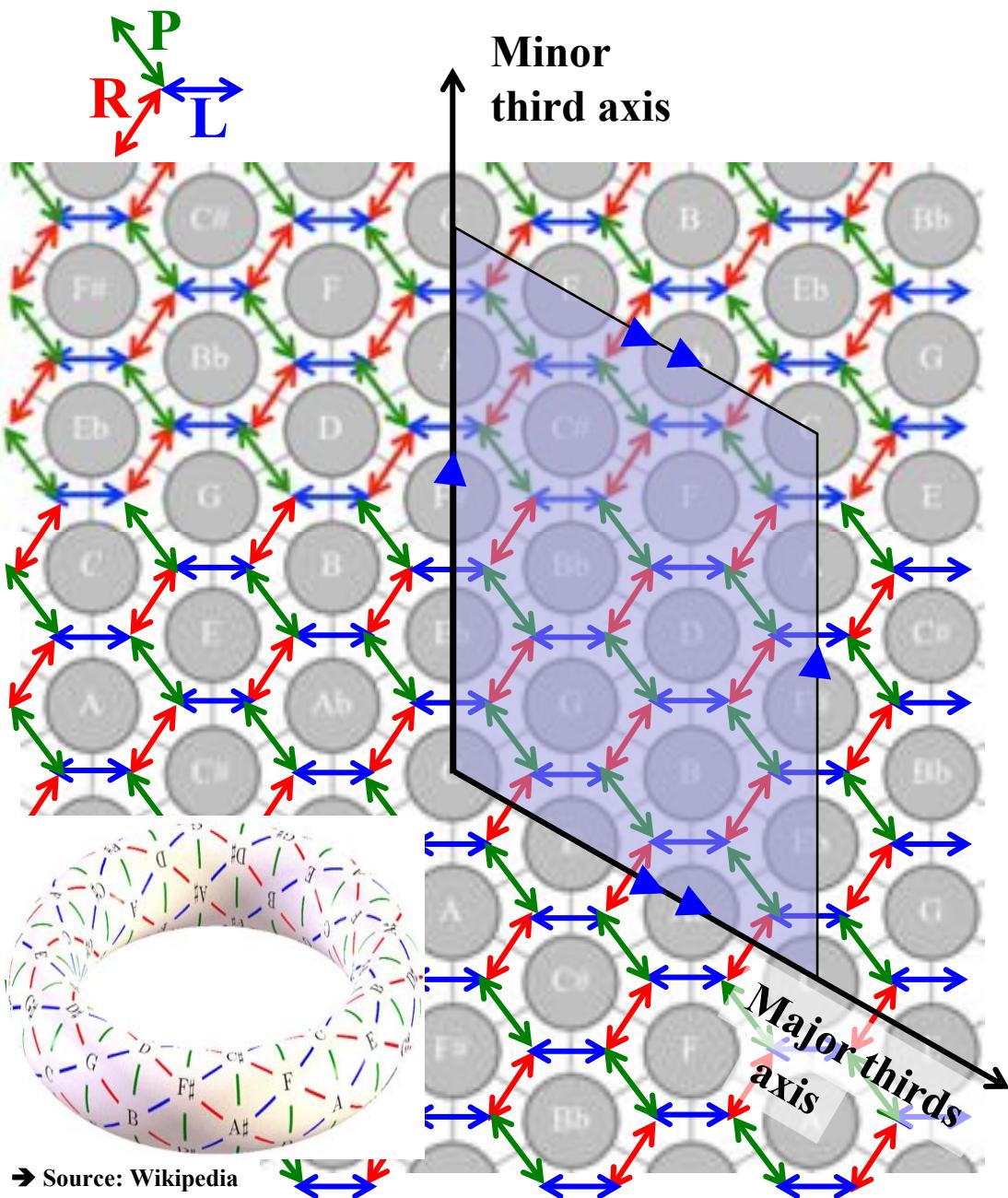
The Tonnetz, its symmetries and its topological structure



The Tonnetz, its symmetries and its topological structure



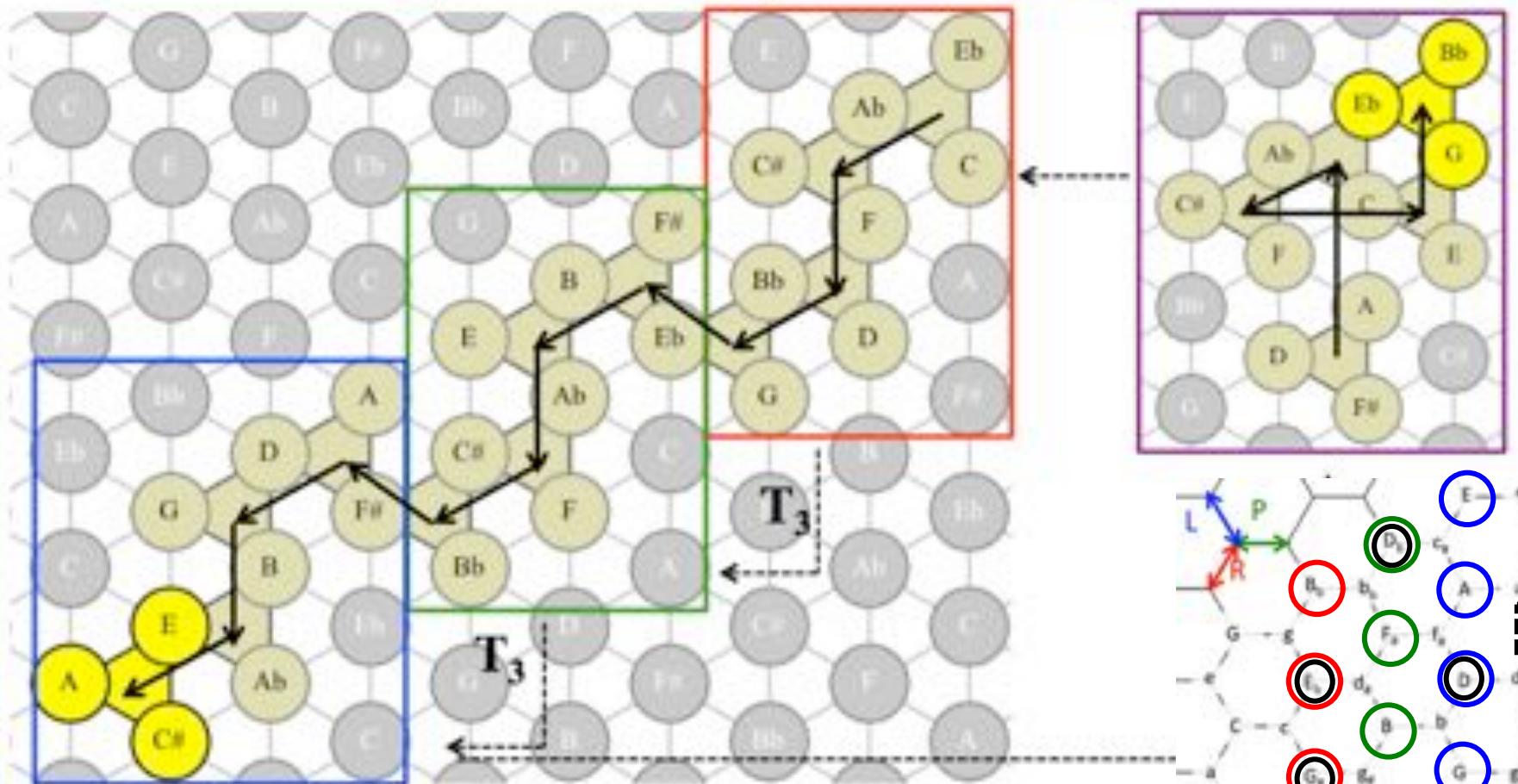
The *Tonnetz*, its symmetries and its topological structure



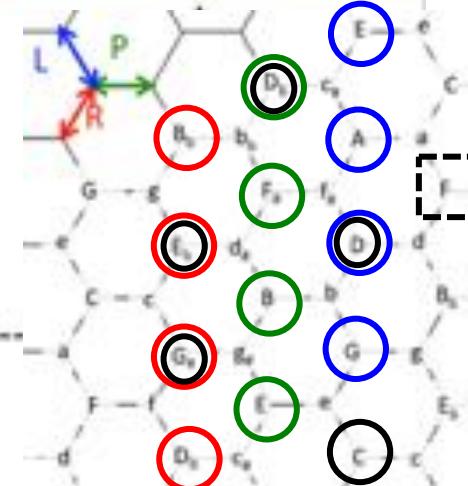


Symmetries in Paolo Conte's *Madeleine*

La_b Re_b Si_b Mi_b Si Mi Re_b Fa_# Re Sol Mi La Re La_b Re_b Do Mi_b



Almost total covering of the major-chords space



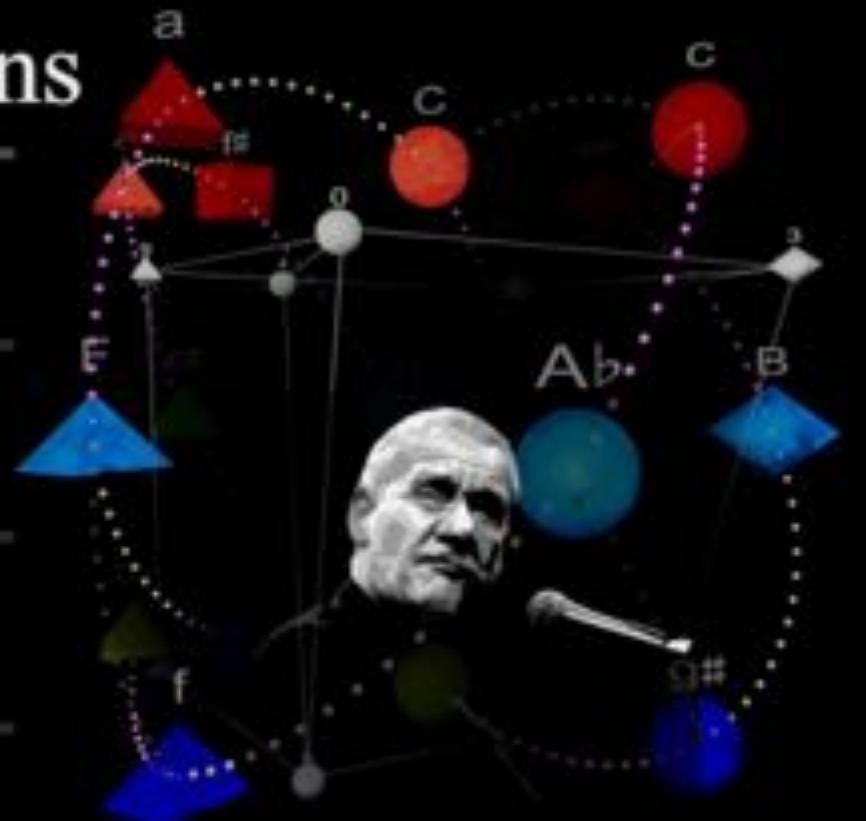
Harmonic Progressions

In Paolo Conte

Madeleine



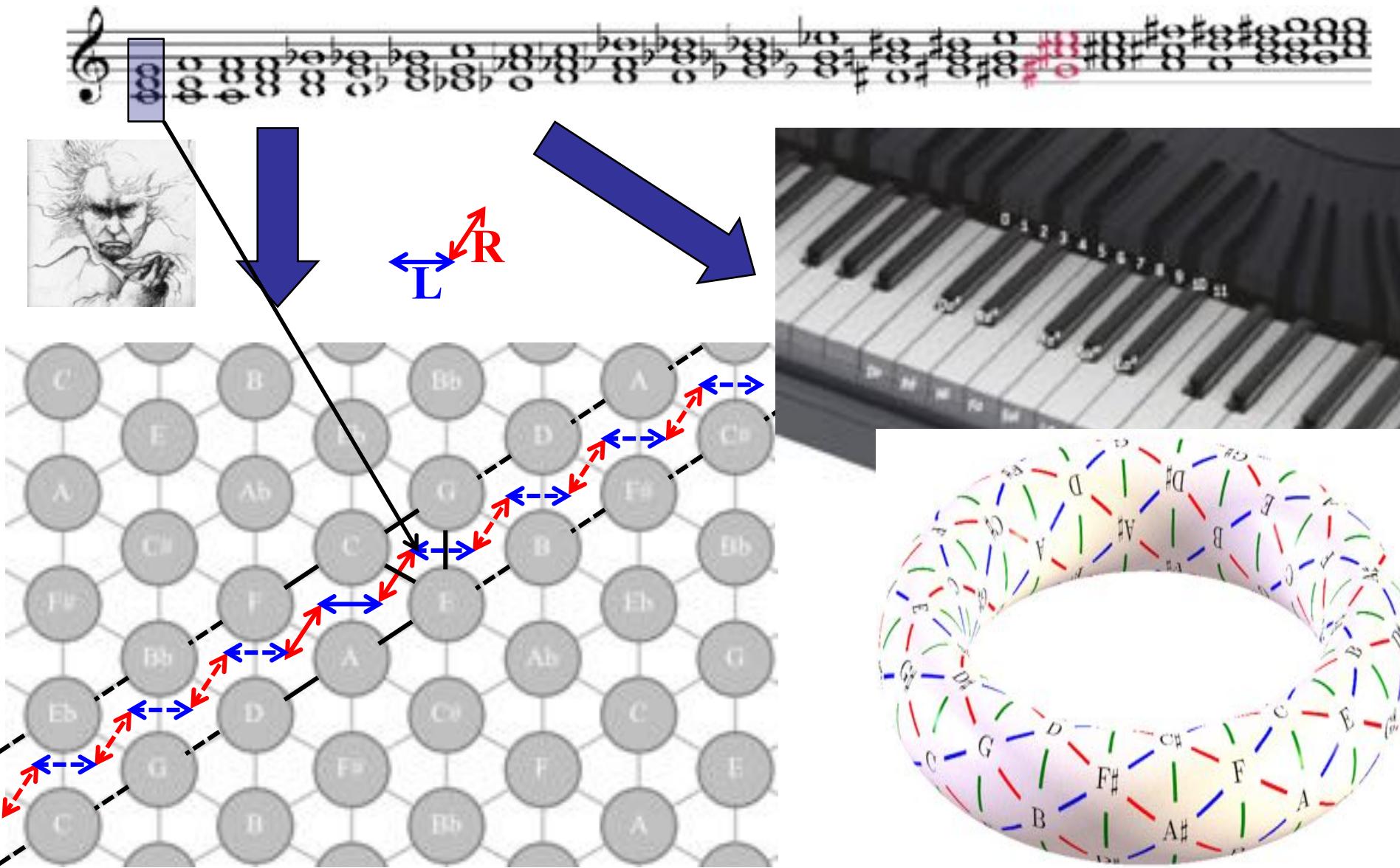
Gilles Baroin



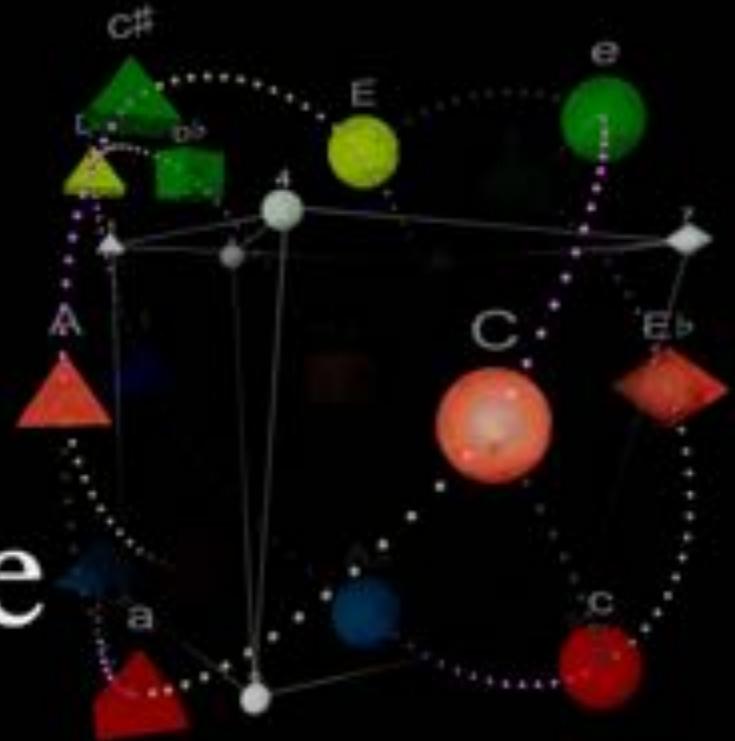
*Supervision Moreno Andreatta
Modélisation Gilles Baroin 2016*

→ <http://planetes.info/>

A zig-zag harmonic progression

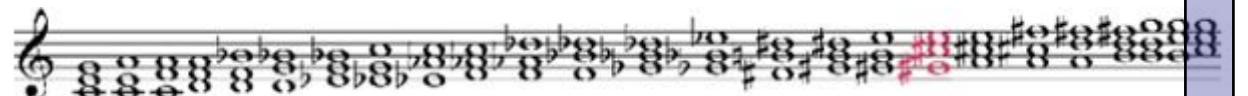


Beethoven and the Hypersphere *(and the Tonnetz)*



Gilles Baroin 2016
www.MatheMusic.net

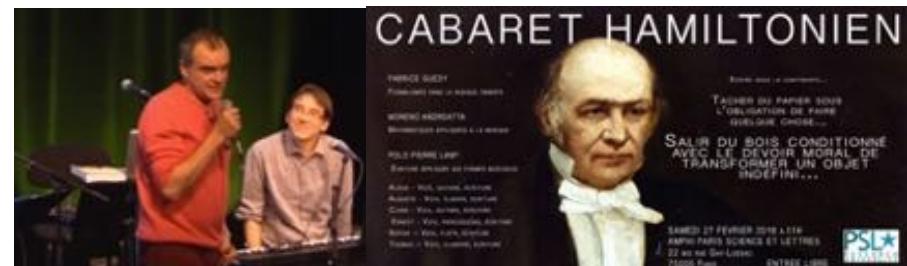
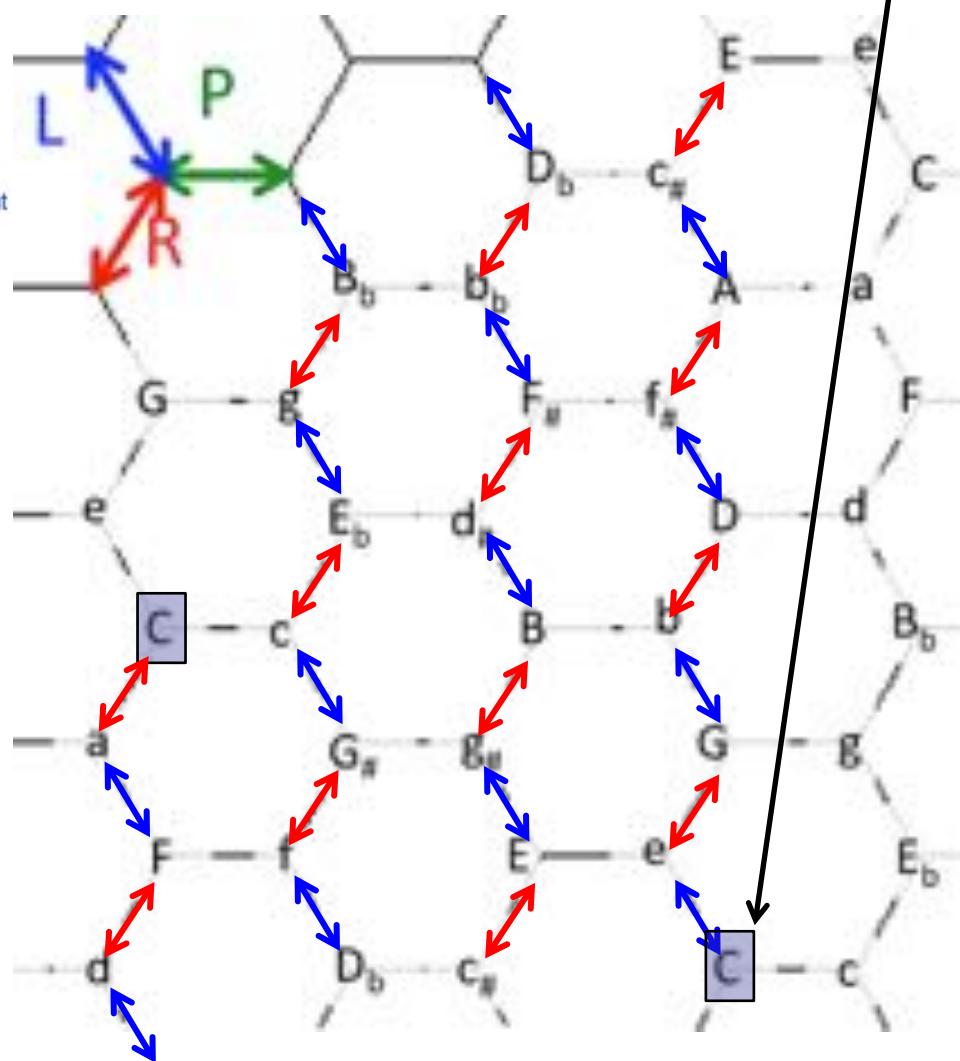
Reading Beethoven backwards



Le Blé en Herbe

(Palo/Moreno/Dieu)

- | | |
|--|--|
| Plonger comme un enfant, cheveux au vent | Croiser matin dans l'herbe folle |
| Sous l'océan du blé en herbe | Deux tourterelles qui s'envolent |
| Marée d'épis couleur d'amande | Suivre les jeux des hirondelles |
| Qui tendent à caresser le ciel | Sur le paysage éternel |
| | Nager comme un enfant, cheveux au vent |
| Algues tendres de mille plages | Sous l'océan |
| Frôlant le ventre des nuages | Du blé en herbe |
| Cheveux de pluie, dos de poissons | |
| Qui frissonnent à l'unisson | Marée de fruits au goût amer |
| | Acide et salée comme la mer |
| Suivre le bord des continents | |
| Dans l'océan du blé en herbe | Vers l'ilôt d'un petit village |
| Pêcher le corail du pavot | Vers un château d'eau sur la plage |
| Dans le sang des coquelicots | Quand tout s'éteint avant l'orage |
| | Quand se lève le vent du large |
| | Sur le blé vert |

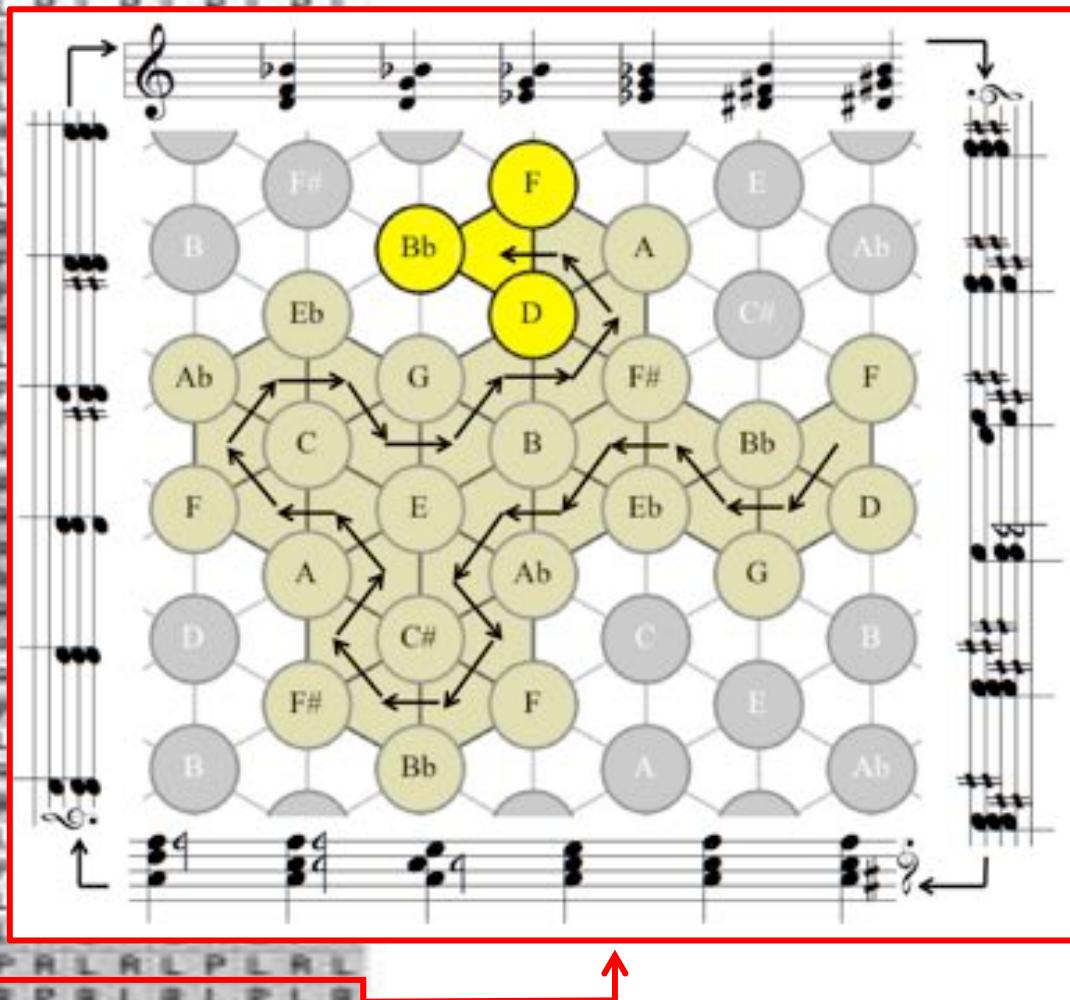
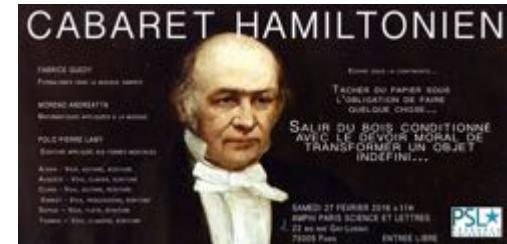


The collection of 124 Hamiltonian Cycles

ACTIONS

Math'n'pop

Aprile (d'après Gabriele D'Annunzio)



G. Albini & S. Antonini, University of Pavia, 2008

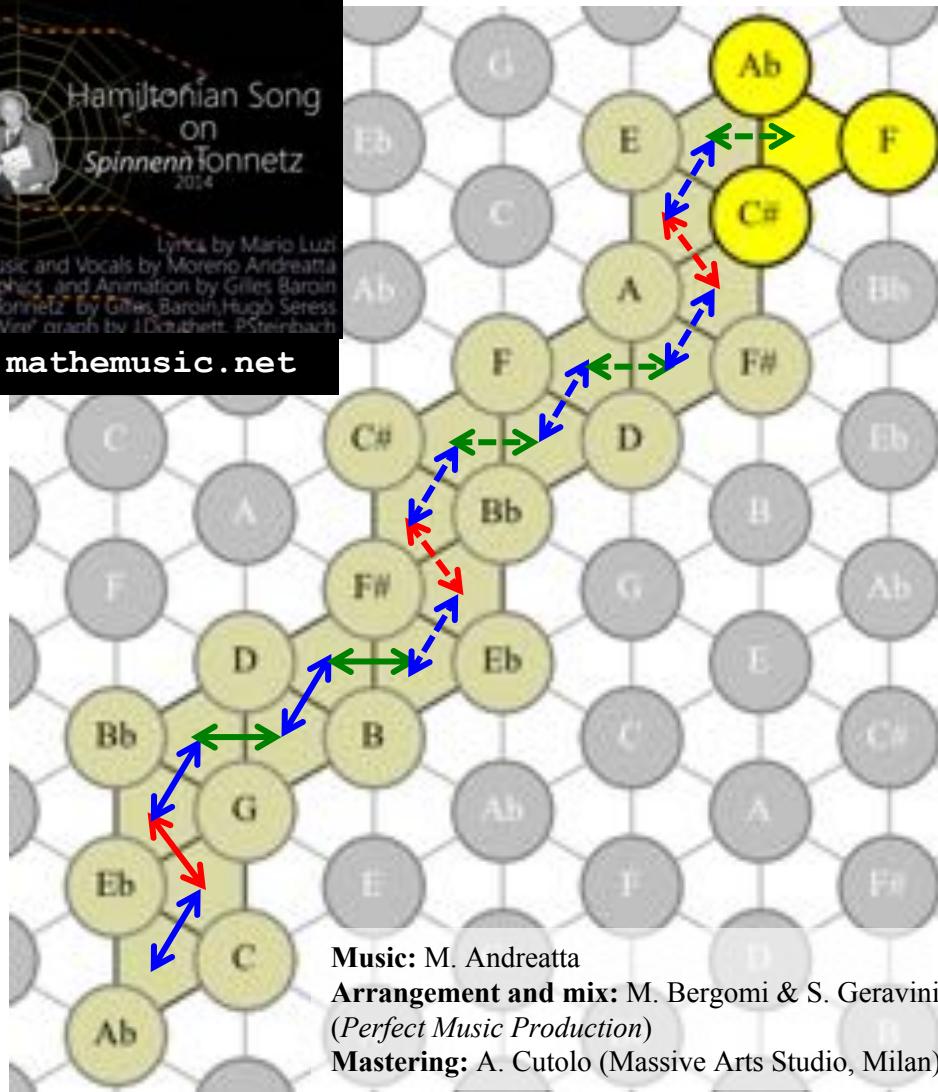
Hamiltonian Cycles with inner periodicities

L P L P L R ...
→ P L P L R L ...
L P L R L P ...
P L R L P L ...
L R L P L P ...
R L P L P L ...



<http://www.mathemusic.net>

min. 1'02"



Music: M. Andreatta
Arrangement and mix: M. Bergomi & S. Geravini
(*Perfect Music Production*)
Mastering: A. Cutolo (Massive Arts Studio, Milan)

La sera non è più la tua canzone (Mario Luzi, 1945, in *Poesie sparse*)

**La sera non è più la tua canzone,
è questa roccia d'ombra traforata
dai lumi e dalle voci senza fine,
la quiete d'una cosa già pensata.**

**Ah questa luce viva e chiara viene
solo da te, sei tu così vicina
al vero d'una cosa conosciuta,
per nome hai una parola ch'è passata
nell'intimo del cuore e s'è perduta.**

Caduto è più che un segno della vita,
riposi, dal viaggio sei tornata
dentro di te, sei scesa in questa pura
sostanza così tua, così romita
nel silenzio dell'essere, (compiuta).

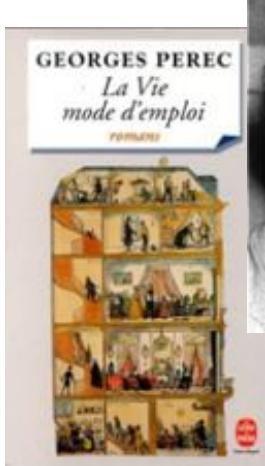
**L'aria tace ed il tempo dietro a te
si leva come un'arida montagna
dove vaga il tuo spirto e si perde,
un vento raro scivola e ristagna.**



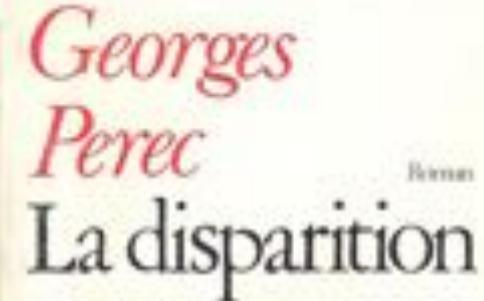
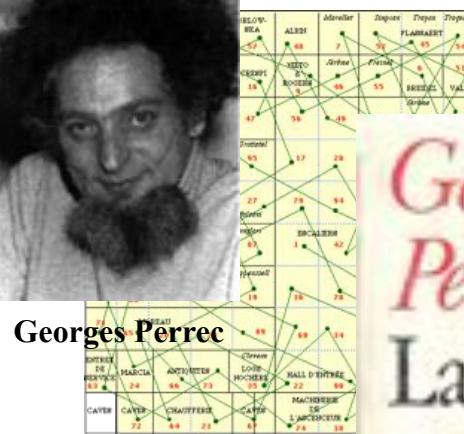
The use of constraints in arts



Cent mille milliards de poèmes, 1961



La vie mode d'emploi,



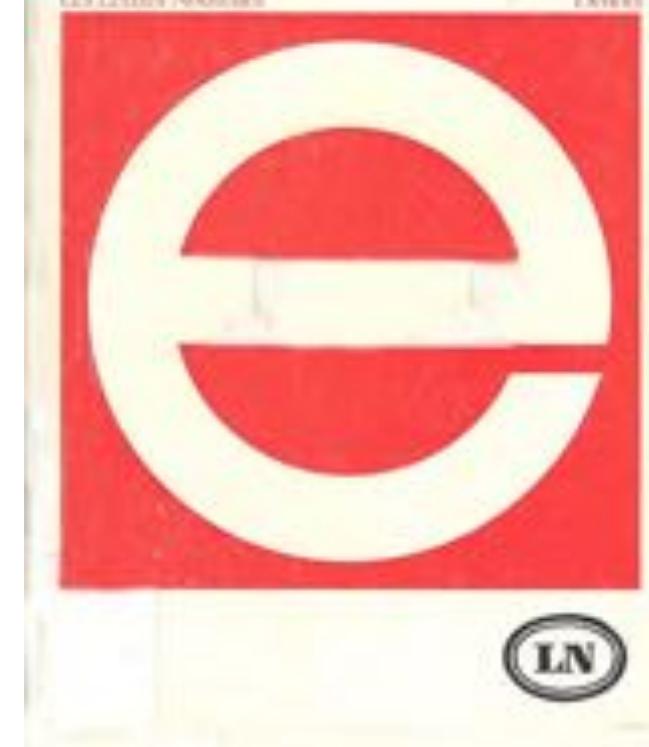
Denoel



Raymond Queneau

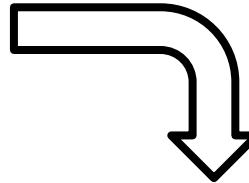


Italo Calvino
Il castello dei destini incrociati, 1969



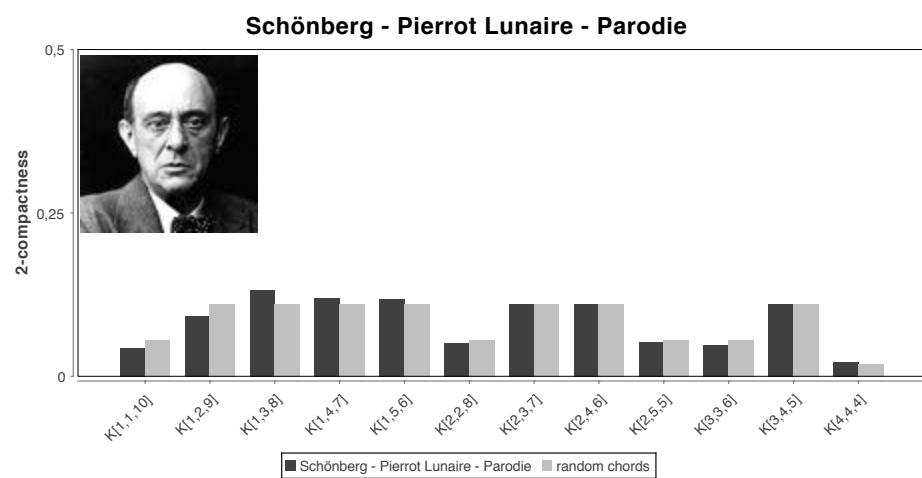
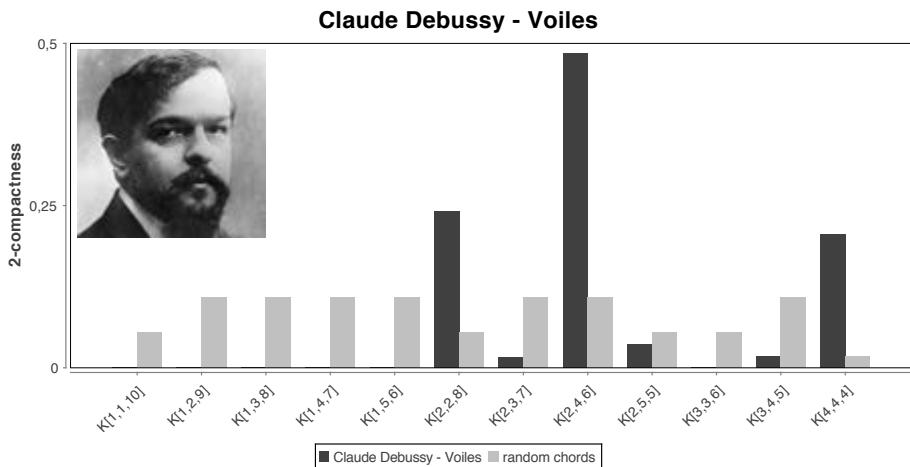
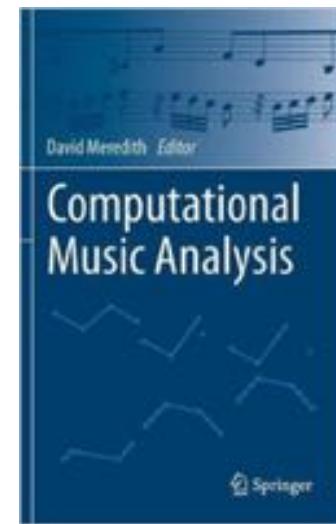
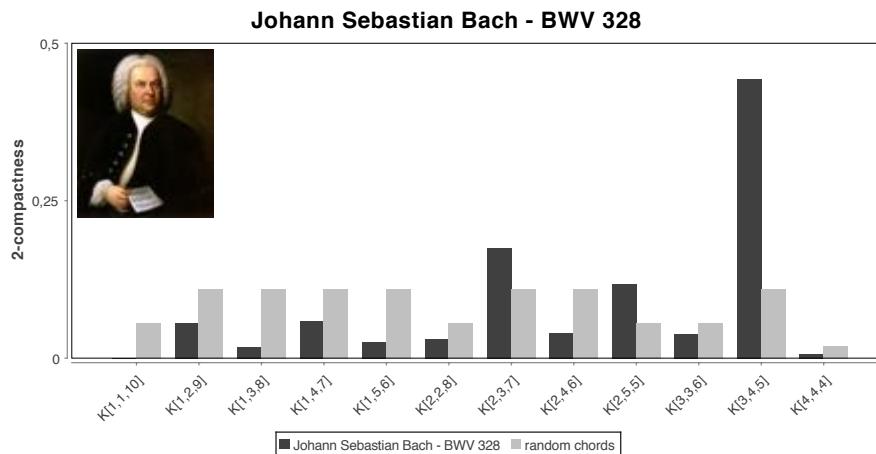
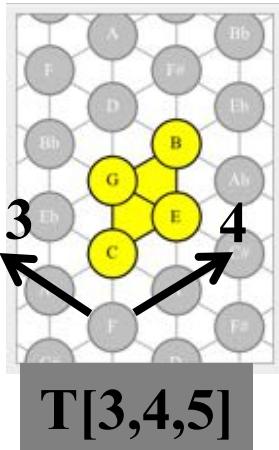
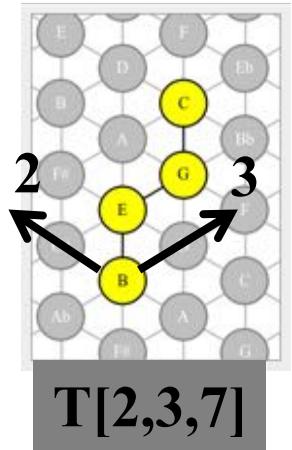
OuLiPo (Ouvroir de Littérature Potentielle)

From the OuLiPo to the OuMuPo (Ouvroir de Musique Potentielle)



Valentin Villenave	Mike Solomon	Jean-François Piete	Martin Granger	Joseph Boisseau	Moreno Andreatta	Tom Johnson	

The geometric character of musical logic

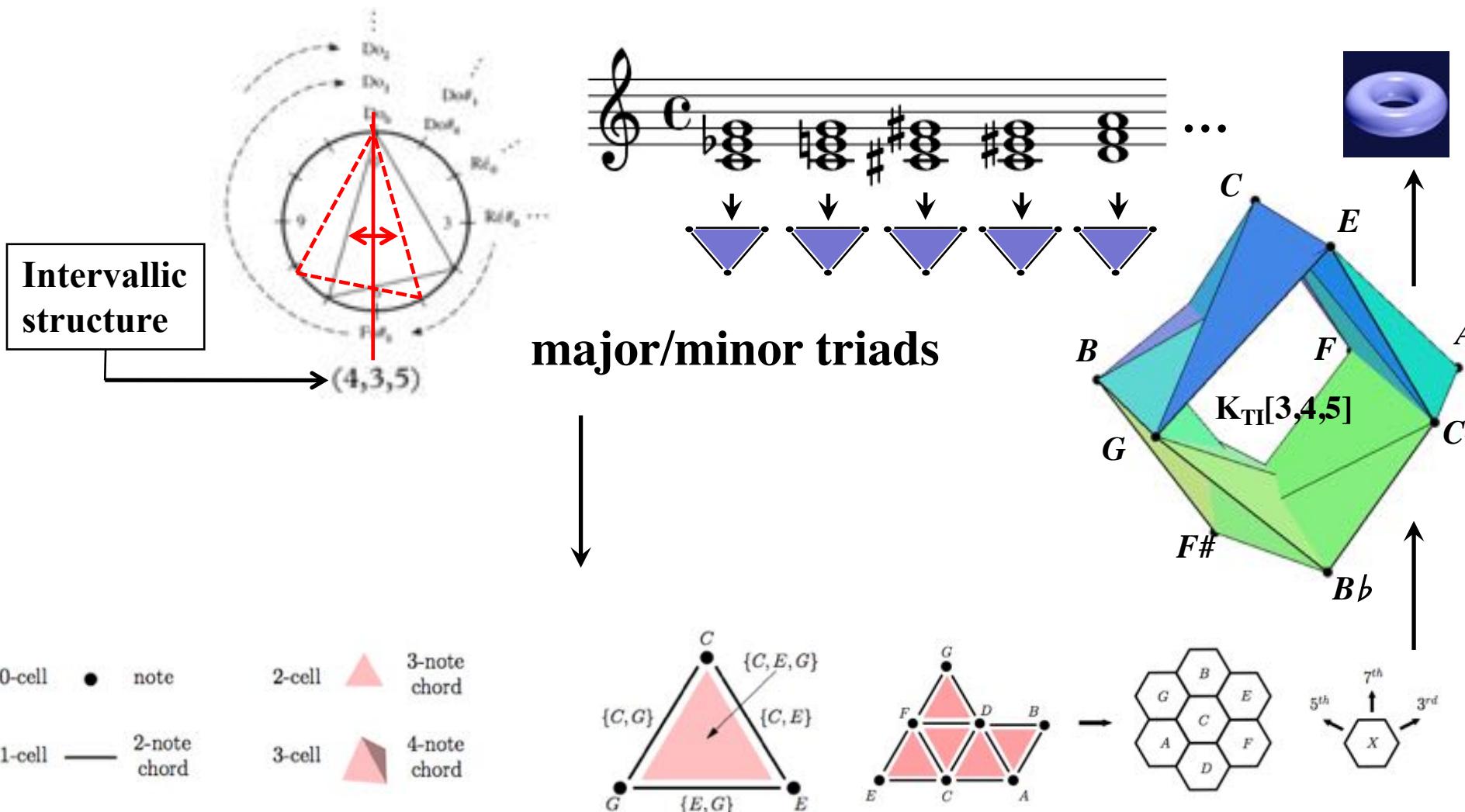


The Tonnetz as a simplicial complex

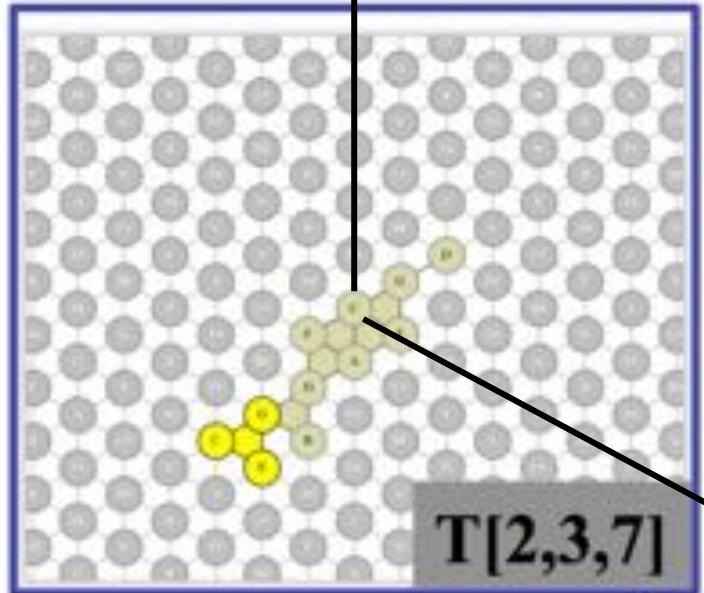
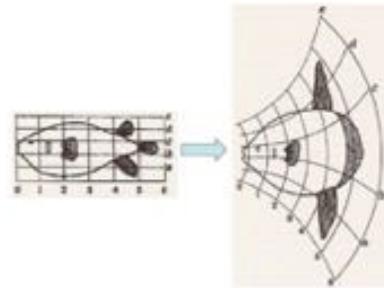
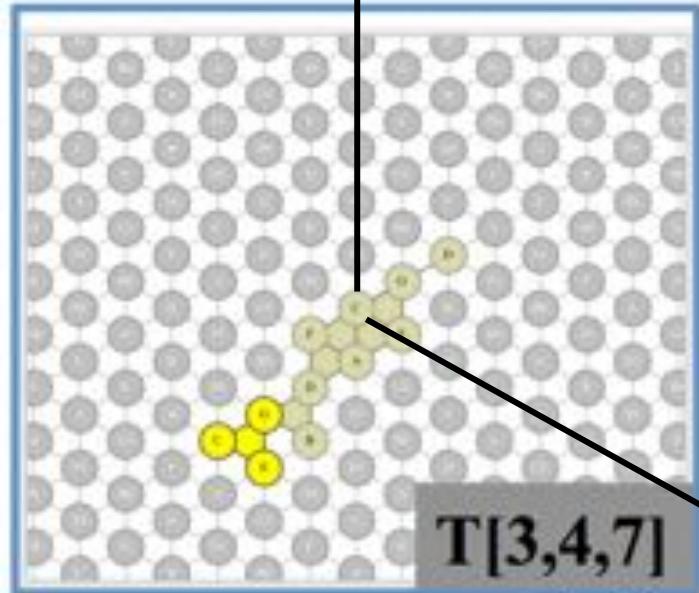
L. Bigo, *Représentation symboliques musicales et calcul spatial*, PhD, Ircam / LACL, 2013



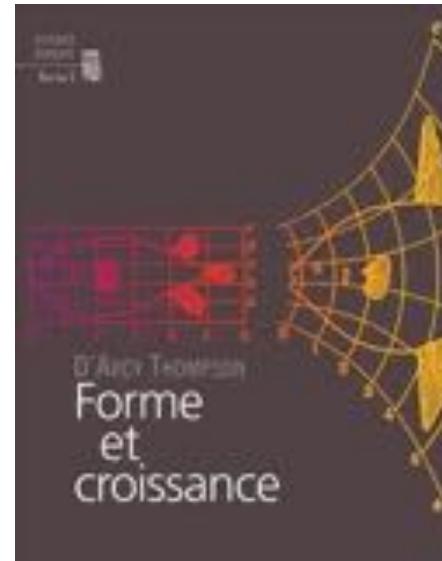
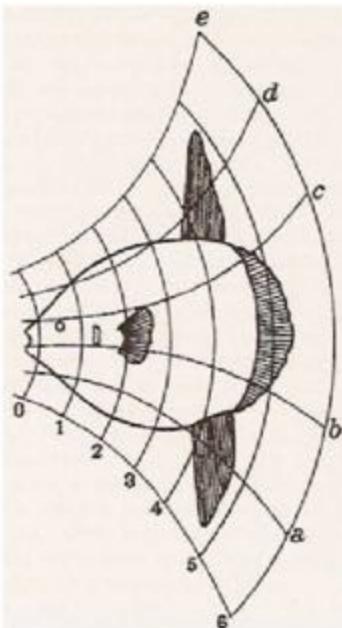
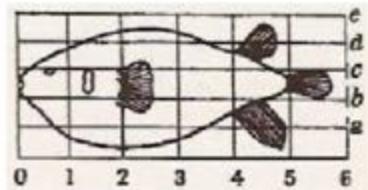
- Assembling chords related by some equivalence relation
 - Transposition/inversion: Dihedral group action on $P(\mathbb{Z}_n)$



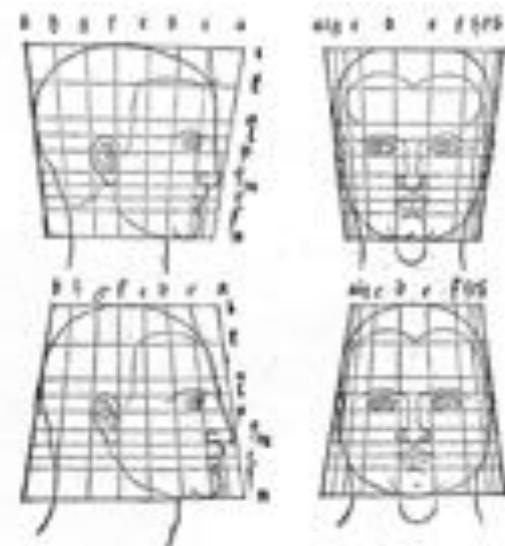
The musical style...is the space!



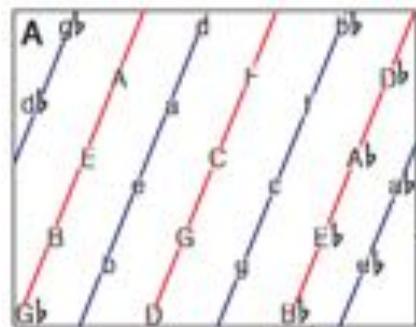
The morphological vs the mathematical genealogy of the structuralism



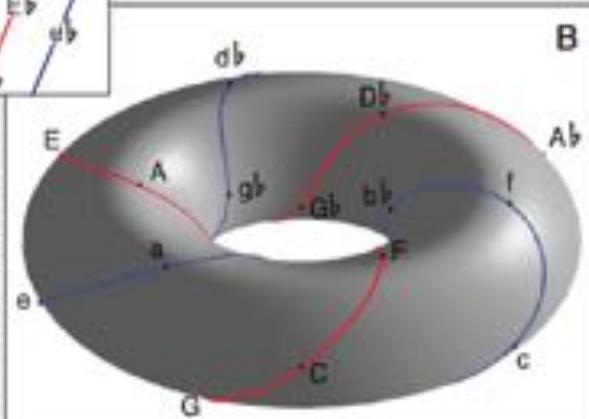
“[The notion of **transformation**] comes from a work which played for me a very important role and which I have read during the war in the United States : *On Growth and Form*, in two volumes, by **D'Arcy Wentworth Thompson**, originally published in 1917. The author (...) proposes an interpretation of the visible transformations between the species (animals and vegetables) within a same gender. This was fascinating, in particular because I was quickly realizing that this perspective had a long tradition: behind Thompson, there was **Goethe's** botany and behind Goethe, **Albert Dürer** with his *Treatise of human proportions*” (Lévi-Strauss, conversation with Eribon, 1988).



Neurosciences and Tonnetz



Mental key maps. (A) Unfolded version of the key map, with opposite edges to be considered matched. There is one circle of fifths for major keys (red) and one for minor keys (blue), each

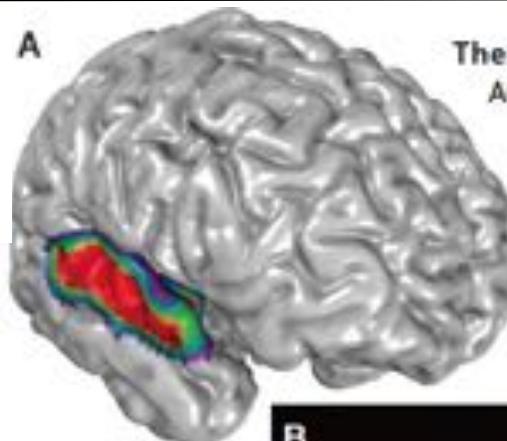


wrapping the torus three times. In this way, every major key is flanked by its relative minor on one side (for example, C major and a minor) and its parallel minor on the other (for example, C major and c minor). (B) Musical keys as points on the surface of a torus.

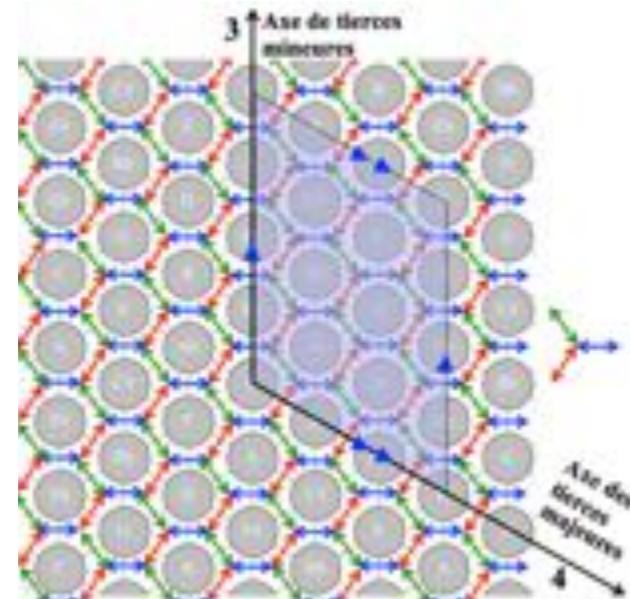
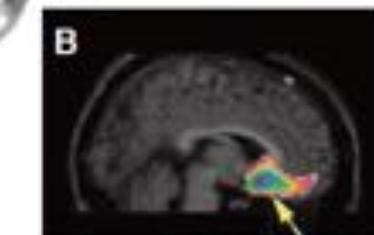
PERSPECTIVES: NEUROSCIENCE

Mental Models and Musical Minds

Robert J. Zatorre and Carol L. Krumhansl



The sensation of music. (A) Auditory cortical areas in the superior temporal gyrus that respond to musical stimuli. Regions that are most strongly activated are shown in red. (B) Metabolic activity in the ventromedial region of the frontal lobe increases as a tonal stimulus becomes more consonant.



Acotto E. et M. Andreatta (2012), « Between Mind and Mathematics. Different Kinds of Computational Representations of Music », *Mathematics and Social Sciences*, n° 199, 2012(3), p. 9-26.

Spatial music analysis via *Hexachord*

The image shows a composite screenshot of the Hexachord software interface and a copy of the Computer Music Journal.

Top Left: A 3D geometric visualization of a polyhedron-like structure composed of green and blue faces.

Top Center: A hexagonal grid visualization titled "Tessellation A[3,3,4,2]". It features a central yellow cluster of hexagons labeled with letters (B, E, C, F, D, G, H, I, J, K, L) surrounded by a larger gray hexagonal grid.

Top Right: A control panel for a MIDI file named "bwv0281.mid". It includes a tempo slider (set to 10), play/stop buttons, and dropdown menus for "Chromatic complexes" (K[2,3,7]) and "Heptatonic complexes" (CM). Other buttons include "Trace off", "Harmonization DN", and "Display graph".

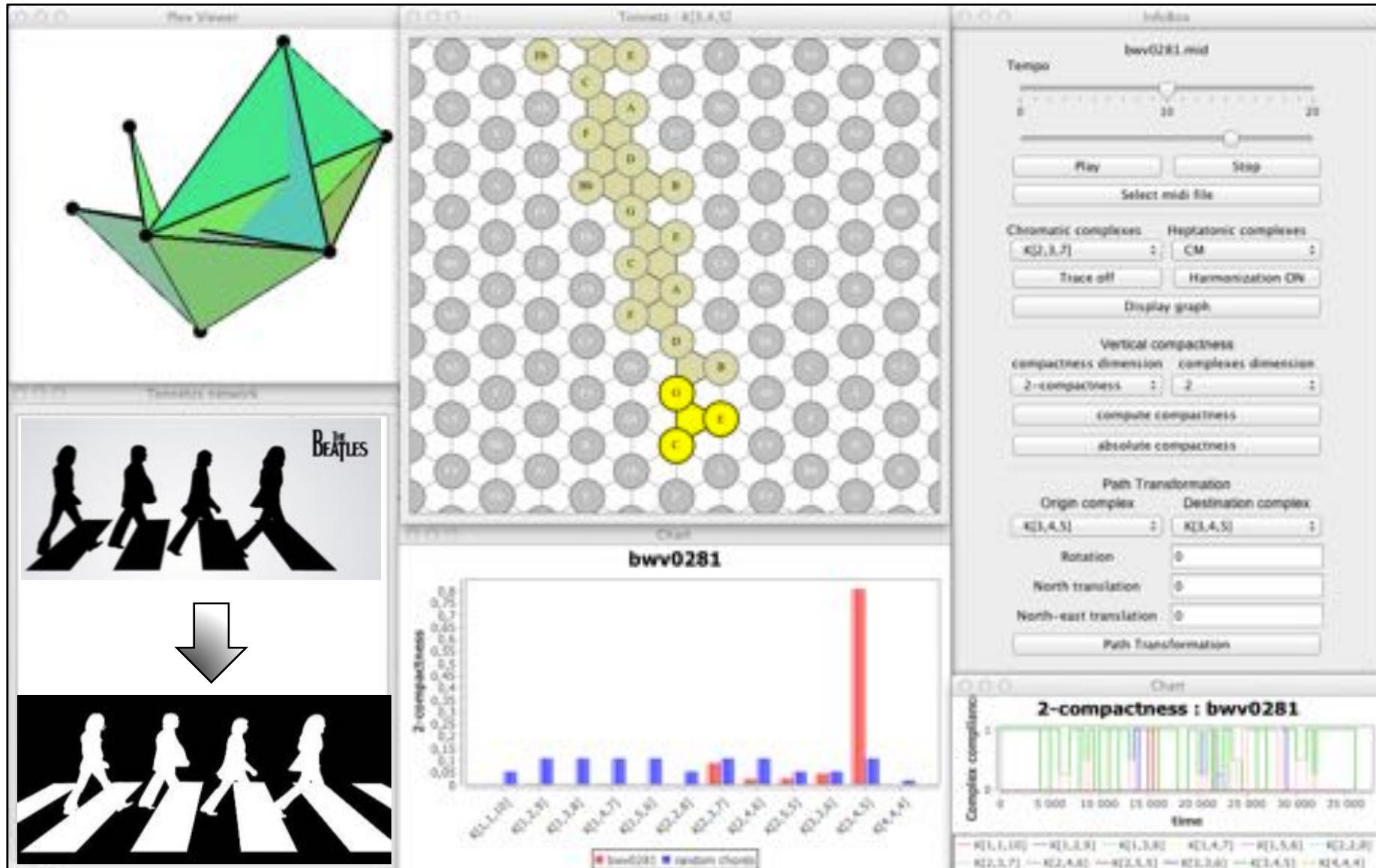
Bottom Left: A screenshot of the "Computer Music Journal" journal cover. The cover features a green background with a white header and footer. The main title "Computer Music Journal" is in large white font. Below it, there's a small image of a person playing a keyboard and some text about the journal's focus on digital music and sound processing.

Bottom Center: A bar chart titled "bwv0281" showing "2-compactness". The y-axis ranges from 0 to 0.05. The x-axis lists time points: 0:1.00, 0:1.50, 0:2.00, 0:2.50, 0:3.00, 0:3.50, 0:4.00, 0:4.50, 0:5.00, 0:5.50, 0:6.00, 0:6.50, 0:7.00, 0:7.50, 0:8.00, 0:8.50, 0:9.00, 0:9.50, 0:10.00. The bars are colored red and blue. A legend at the bottom indicates that red bars represent "bwv0281" and blue bars represent "random choice".

Bottom Right: A timeline visualization titled "2-compactness : bwv0281". The y-axis is "Complex compactness" (0 to 1) and the x-axis is "time" (0 to 35,000). The visualization shows a series of vertical bars in various colors (green, blue, red, purple) representing different complex compactness values over time.

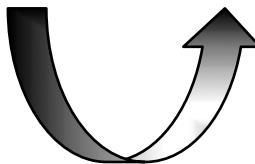
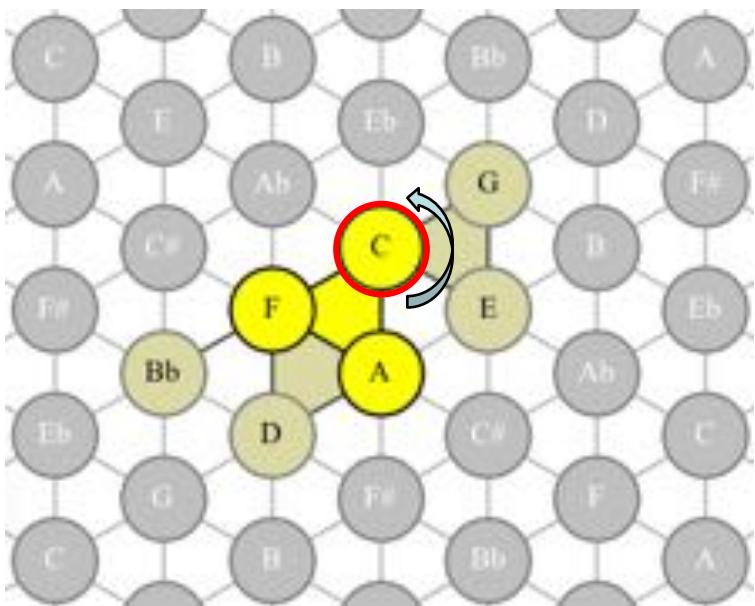
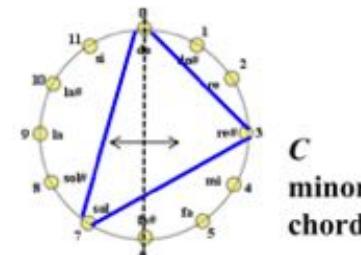
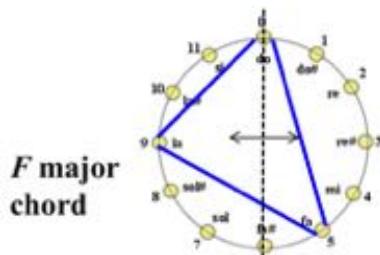
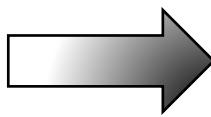
→ <http://www.lacl.fr/~lbigo/hexachord>

Keeping the space...but changing the trajectory!

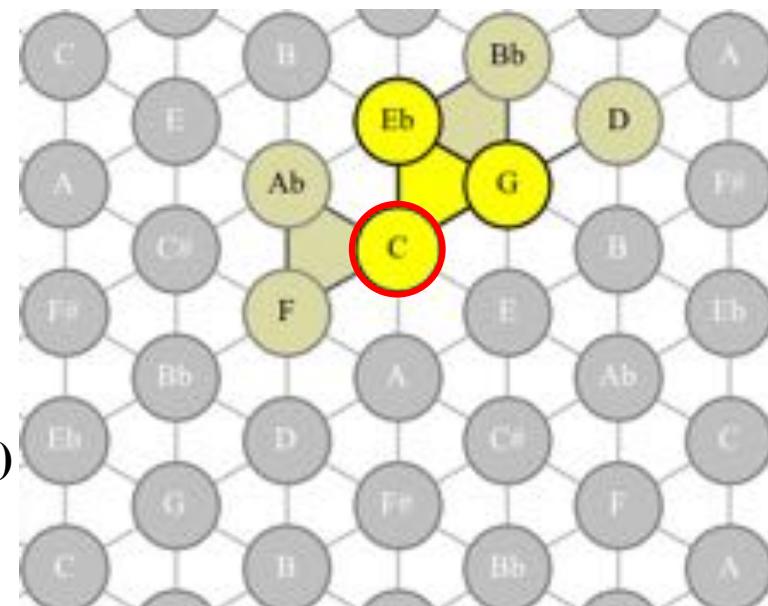


→ <http://www.lacl.fr/~lbigo/hexachord>

Keeping the space...but changing the trajectory!



Rotation
(autour du *do*)



Rotational symmetry applied to traditional Brazilian music

Sonoroso

K. Kostello

Brejeiro

Ernesto Nazareth

Musica

Doce de Côco

José do Canto

Violino

Violoncello

Piano

Double Bass

Flute

Saxophone

Tenor Saxophone

Trumpet

French Horn

Trombone

Tuba

Drums

Maracas

Aeroporto do Galeão

Almeida Carrilho

Apanhei-te Cavaquinho

Ernesto Nazareth
Baldoman



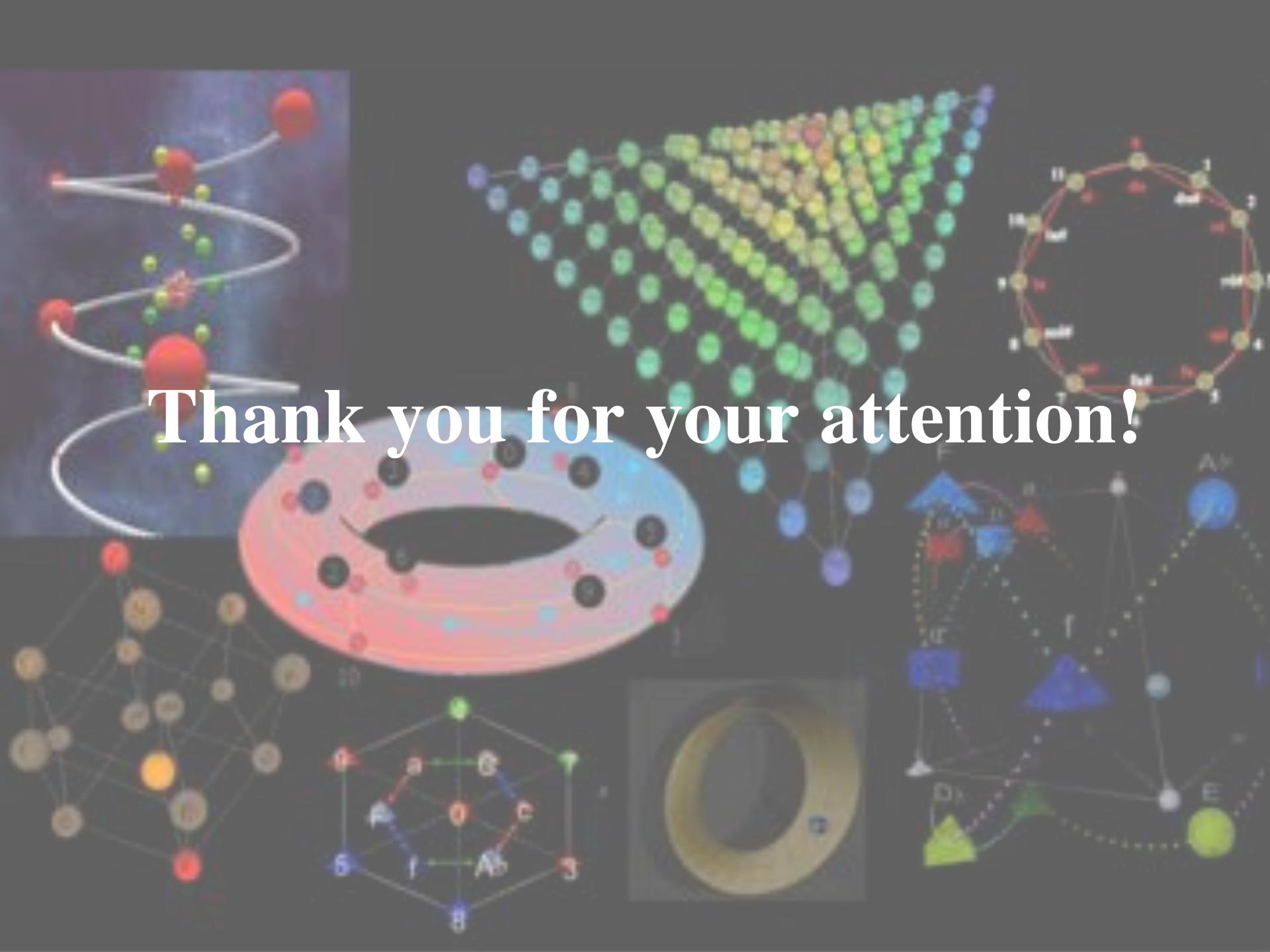
Escovado
Rengo Brasileiro

Ernesto Nazareth
1903

Piano

Violin

Cello



Thank you for your attention!