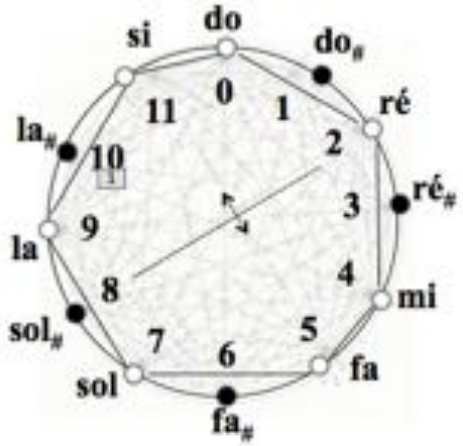
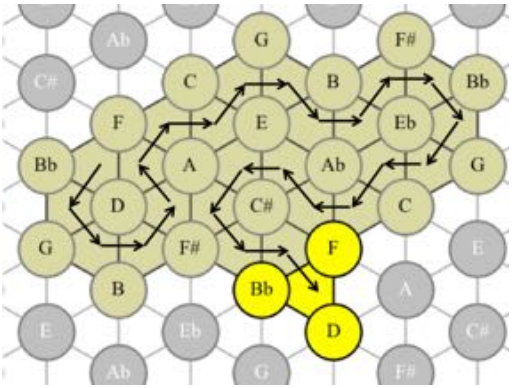
Algebra and Geometry in Music and Musicology



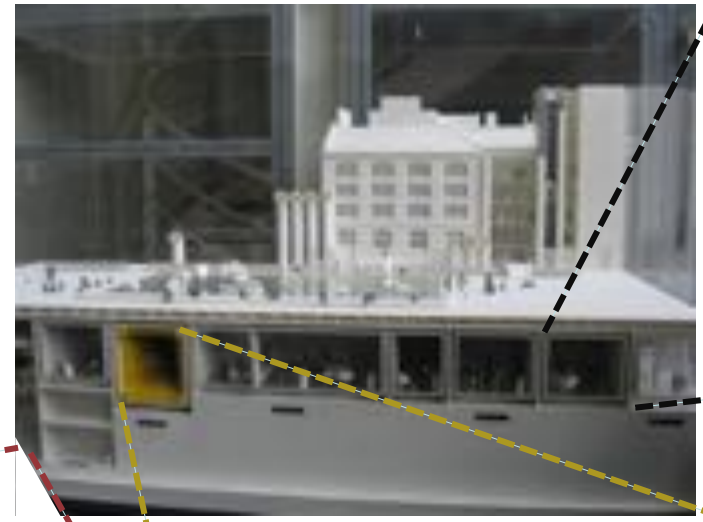
Séminaire PMMH / ESPCI

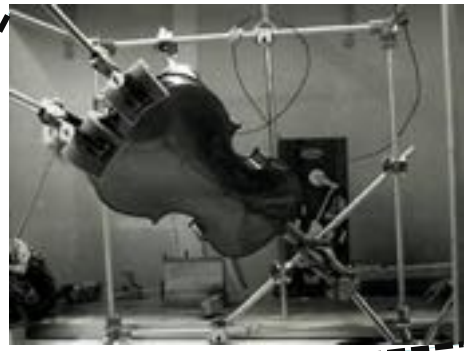
9 Juin 2017



Moreno Andreatta
Music Representations Team
IRCAM / CNRS UMR 9912 / UPMC, Paris
IRMA & GREAM, University of Strasbourg

The musical and scientific research at IRCAM...





www.ircam.fr

... at the interface between contemporary and pop music

MusiqueLab 2

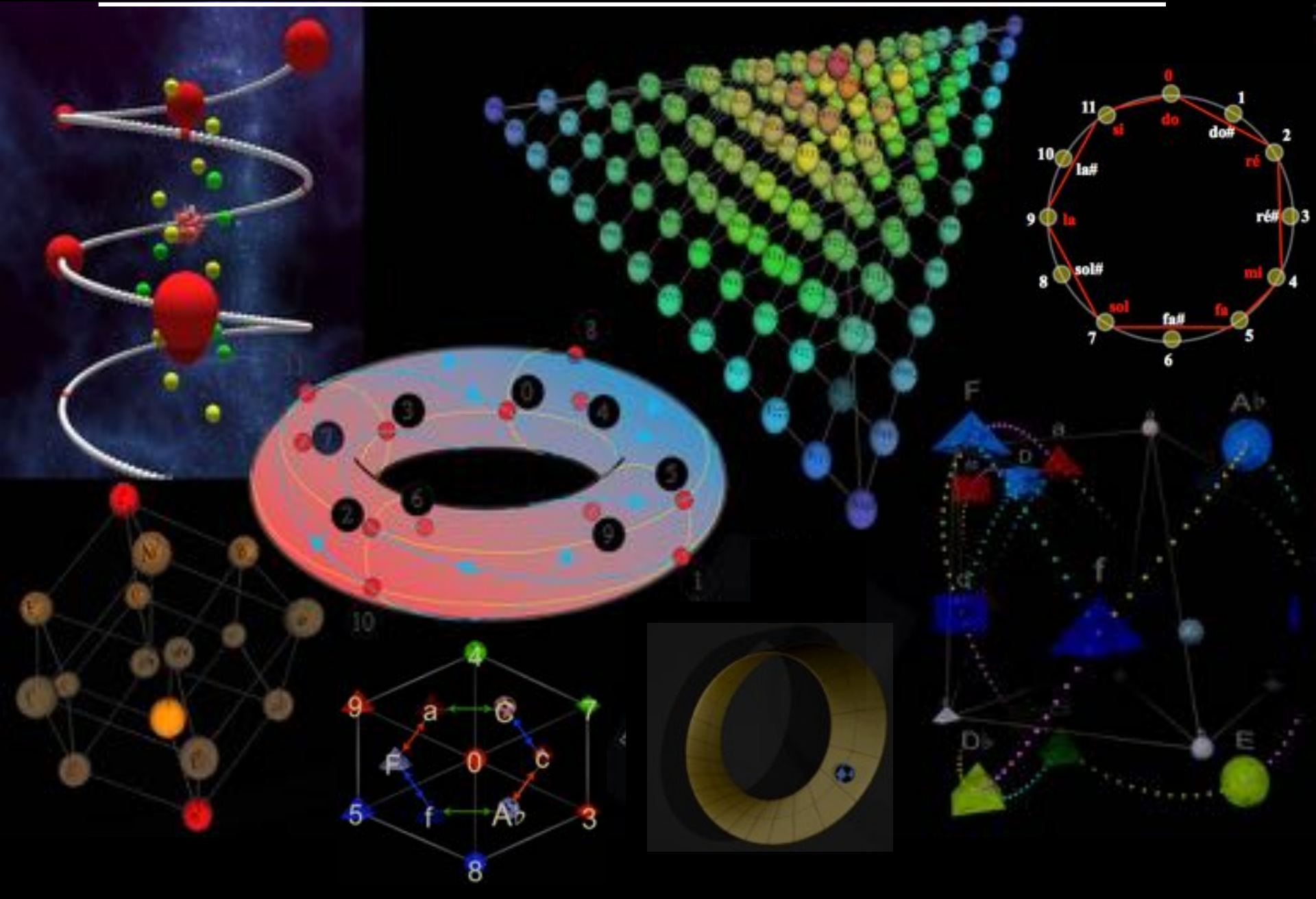
Orig.

OMAX (computer-aided impro)

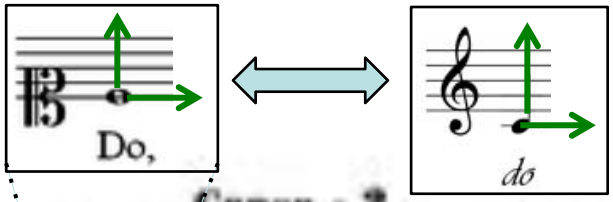
Var. 1 Var. 2

www.ircam.fr

The galaxy of geometrical models at the service of music



Bach's enigmatic canons and geometry

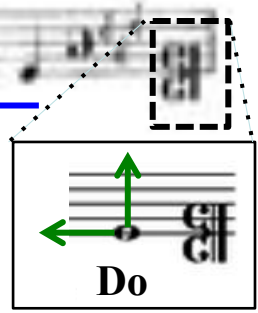


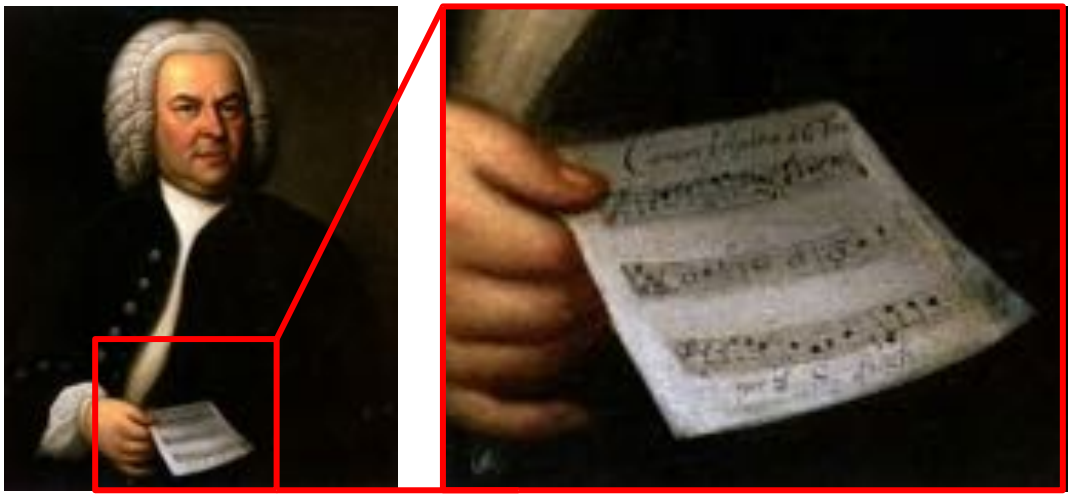
Canones diversi

super thema regium

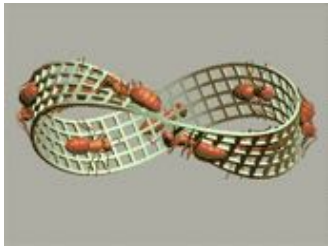
Canon a 2

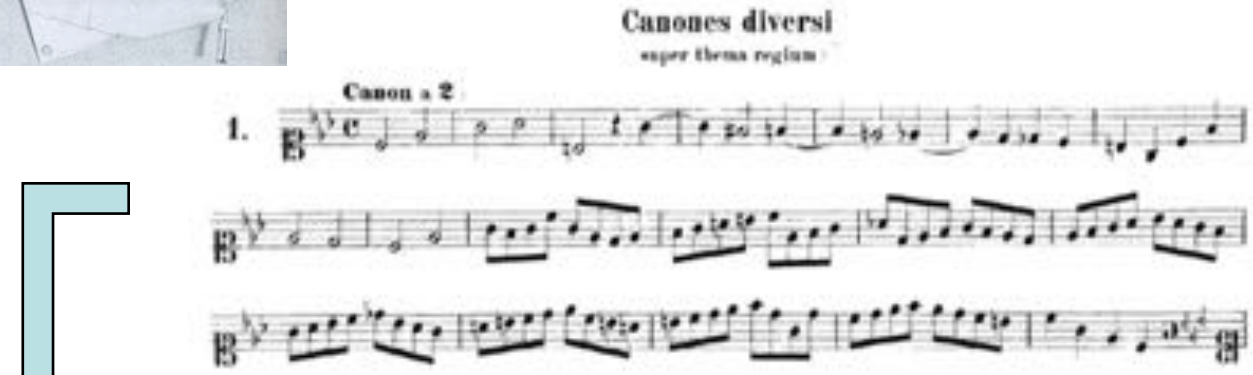
1.

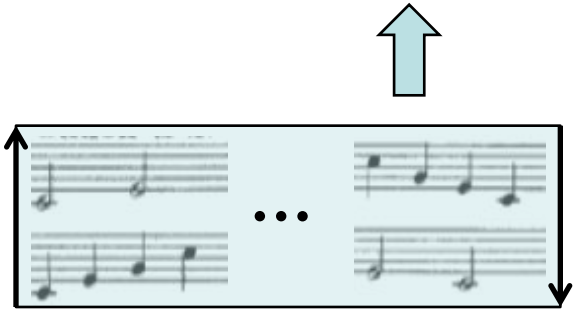


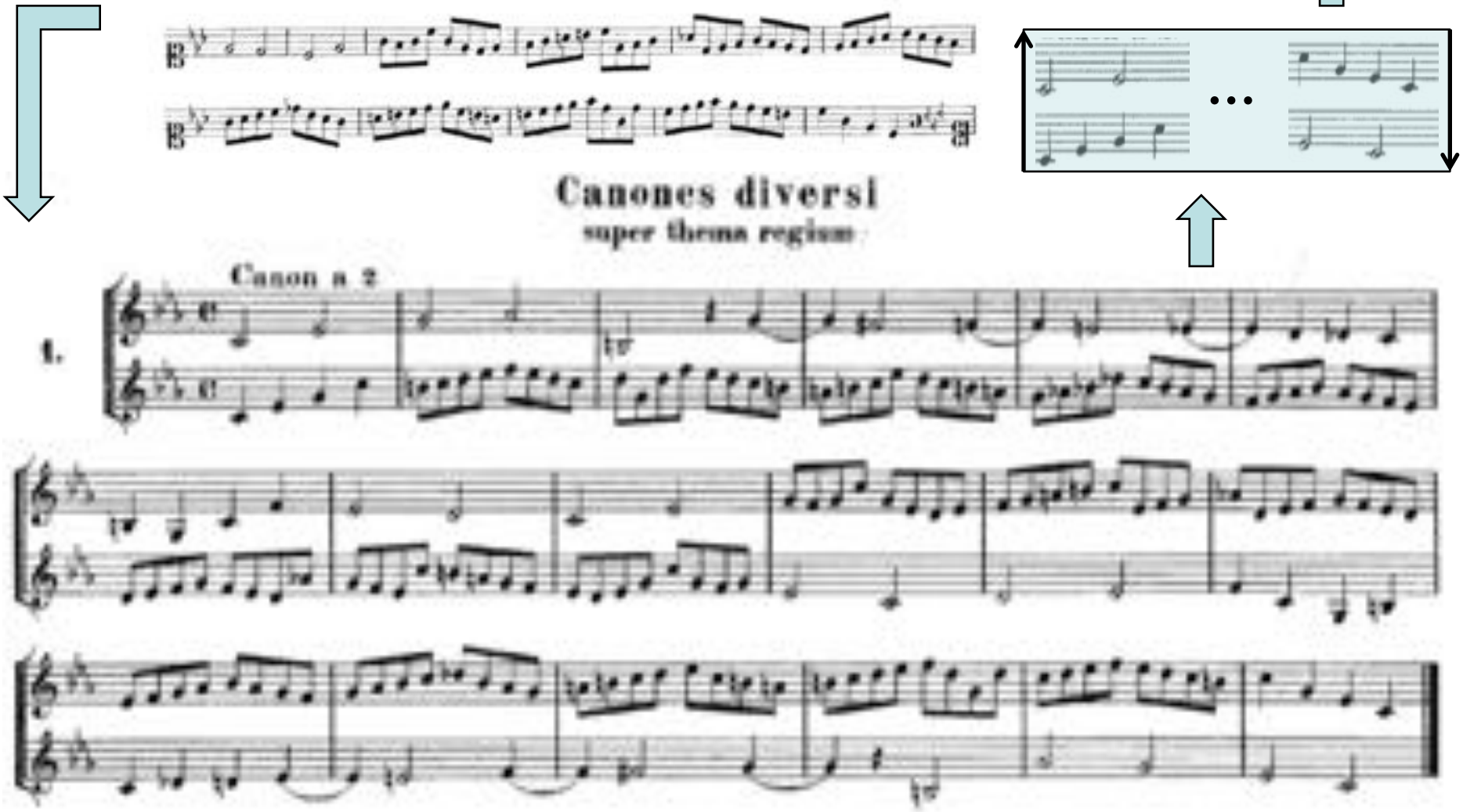


My end is my beginning (but twisted!)









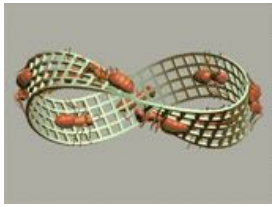
BACH

Musikalisches Opfer

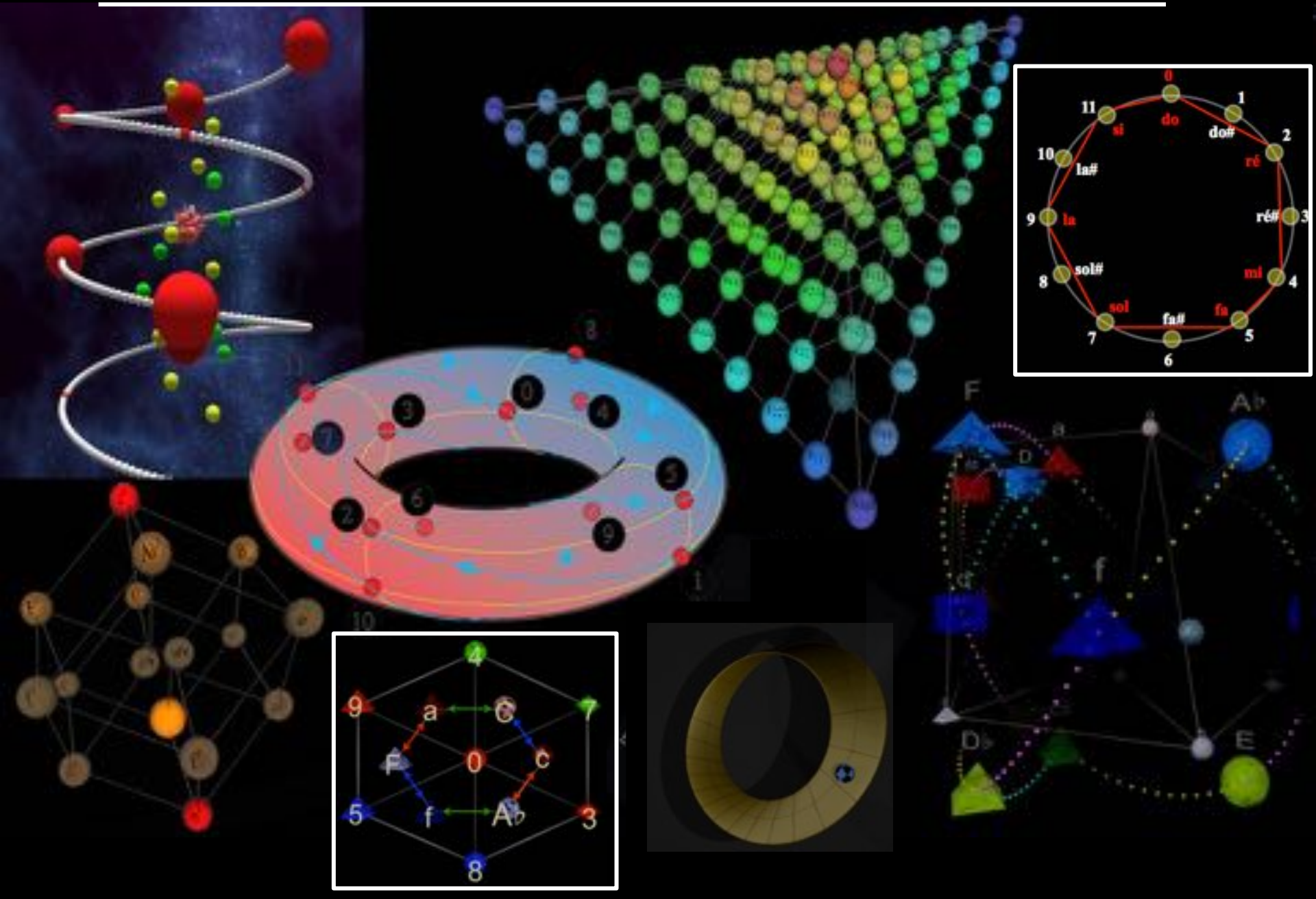
Musical Offering - Offrande Musicale

http://www.josleys.com/Canon/Canon.html

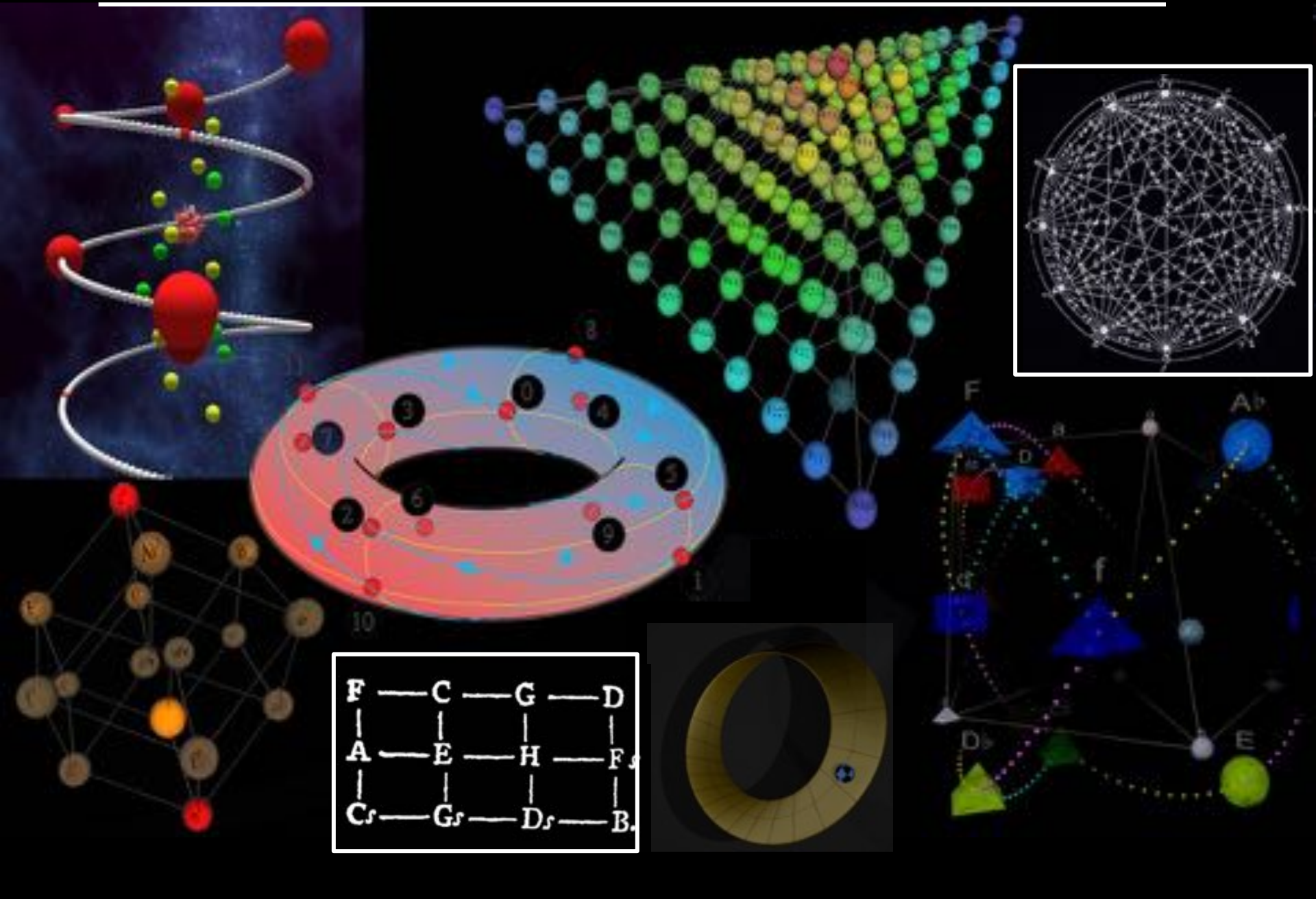
[min. 1’14”]



The galaxy of geometrical models at the service of music



The galaxy of geometrical models at the service of music



Mersenne and the birth of (musical) combinatorics

114 **Marin Mersenne, *Harmonicorum Libri XII*, 1648**

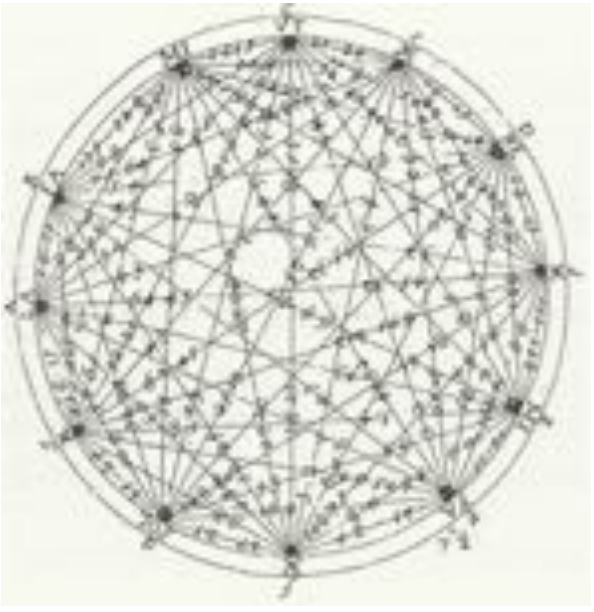
LIBER SEPTIMVS.

DE CANTIBVS, SEV CANTILENIS,

EARVMQ; NVMERO, PARTIBVS, ET SPECIEBVS.

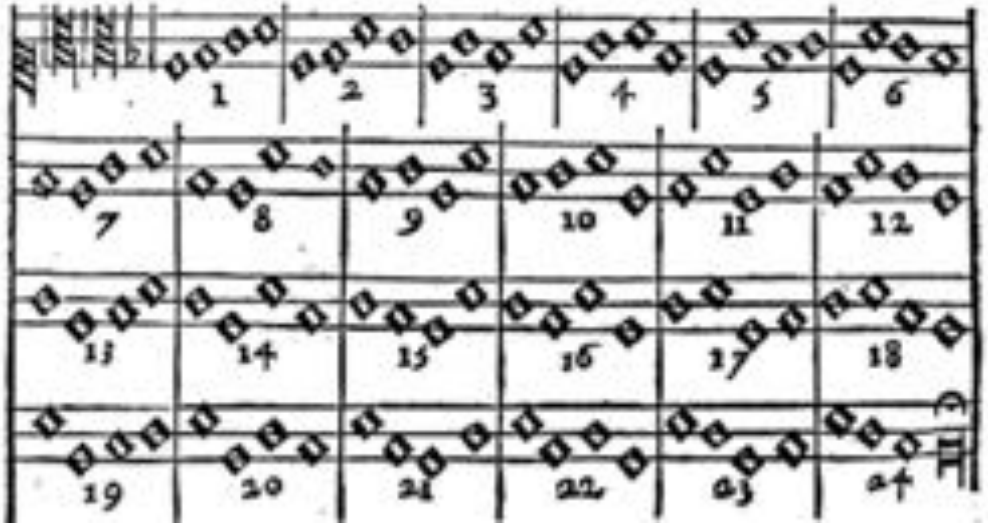
Tabula Combinationis ab 1 ad 22.

I	1
II	2
III	6
IV	24
V	120
VI	720
VII	5040
VIII	40320
IX	362880
X	3628800
XI	39916800
XII	479001600
XIII	6227020800
XIV	87178291200
XV	1307674368000
XVI	20922789888000
XVII	355687428096000
XVIII	6402373705728000
XIX	121645100408832000
XX	2432902008176640000
XXI	51090942171709440000
XXII.	1124000727777607680000



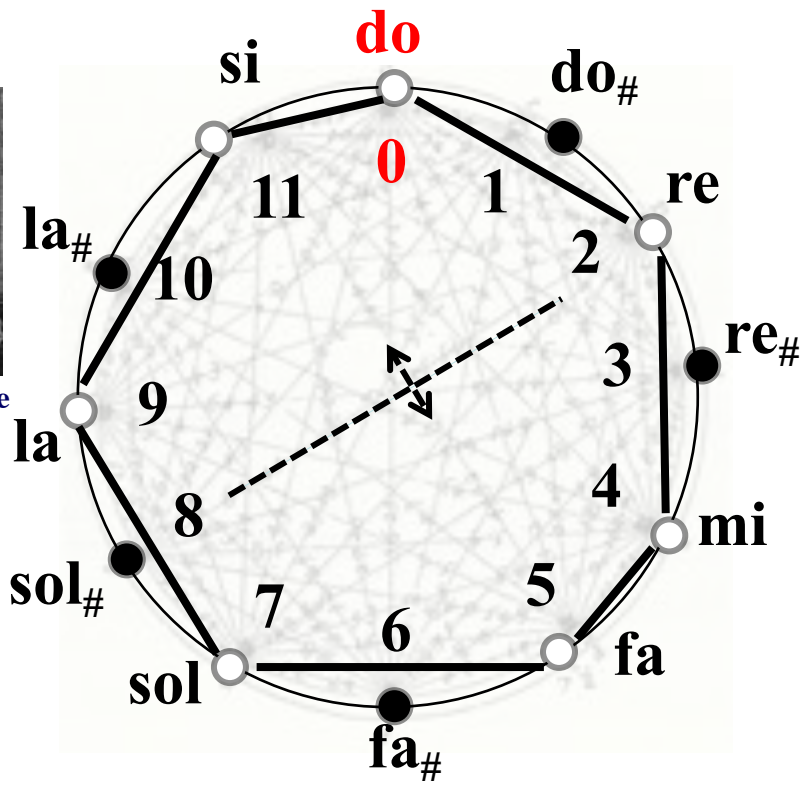
Marin Mersenne

Varietas, seu Combinatio quatuor notarum.

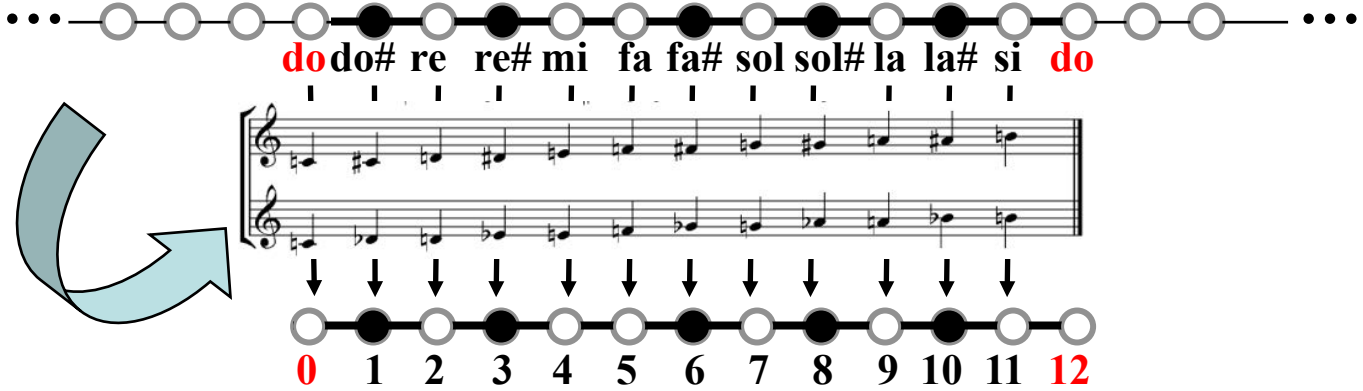


The circular representation of the pitch space

Marin Mersenne

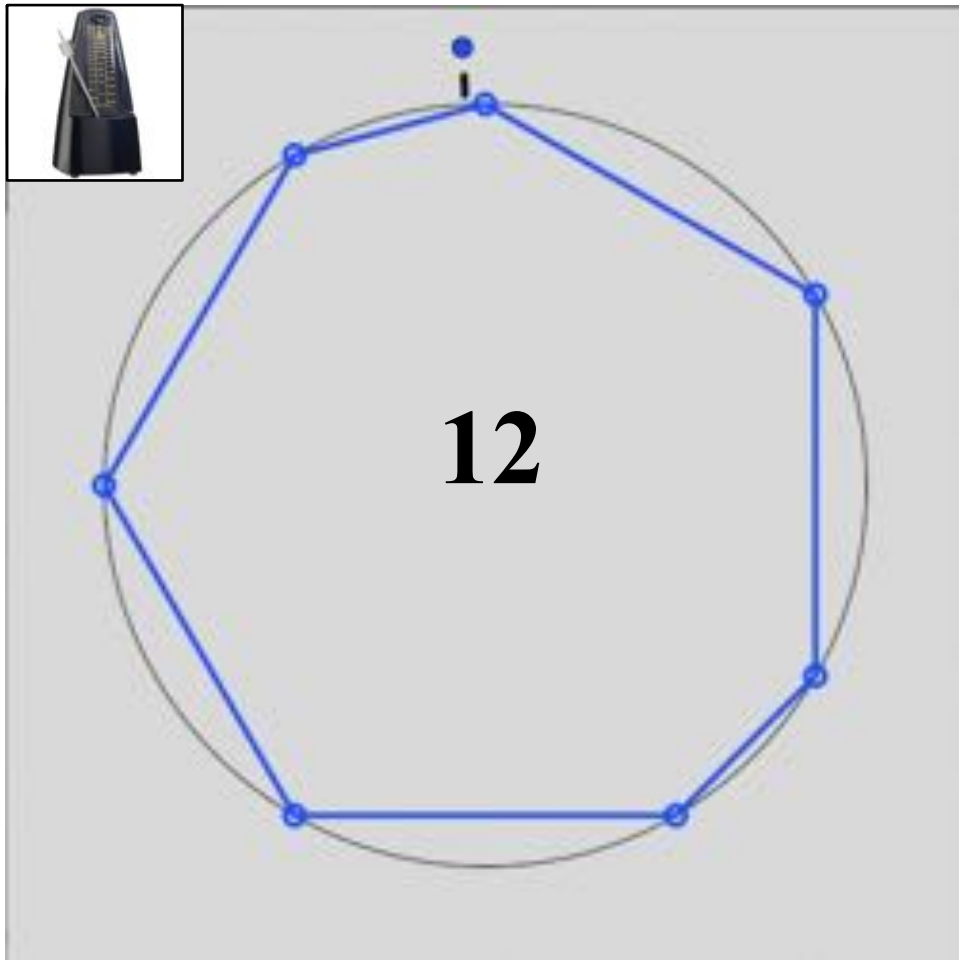


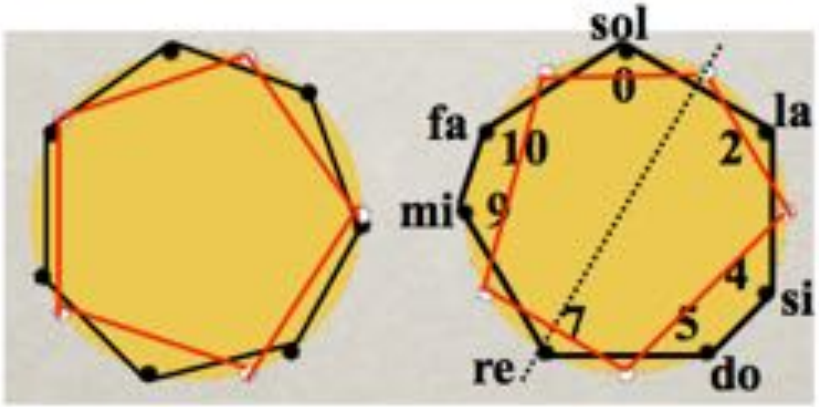
Harmonicorum Libri XII, 1648



➔ DEMO

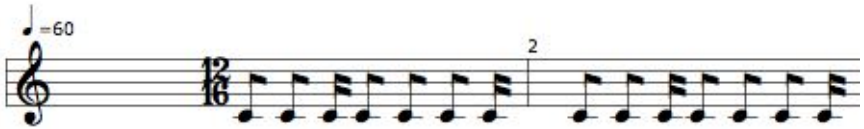
The Pitch/Rhythm Isomorphism



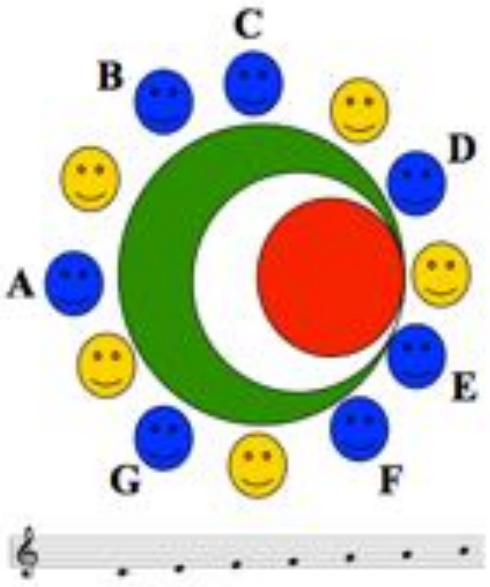


The problem of concentric circles

?



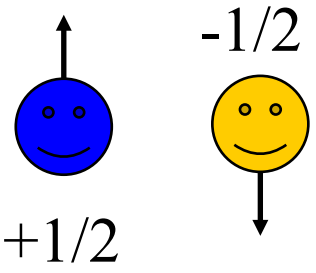
Abadja or Bembé

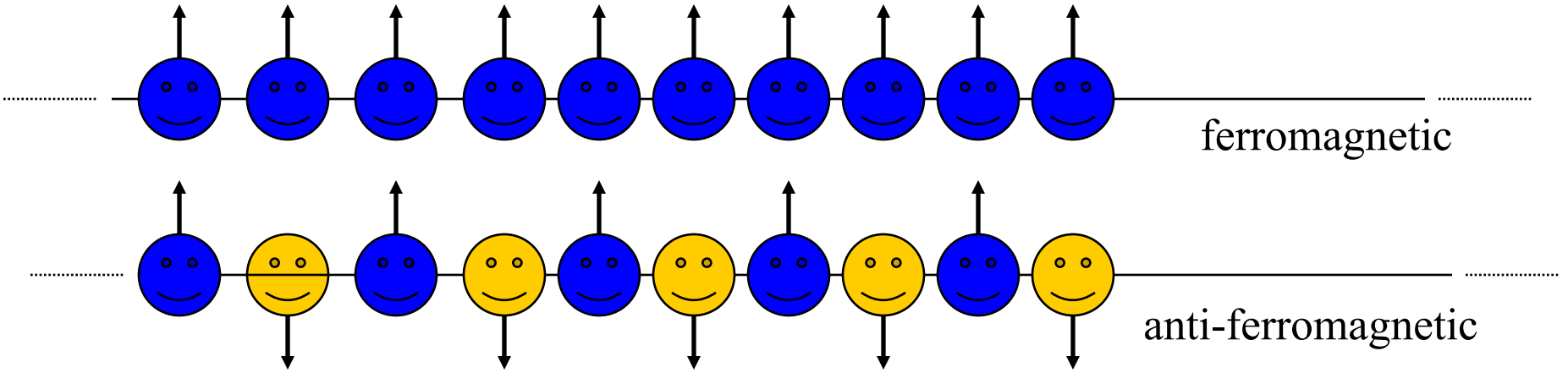


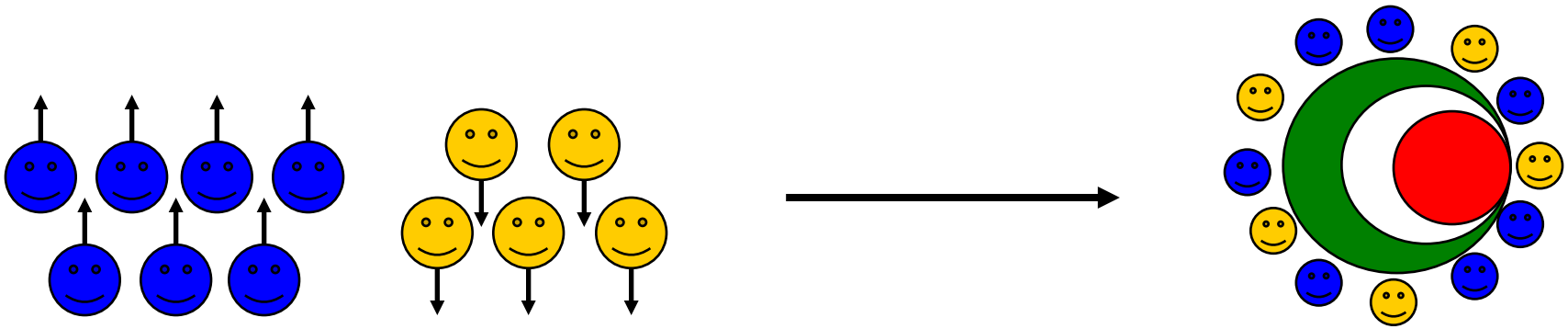
Dinner Table Problem

ME-sets and the Ising antiferromagnetic spin-1/2 model

- Jack Douthett & Richard Krantz, "**Energy extremes and spin configurations for the one-dimensional antiferromagnetic Ising model with arbitrary-range interaction",** *J. Math. Phys.* **37** (7), July 1996
- Jack Douthett & Richard Krantz, "Maximally even sets and configurations: common threads in mathematics, physics, and music", *J. Comb. Optm.* 14, 2007

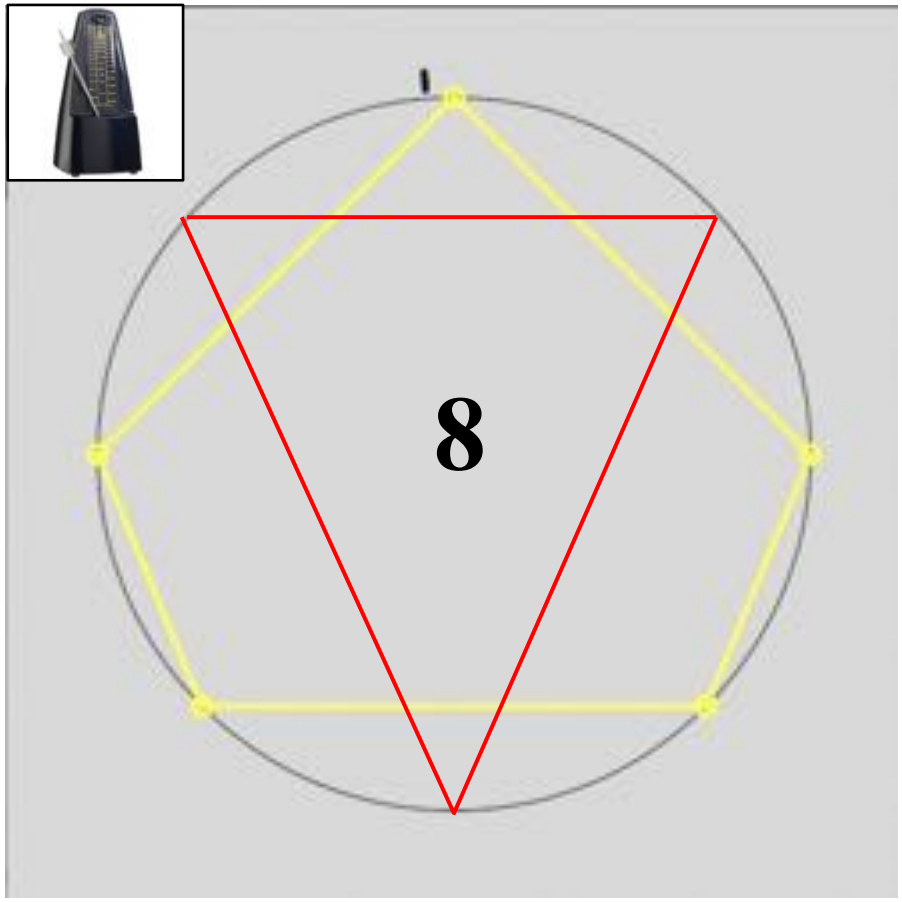


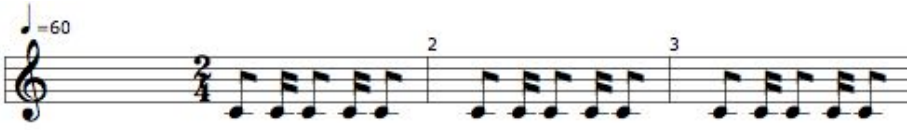




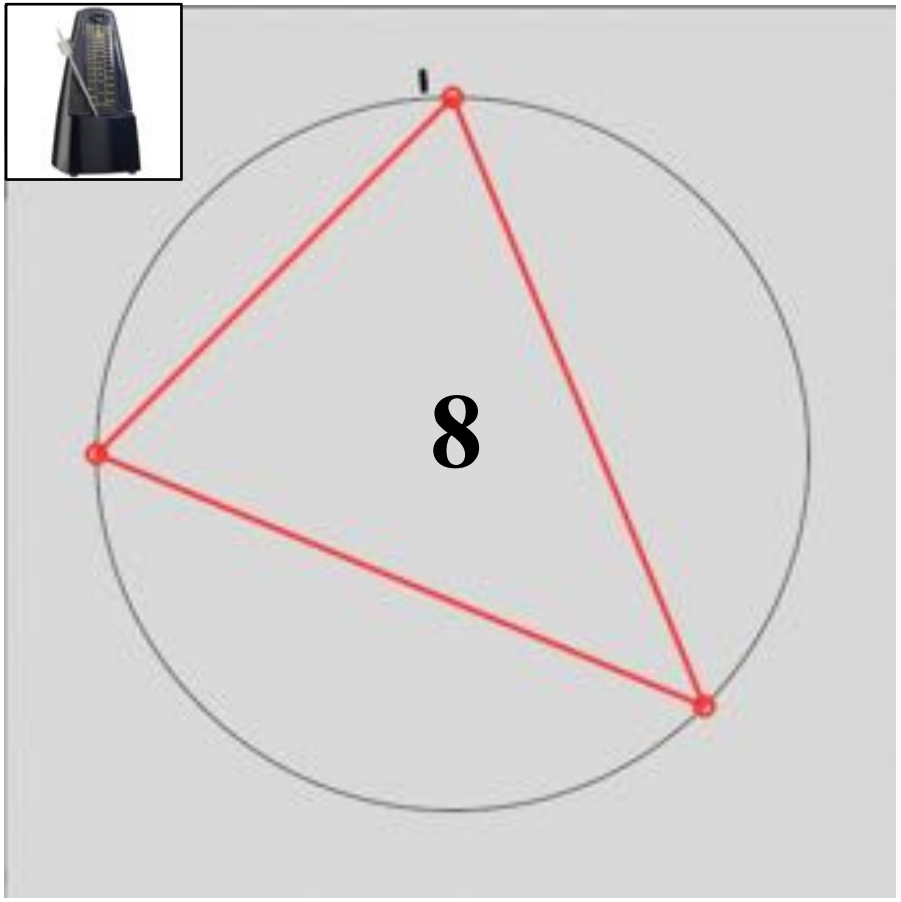
African-cuban ME-rhythms

El cinquillo

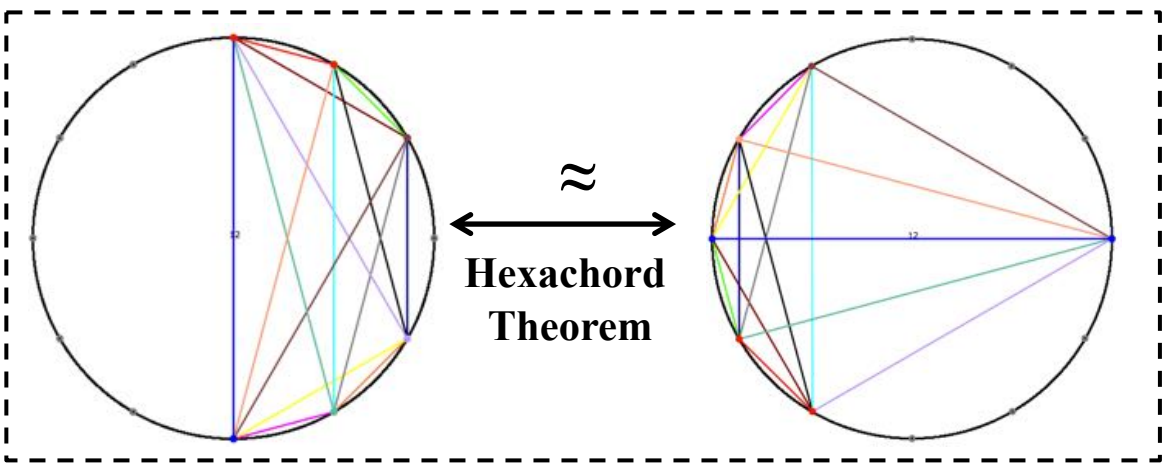


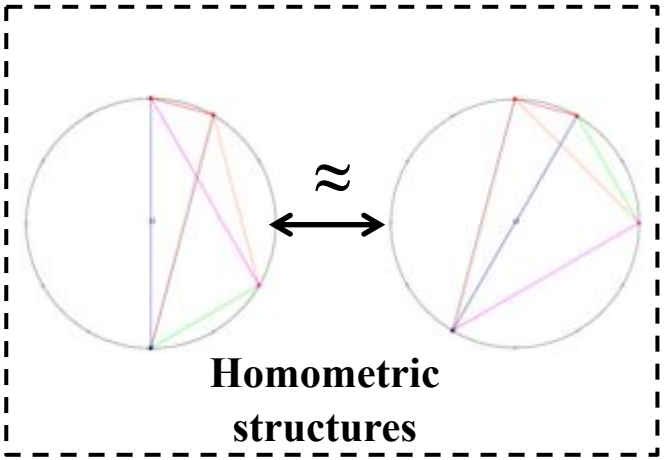


El trecillo



The Discrete Fourier Transform in symbolic MIR





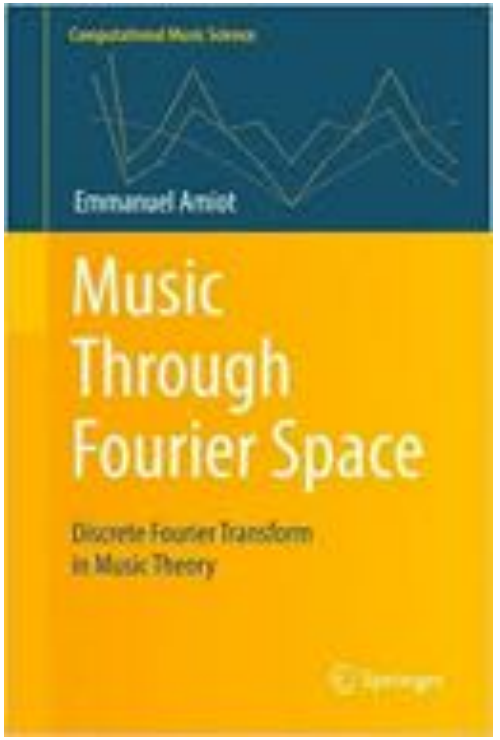
$IC_A = [4, 3, 2, 3, 2, 1\ ] = [4, 3, 2, 3, 2, 1\ ] = IC_{A'}$

$$IC_A(k) = (1_A \star 1_{-A})(k)$$

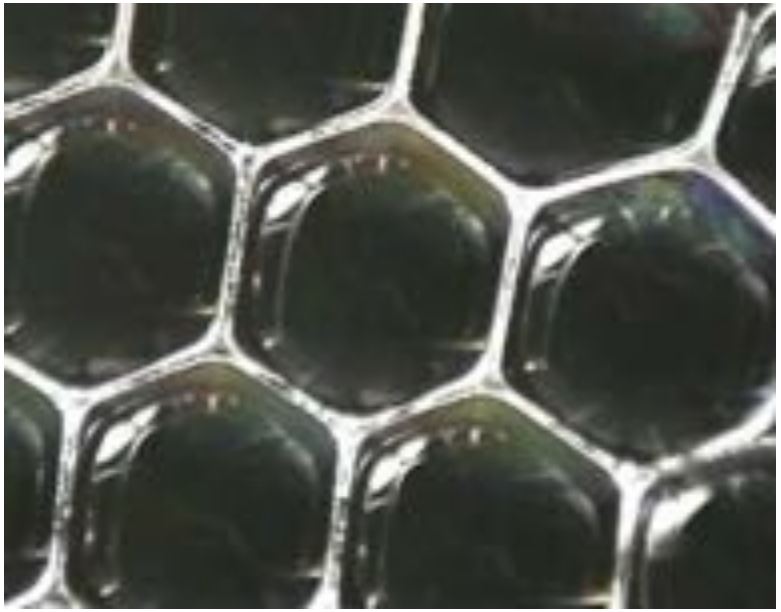
$$\mathcal{F}_A : t \mapsto \sum_{k \in A} e^{-2i\pi kt/c}$$

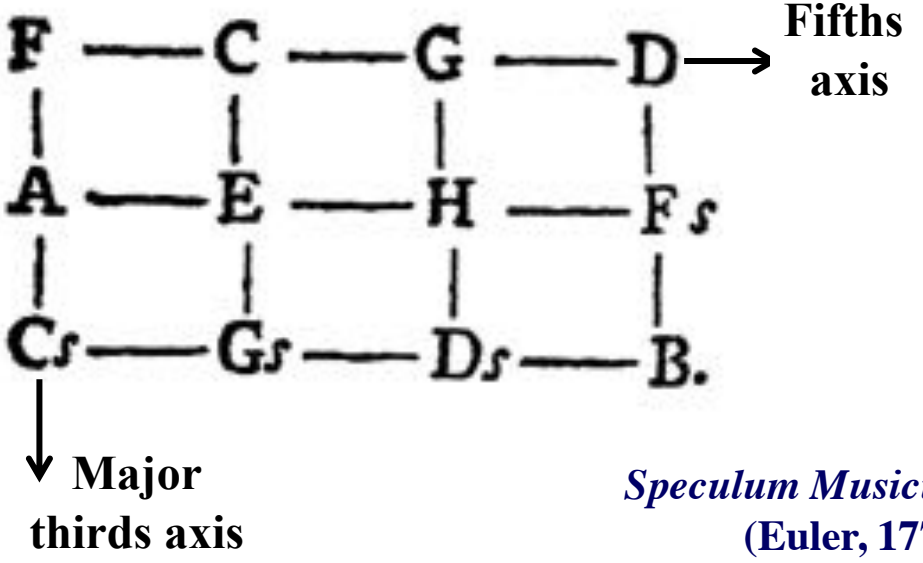
$$\forall k\ \mathcal{F}(\mathrm{IC}_{\mathbb{Z}_c \setminus A})(k) = \mathcal{F}(\mathrm{IC}_A)(k)$$

E. Amiot : « Une preuve élégante du théorème de Babbitt par transformée de Fourier discrète », Quadrature, 61, 2006.



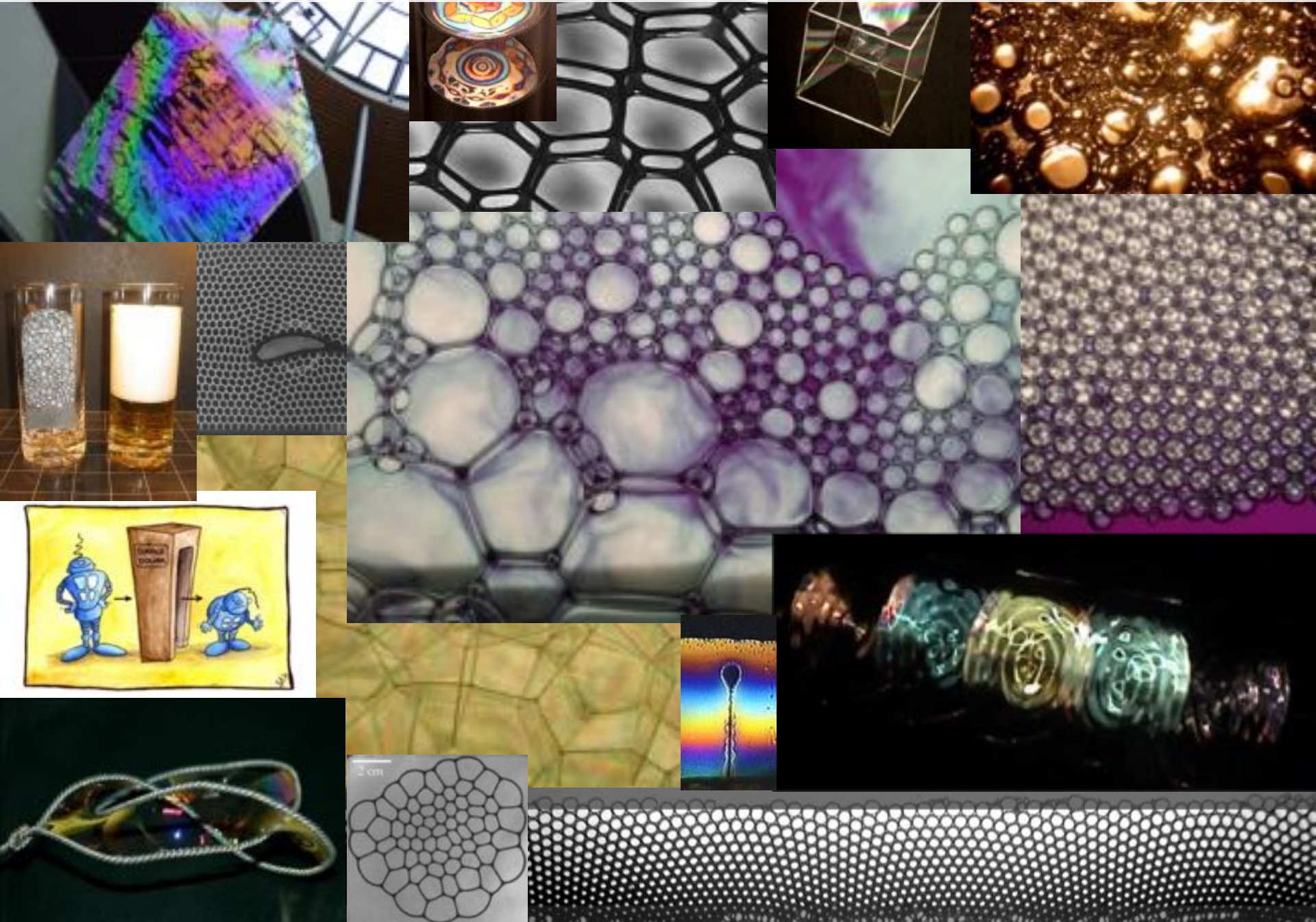
The *Tonnetz*
(or 'honeycomb' hexagonal tiling)



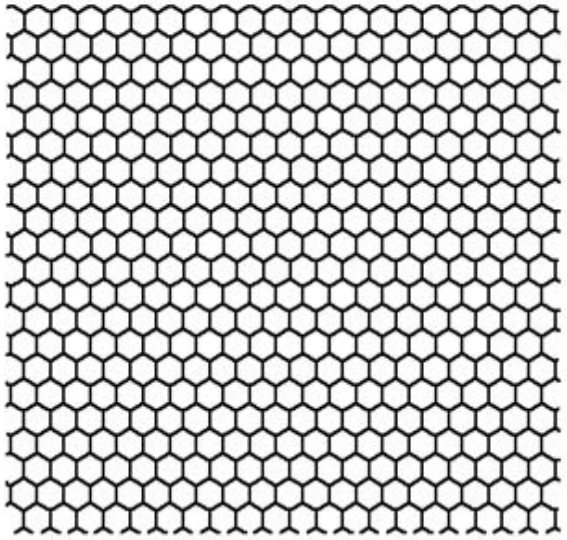


Speculum Musicum
(Euler, 1773)

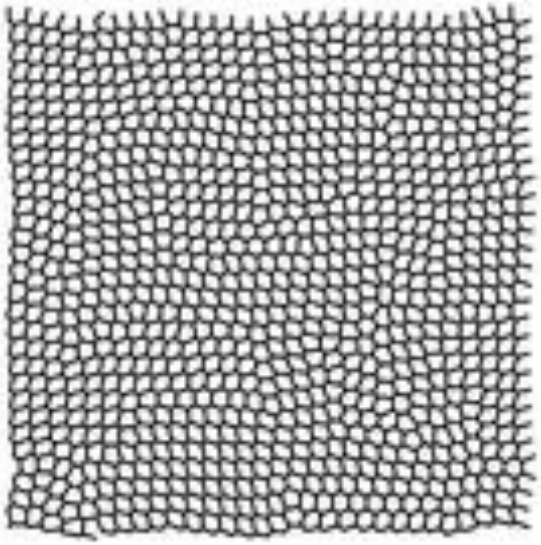
From music analysis to the sonifications of complex systems



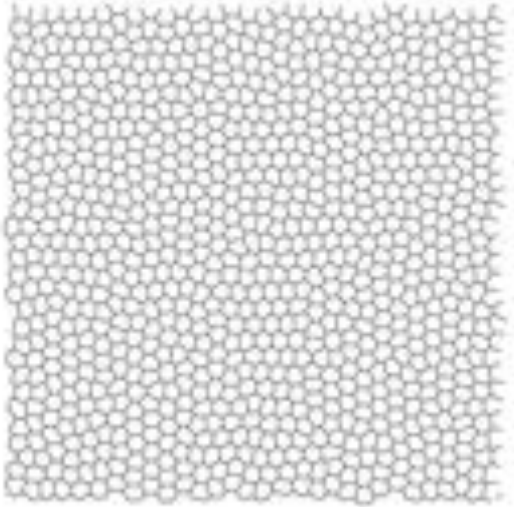
Can you hear the shape of a tiling?



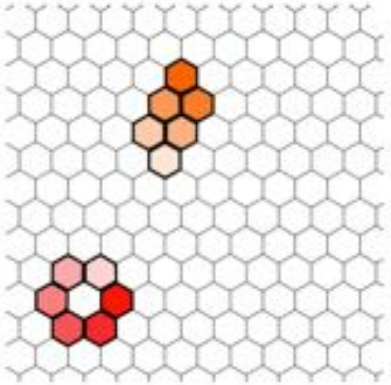
Honeycomb Tonnetz

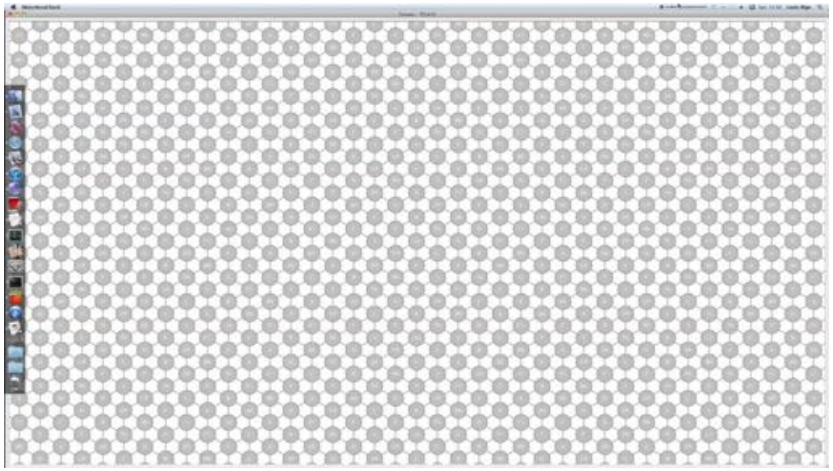


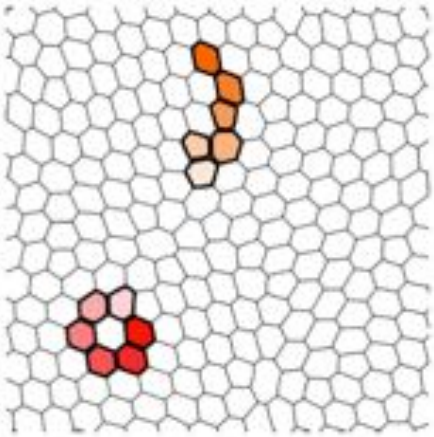
Grain boundaries



Disordered

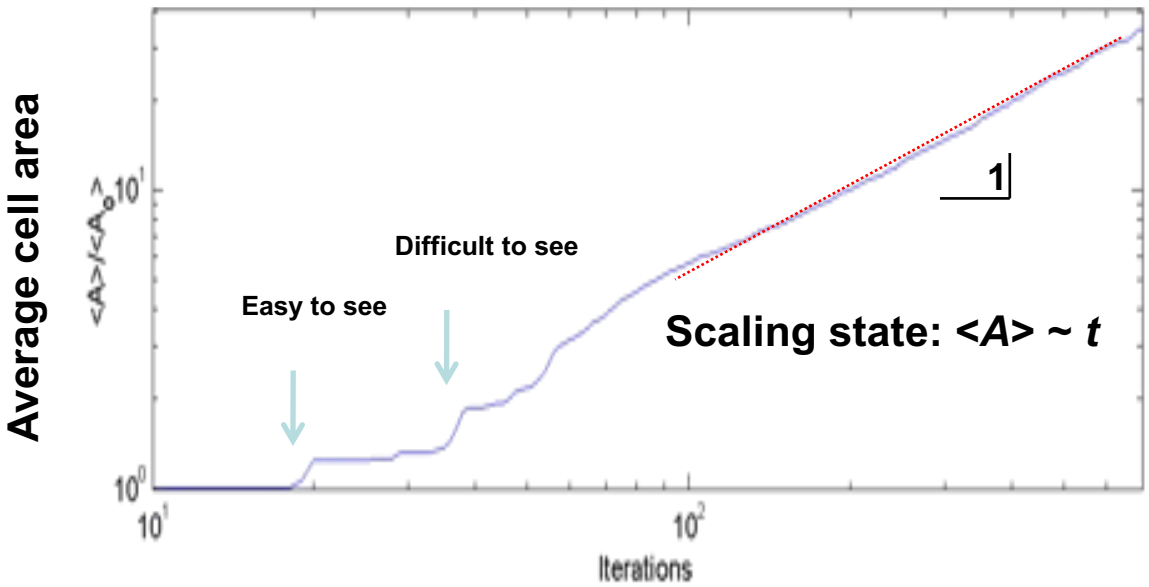


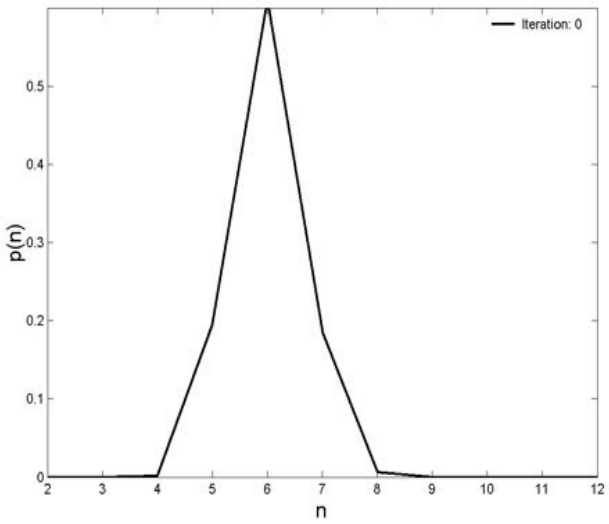




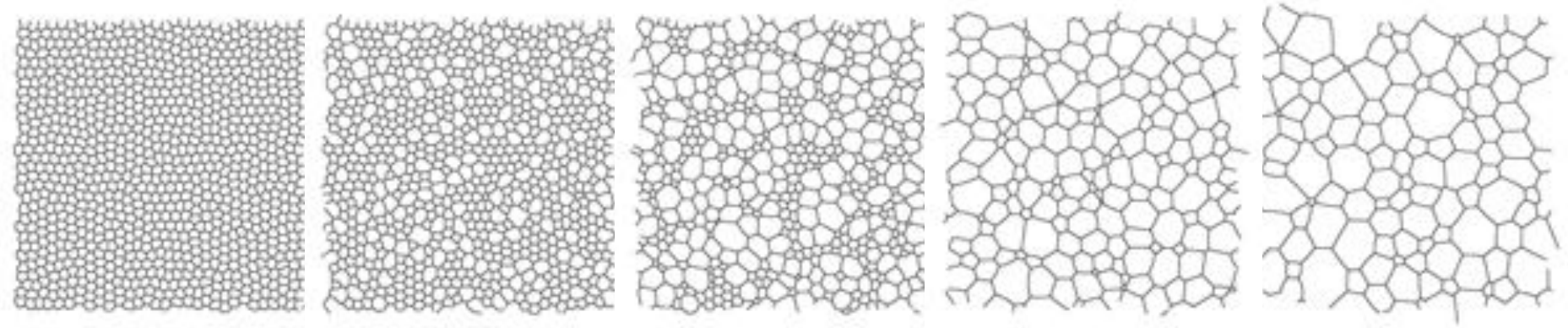
M. Potier, *De la sonification à la « musification » de systèmes complexes*, Master MPRI, Université Paris Diderot, 6 septembre 2012 (codirection avec Wiebke Drenckhan, LPS-CNRS-université Paris Sud)

Can you hear the transition towards the ‘scaling state’?

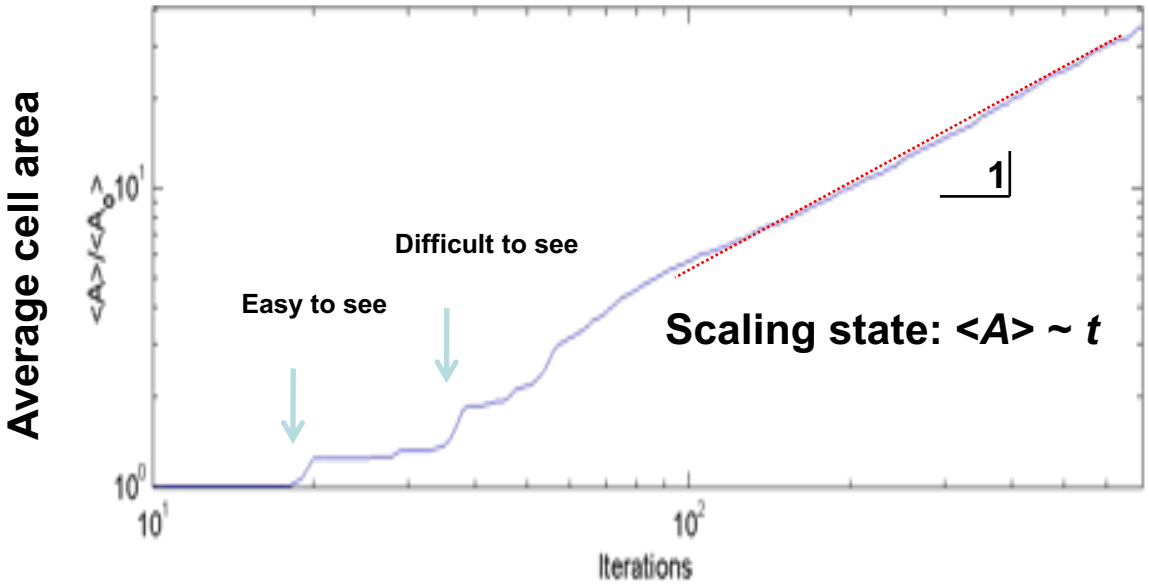


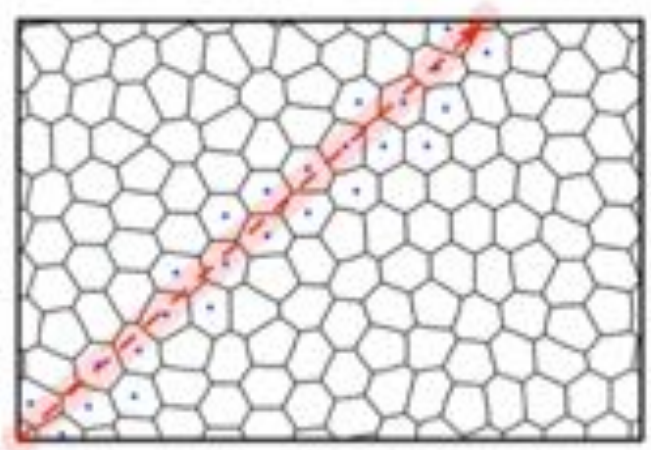


Number *n* of neighbours of cell

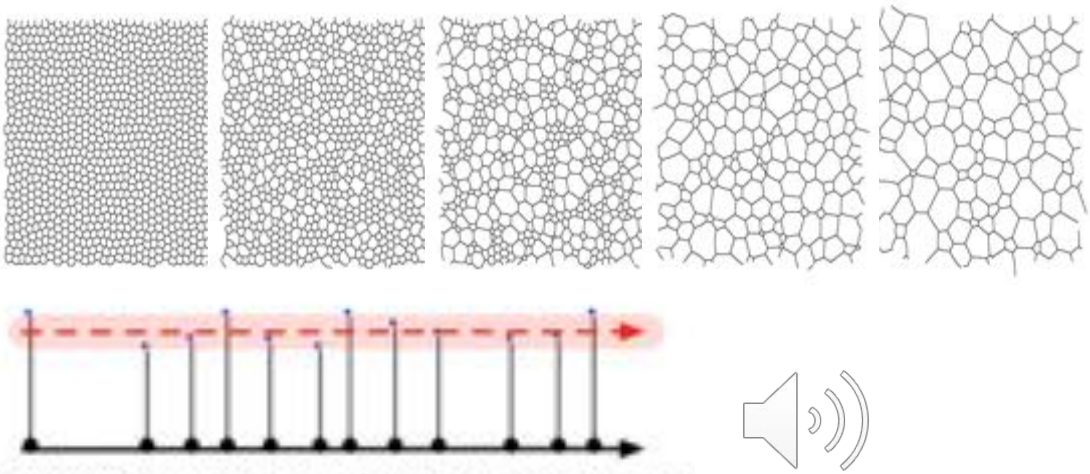


Can you hear the transition towards the ‘scaling state’?



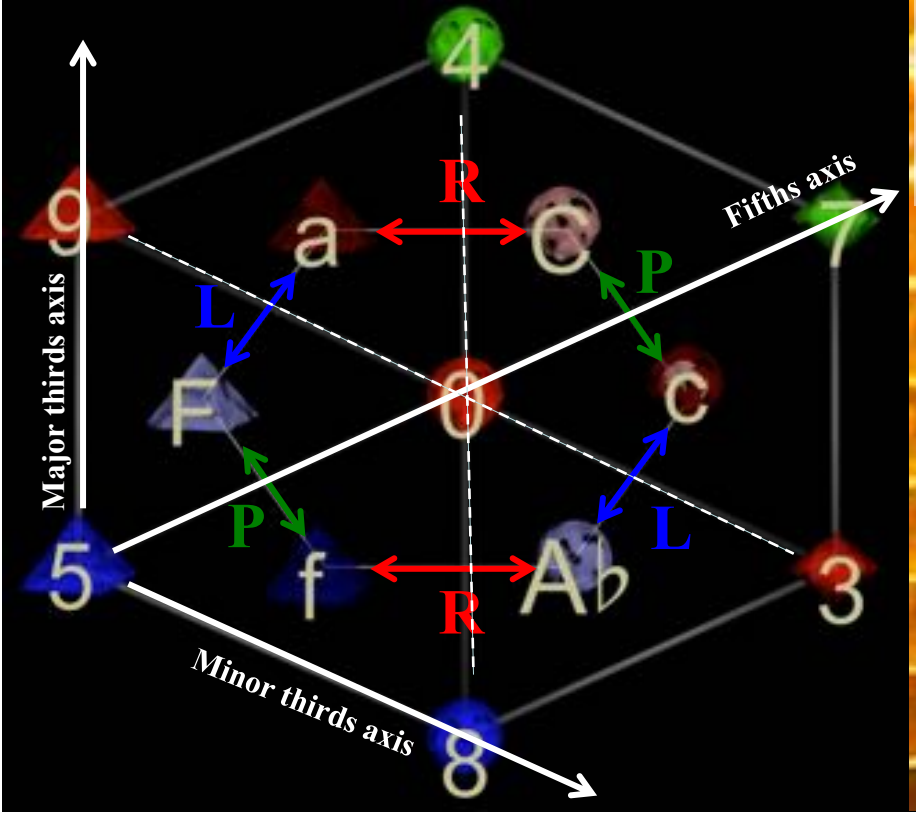


(a) En pointillés, (Δ) traverse la mousse. Les centres des bulles proches sont sélectionnés

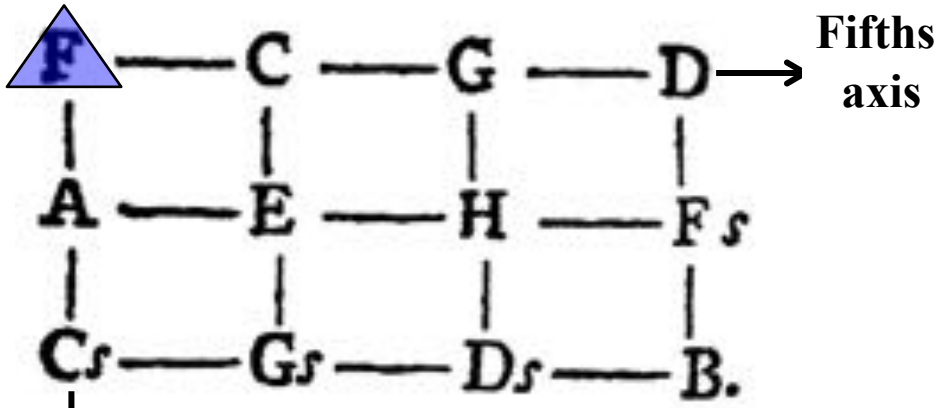


(b) Projection orthogonale des centres de bulles sur (Δ) pour obtenir une phrase rythmique

The *Tonnetz*
(or 'honeycomb' hexagonal tiling)



Gilles Baroin



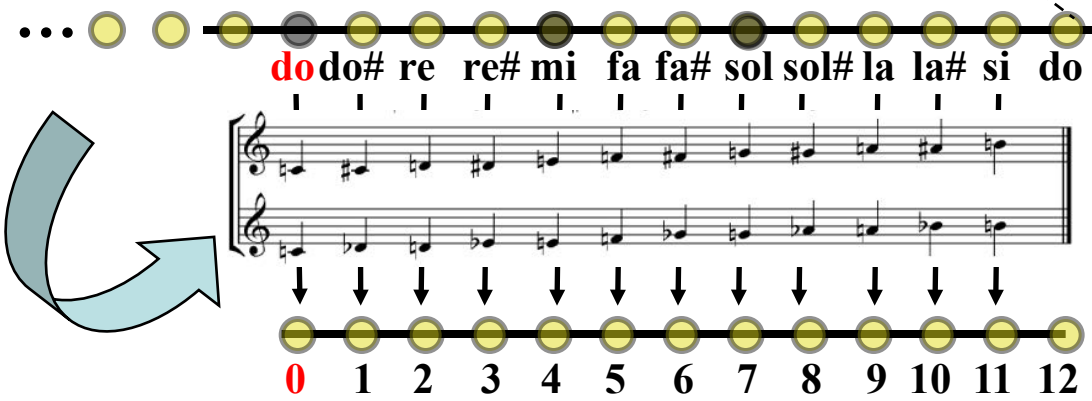
Speculum Musicum
(Euler, 1773)

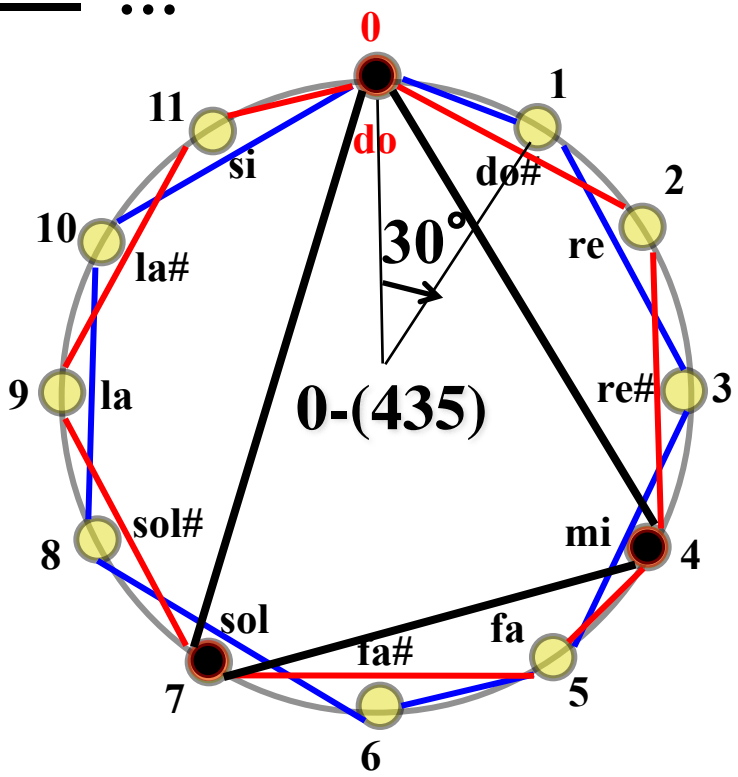
Musical transpositions are additions...

$T_k : x \rightarrow x+k \bmod 12$

***Do* maj** = {0,2,4,5,7,9,11} +1

***Do#* maj** = {1,3,5,6,8,10,0}





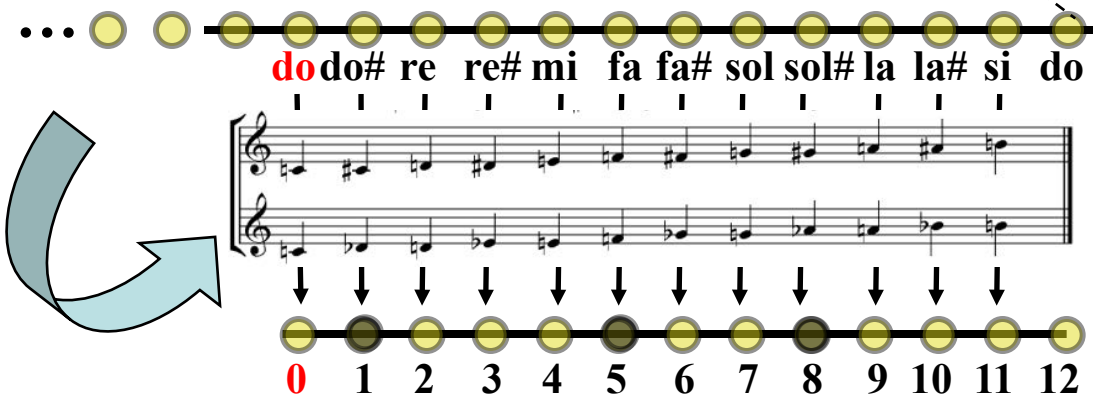
...or rotations !

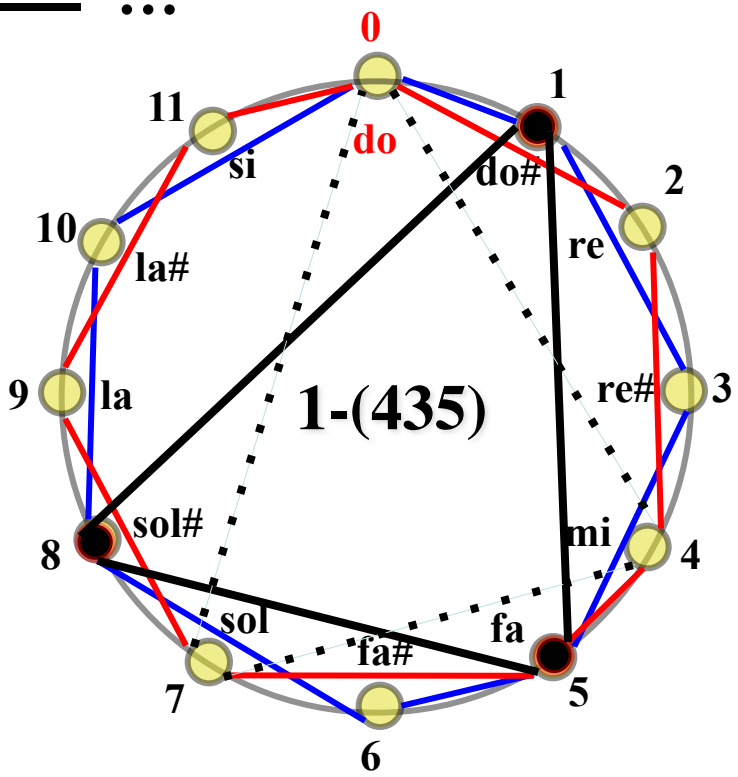
Musical transpositions are additions...

$T_k : x \rightarrow x+k \bmod 12$

***Do* maj** = {0,2,4,5,7,9,11} +1

***Do#* maj** = {1,3,5,6,8,10,0}





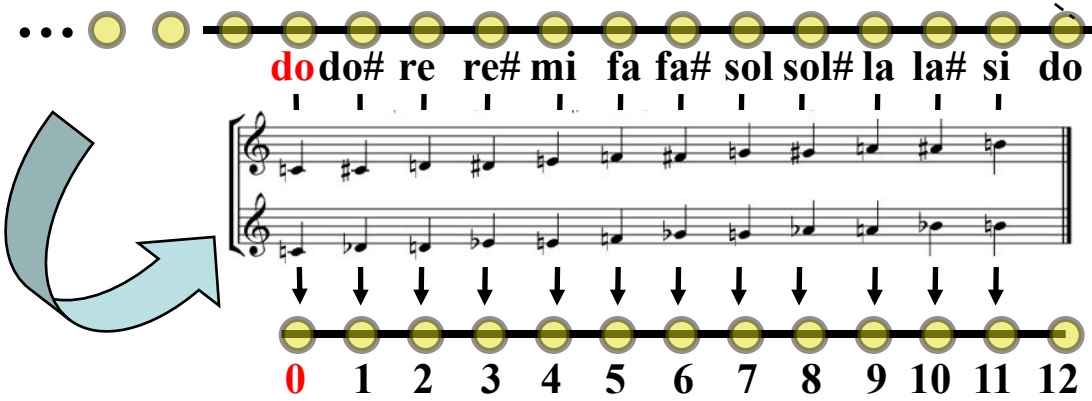
...or rotations !

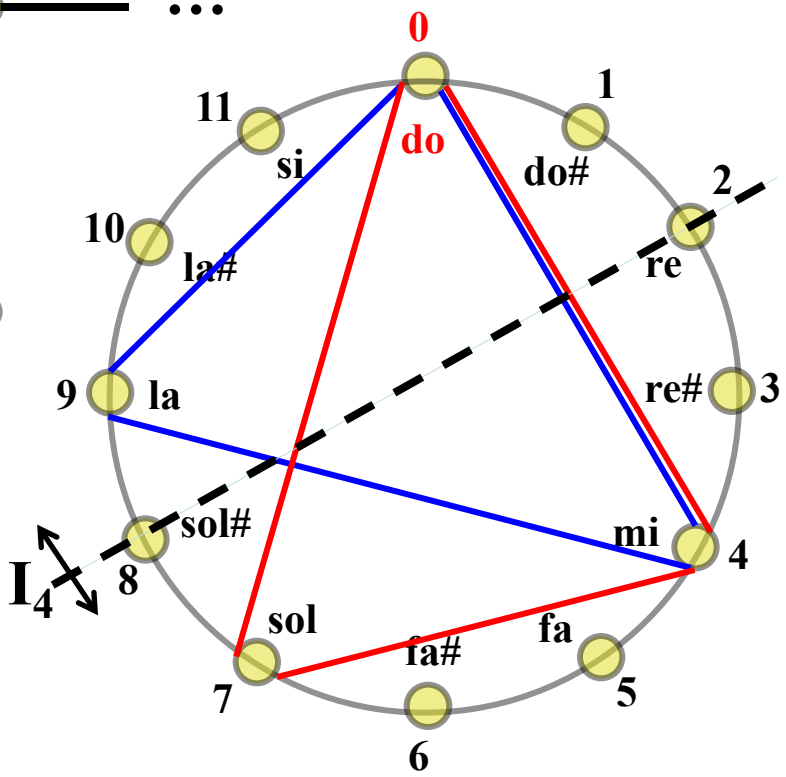
Musical inversions are differences...

I : $x \rightarrow -x$ mod 12

Do maj = {0,4,7}
La min = {0,4,9}

$I_4(x)=4-x$





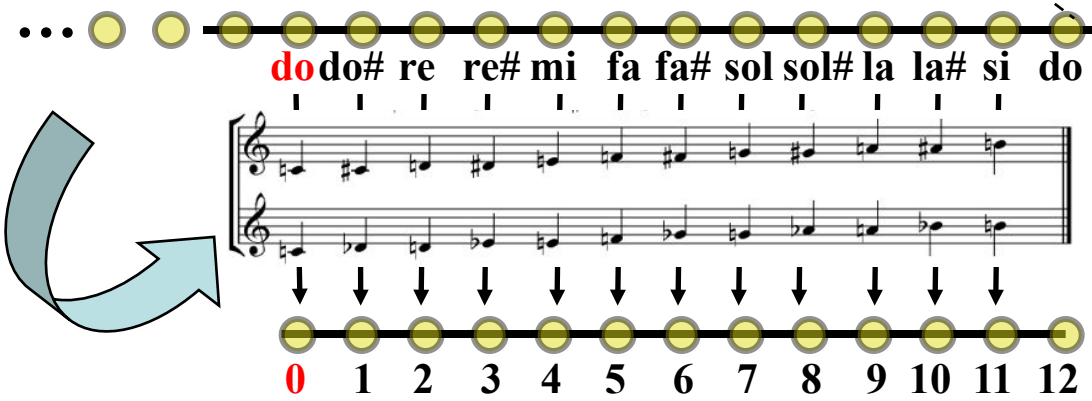
... or axial symmetries!

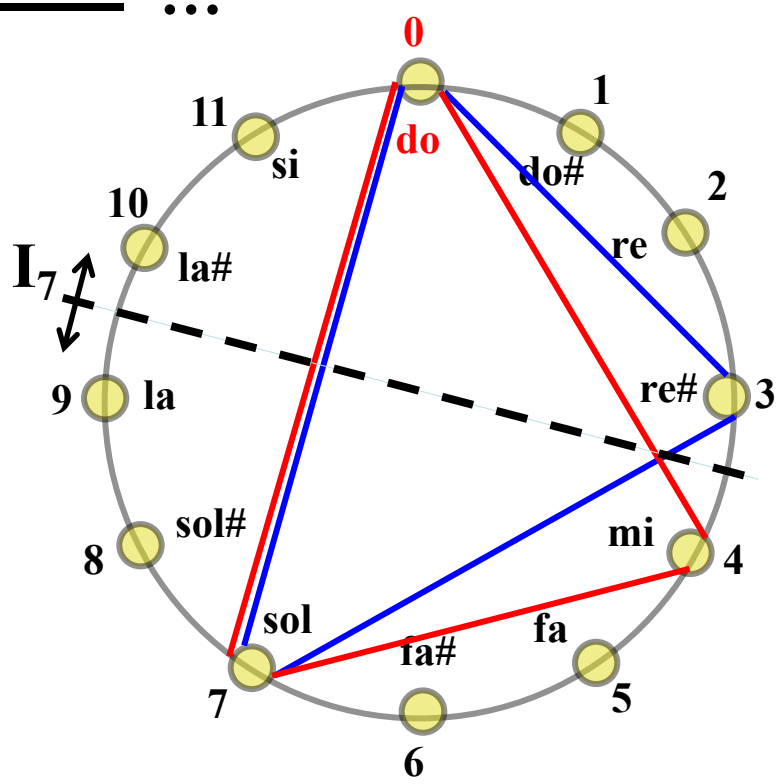
Musical inversions are differences...

$I : x \rightarrow -x \bmod 12$

***Do* maj** $= \{0,4,7\}$
***Do* min** $= \{0,3,7\}$

$I_7(x)=7-x$





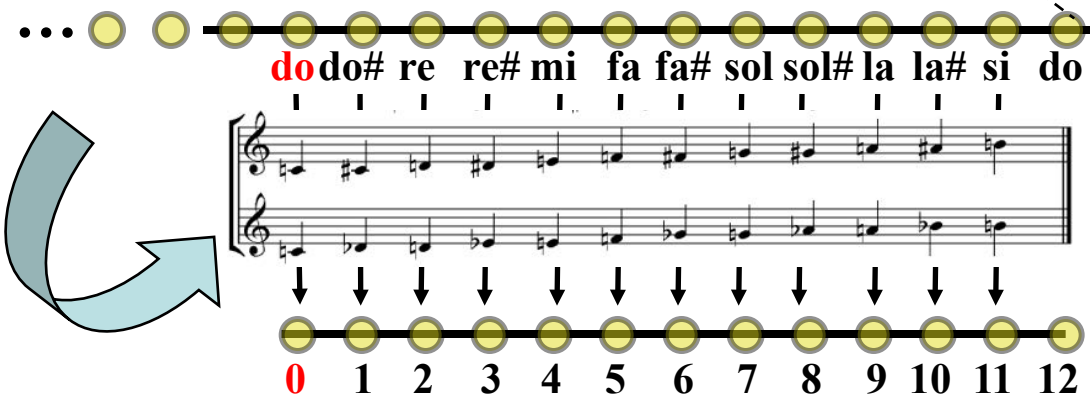
... or axial symmetries!

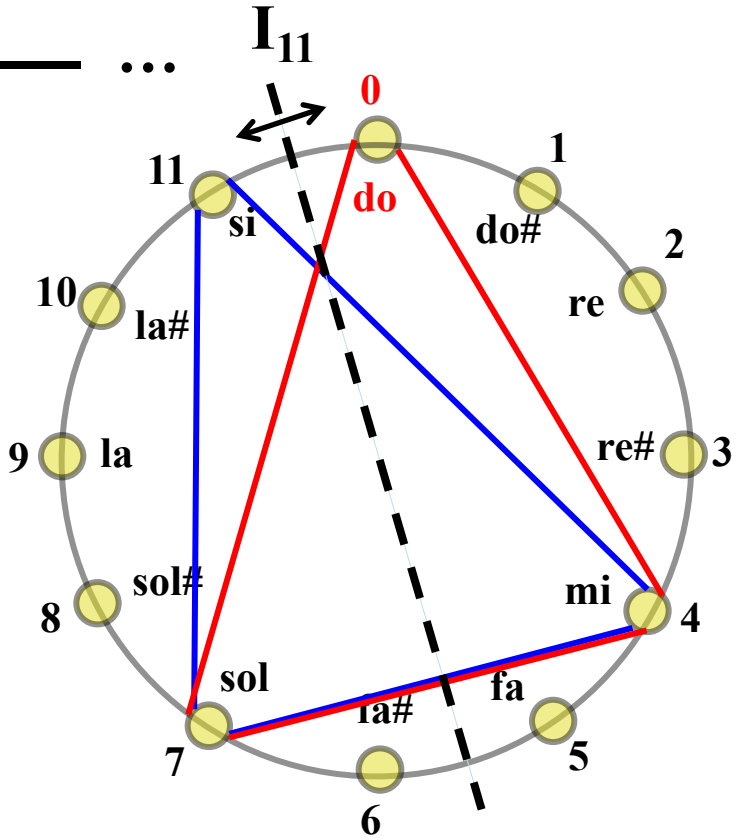
Musical inversions are differences...

I : $x \rightarrow -x$ mod 12

Do maj = {0,4,7}
Mi min = {4,7,11}

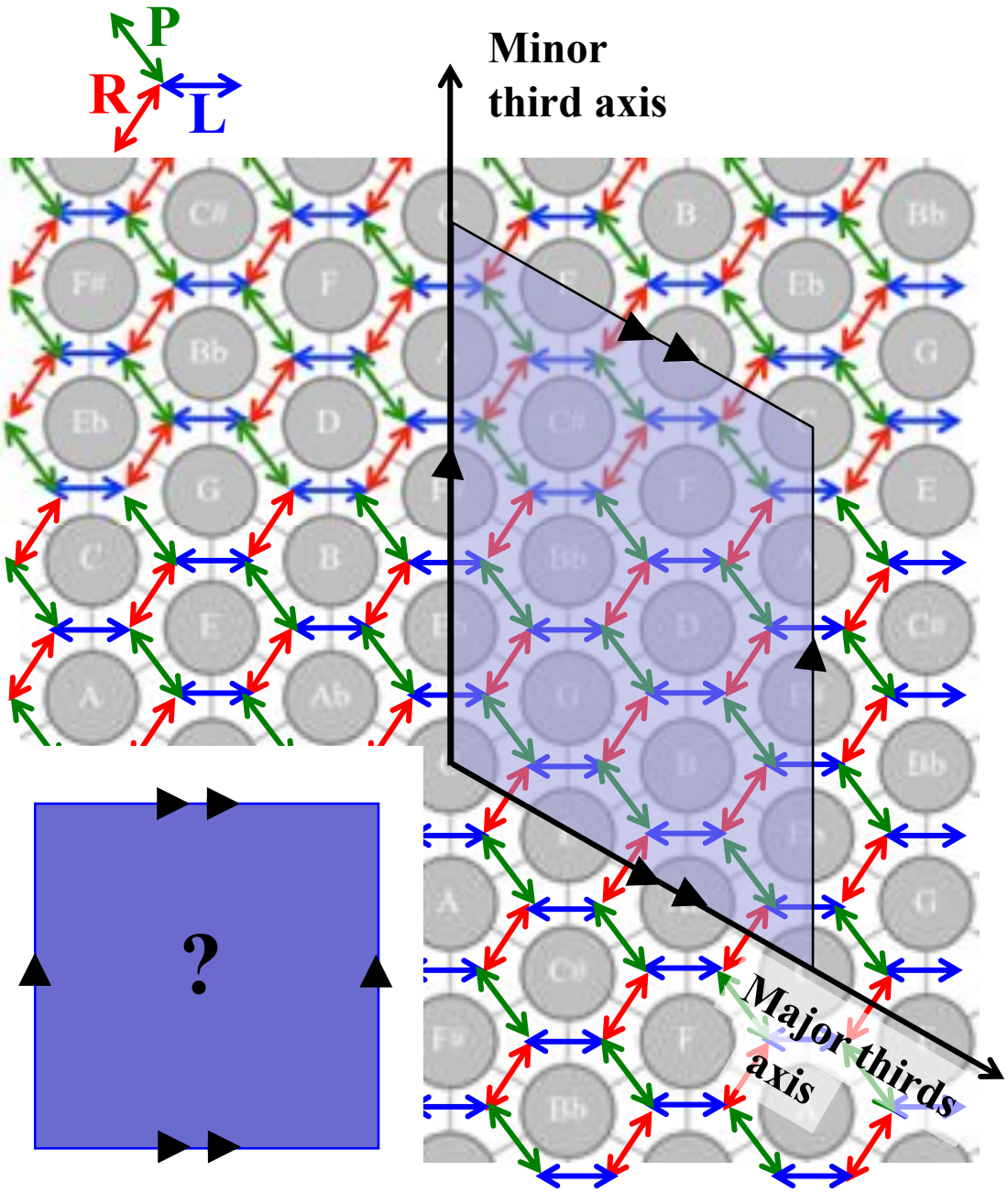
$I_{11}(x)=11-x$



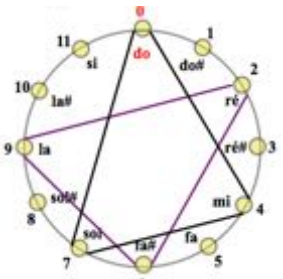


... or axial symmetries!

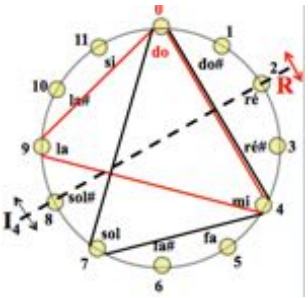
The *Tonnetz*, its symmetries and its topological structure



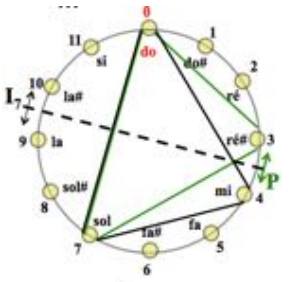
transposition



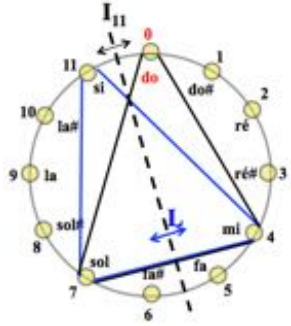
R as relative



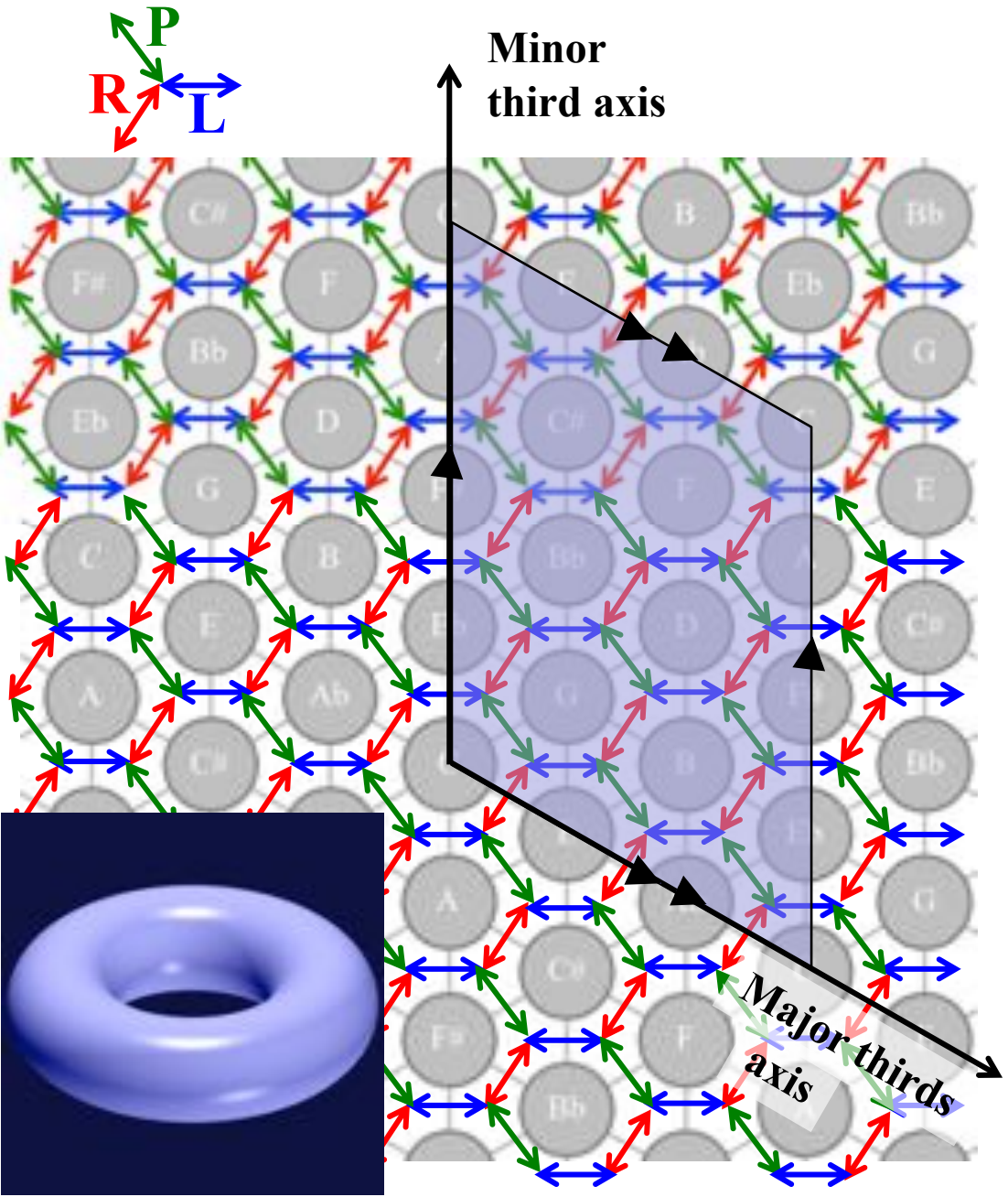
P as parallel

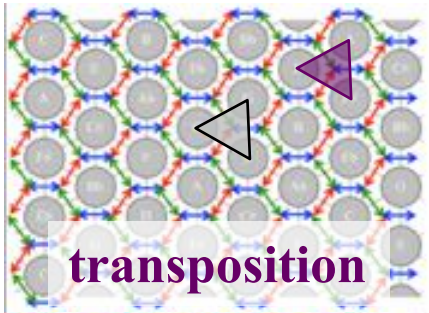


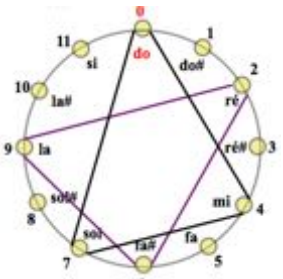
L = Leading Tone

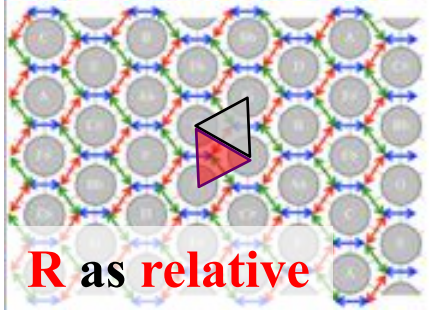


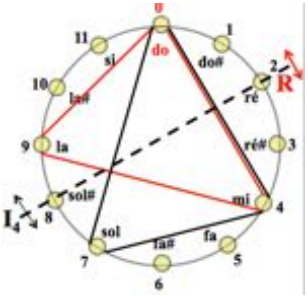
The *Tonnetz*, its symmetries and its topological structure

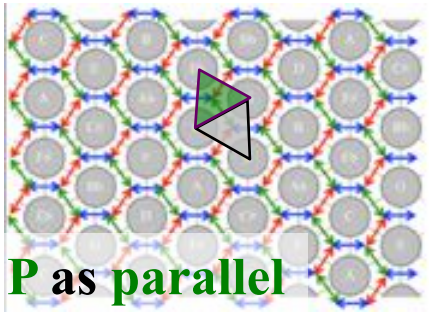


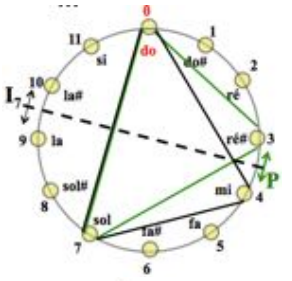


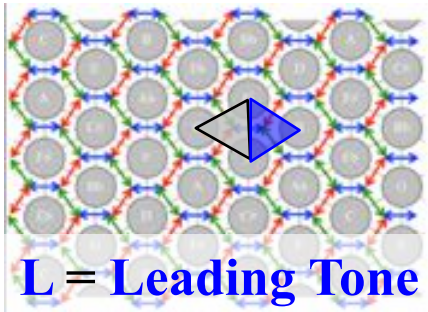


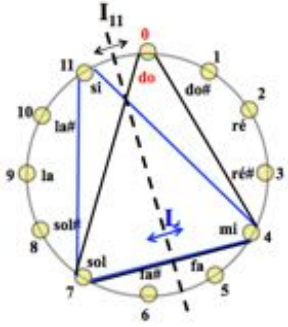




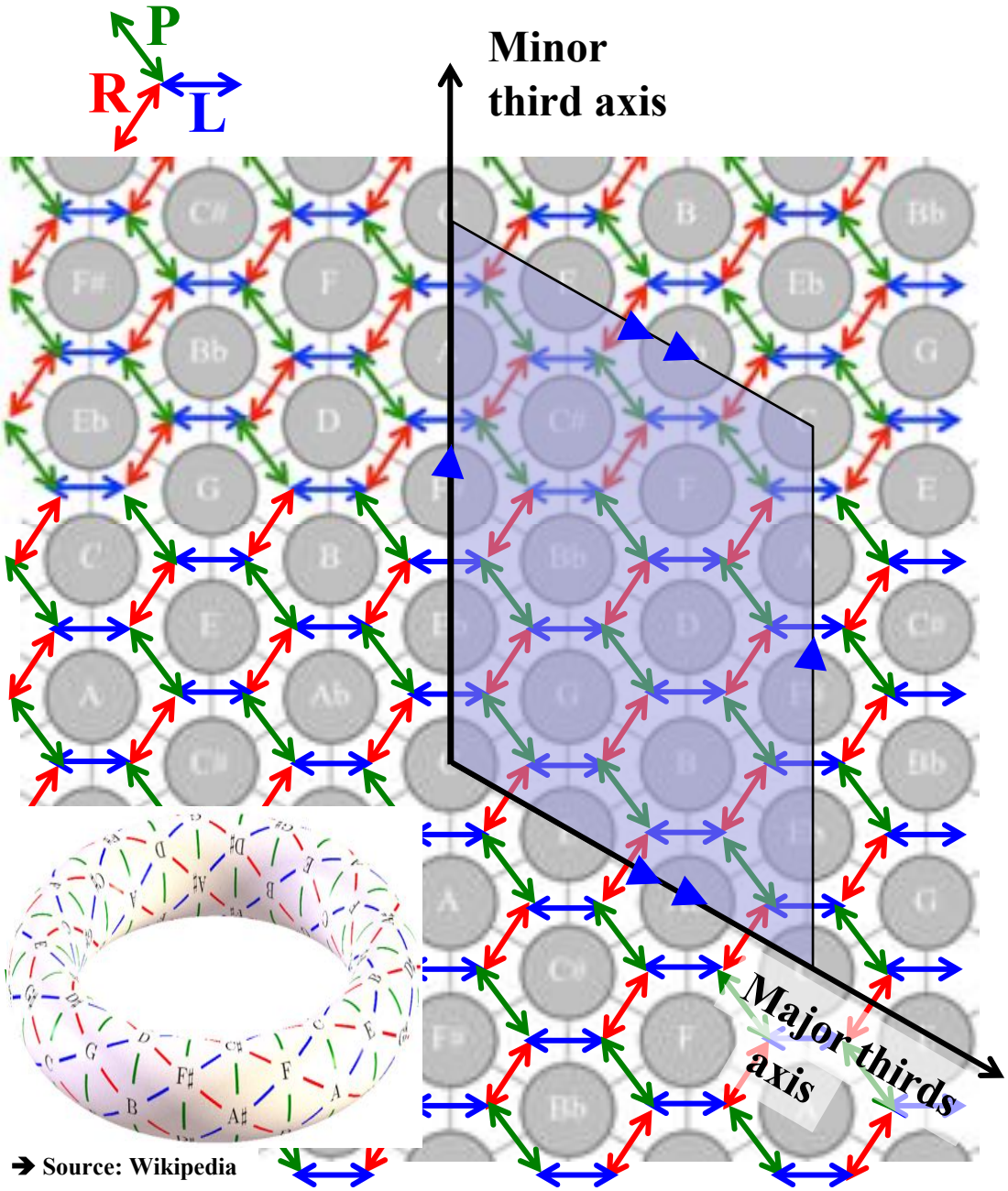


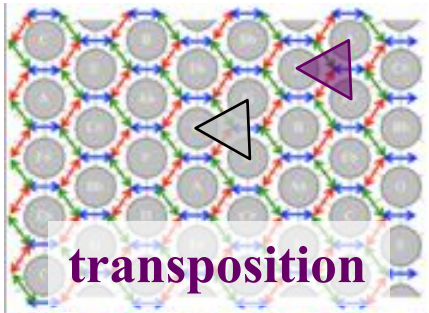


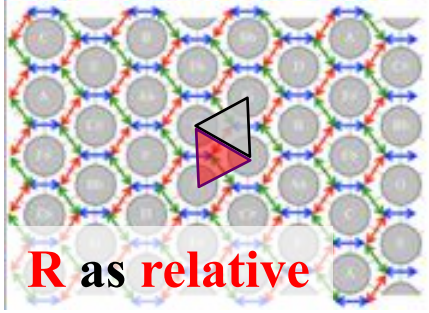


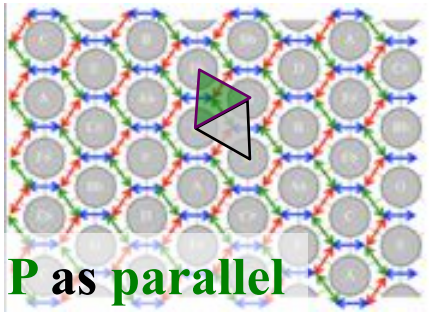


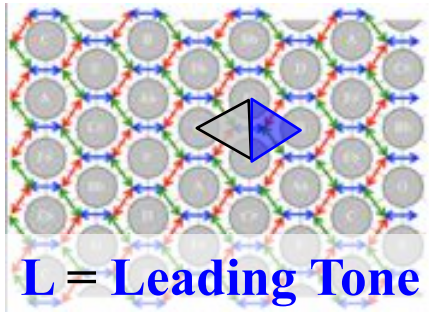
The *Tonnetz*, its symmetries and its topological structure

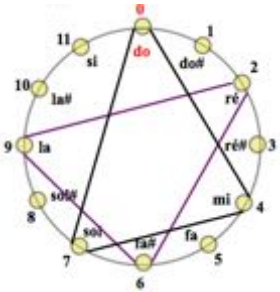


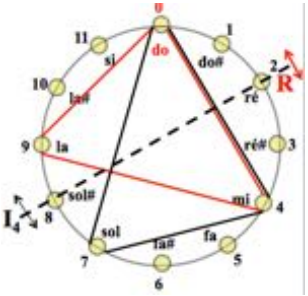


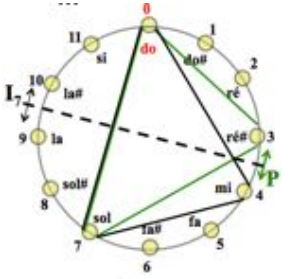


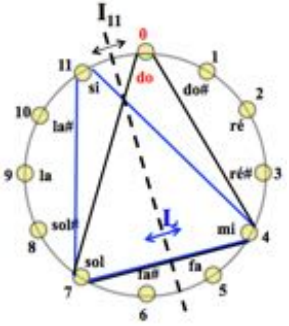








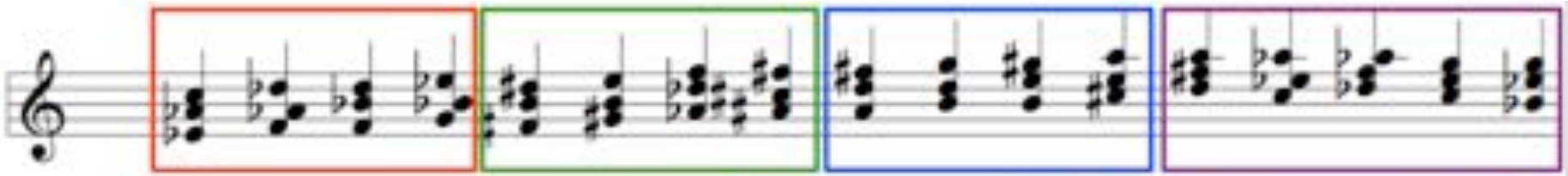


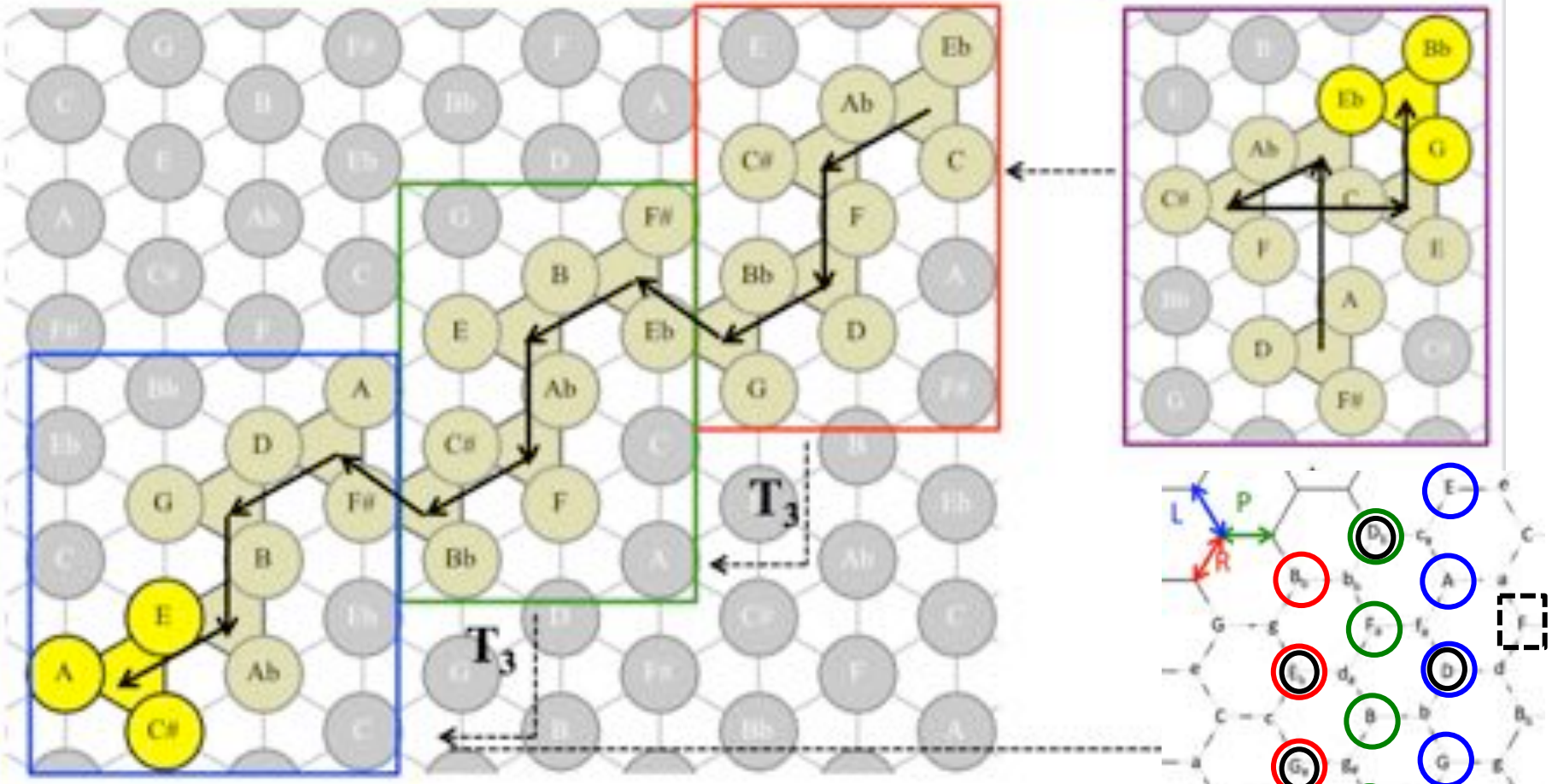


➔ Source: Wikipedia

Symmetries in Paolo Conte's *Madeleine*

La_b Re_b Si_b Mi_b Si Mi Re_b $Fa_\#$ Re Sol Mi La Re La_b Re_b Do Mi_b





Almost total covering of the major-chords space

Harmonic Progressions

In Paolo Conte

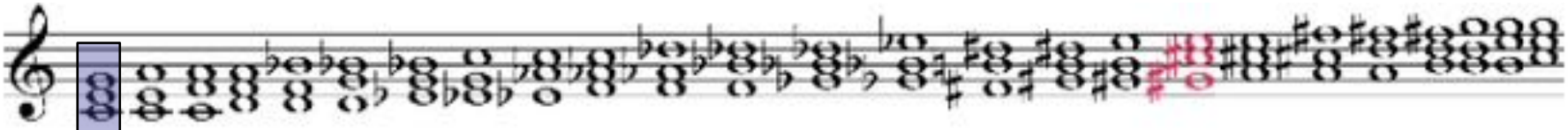
Madeleine

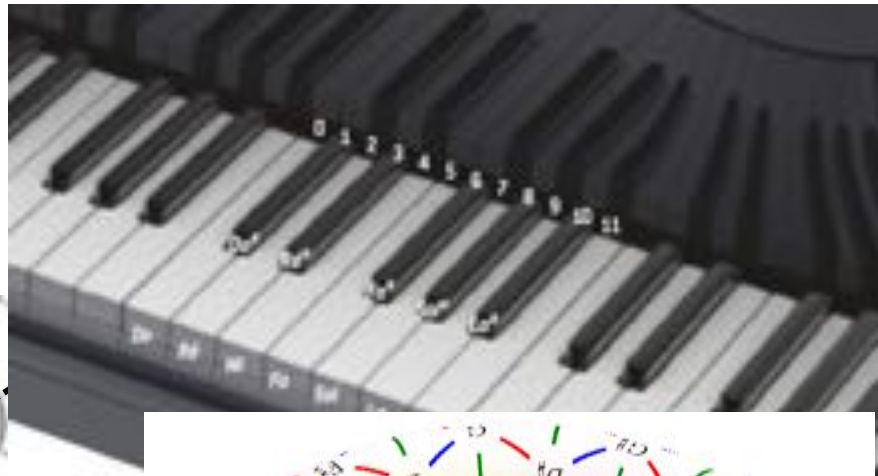
Gilles Baroin

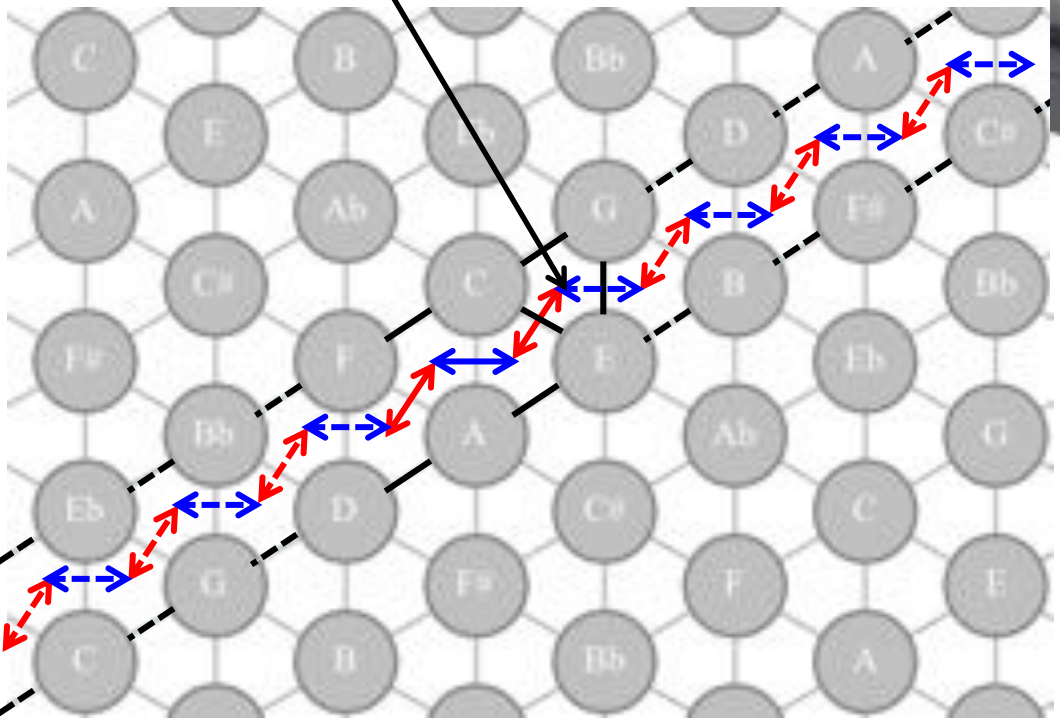
Supervision Moreno Andreatta
Modelisation Gilles Baroin 2016

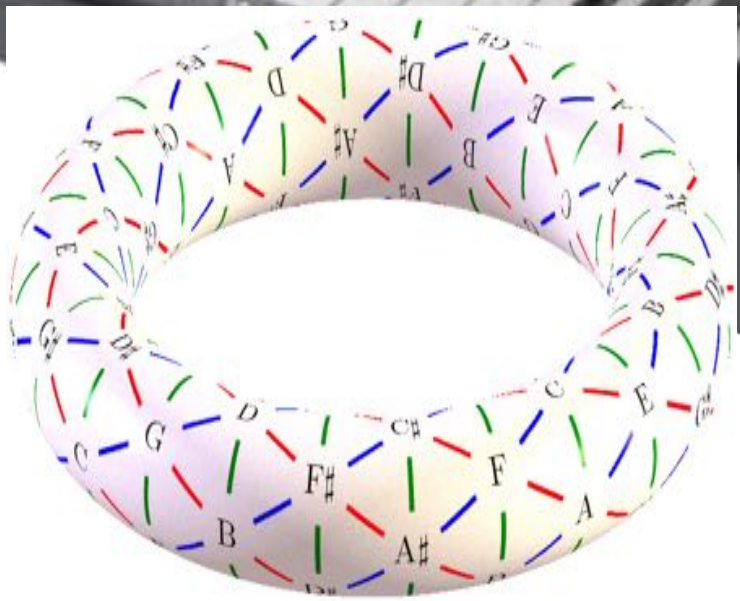
➔ **http://planetes.info/**

A zig-zag harmonic progression





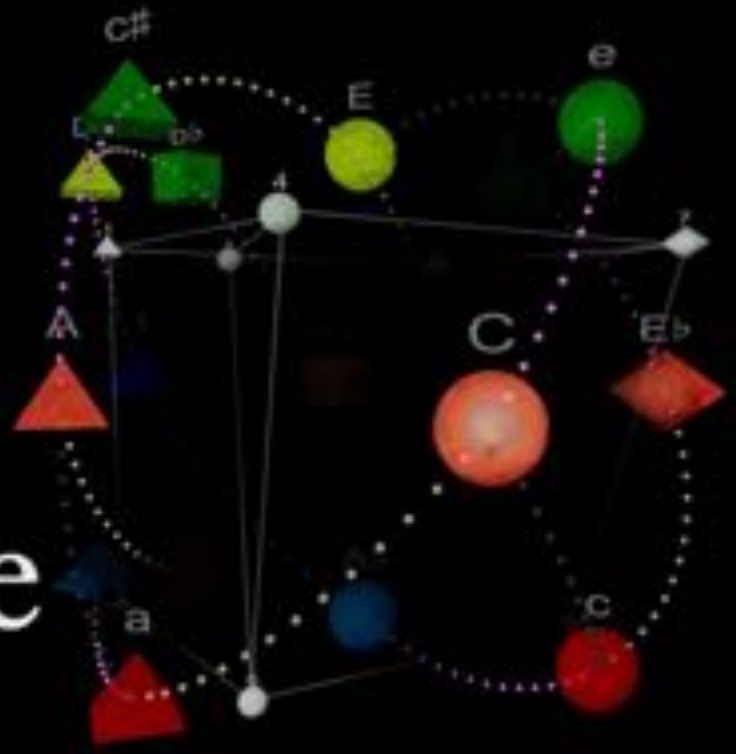




➔ Source : https://upload.wikimedia.org/wikipedia/commons/6/67/TonnetzTorus.gif

Beethoven and the Hypersphere

(and the Tonnetz)

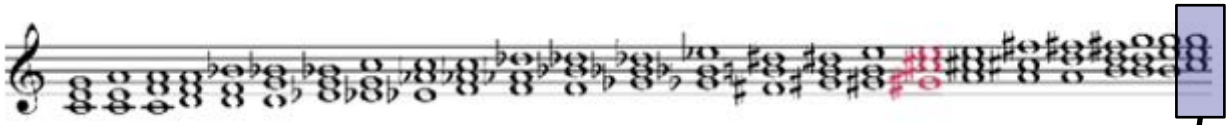


Gilles Baroin 2016
www.MatheMusic.net

Gilles Baroin

➔ **http://planetes.info/**

Reading Beethoven backwards



time

Le Blé en Herbe

(Polo/Moreno/Dieu)

Plonger comme un enfant, cheveux au vent
Sous l'océan du blé en herbe
Marée d'épis couleur d'amande
Qui tendent à caresser le ciel

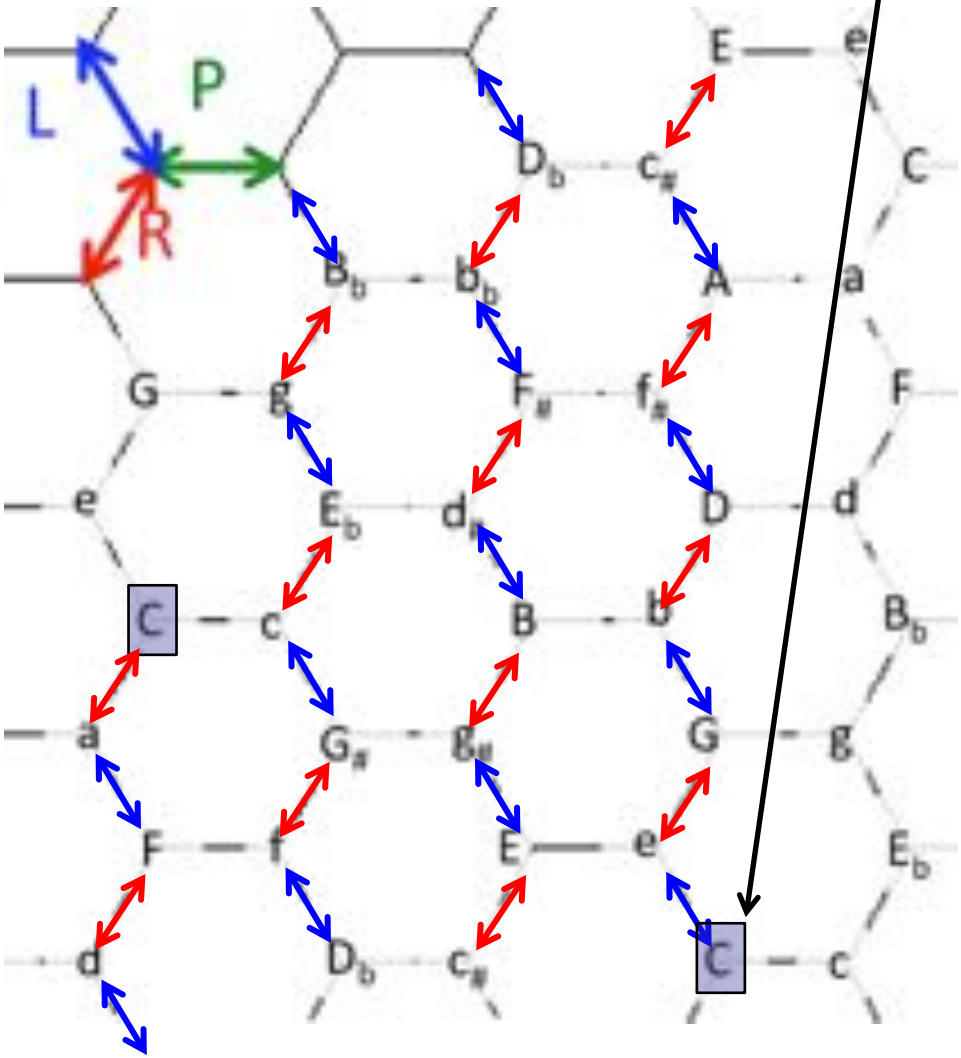
Algues tendres de mille plages
Frôlant le ventre des nuages
Cheveux de pluie, dos de poissons
Qui frissonnent à l'unisson

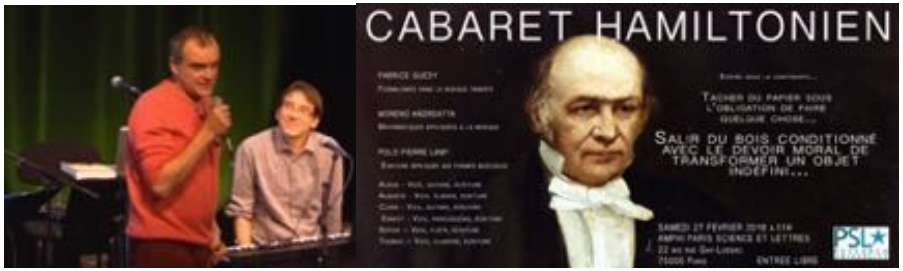
Suivre le bord des continents
Dans l'océan du blé en herbe
Pécher le corail du pavot
Dans le sang des coquelicots

Croiser matin dans l'herbe folle
Deux tourterelles qui s'envolent
Suivre les jeux des hirondelles
Sur le paysage éternel
Nager comme un enfant, cheveux au vent
Sous l'océan
Du blé en herbe

Marée de fruits au goût amer
Acide et salée comme la mer

Vers l'îlôt d'un petit village
Vers un château d'eau sur la plage
Quand tout s'éteint avant l'orage
Quand se lève le vent du large
Sur le blé vert



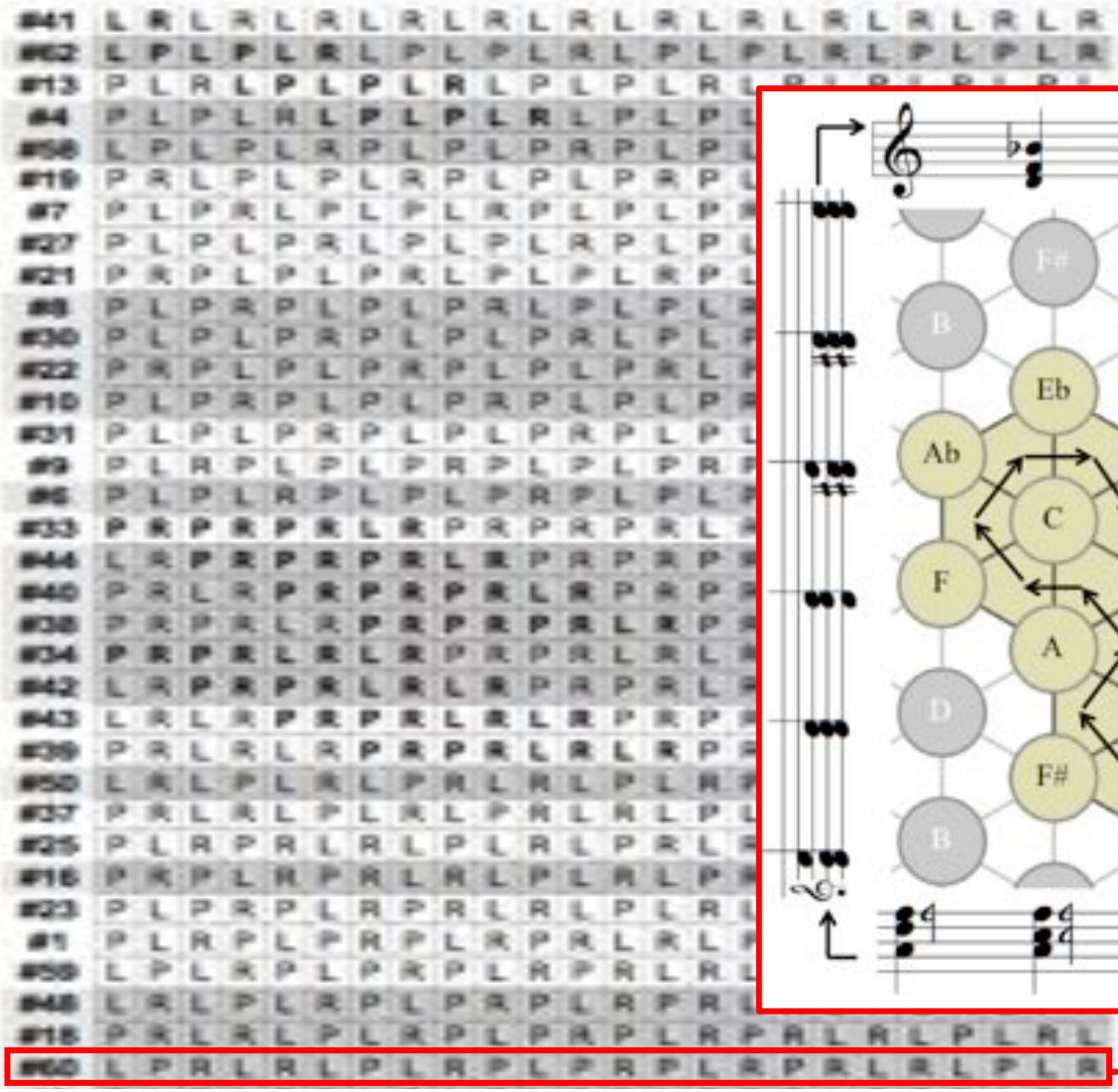


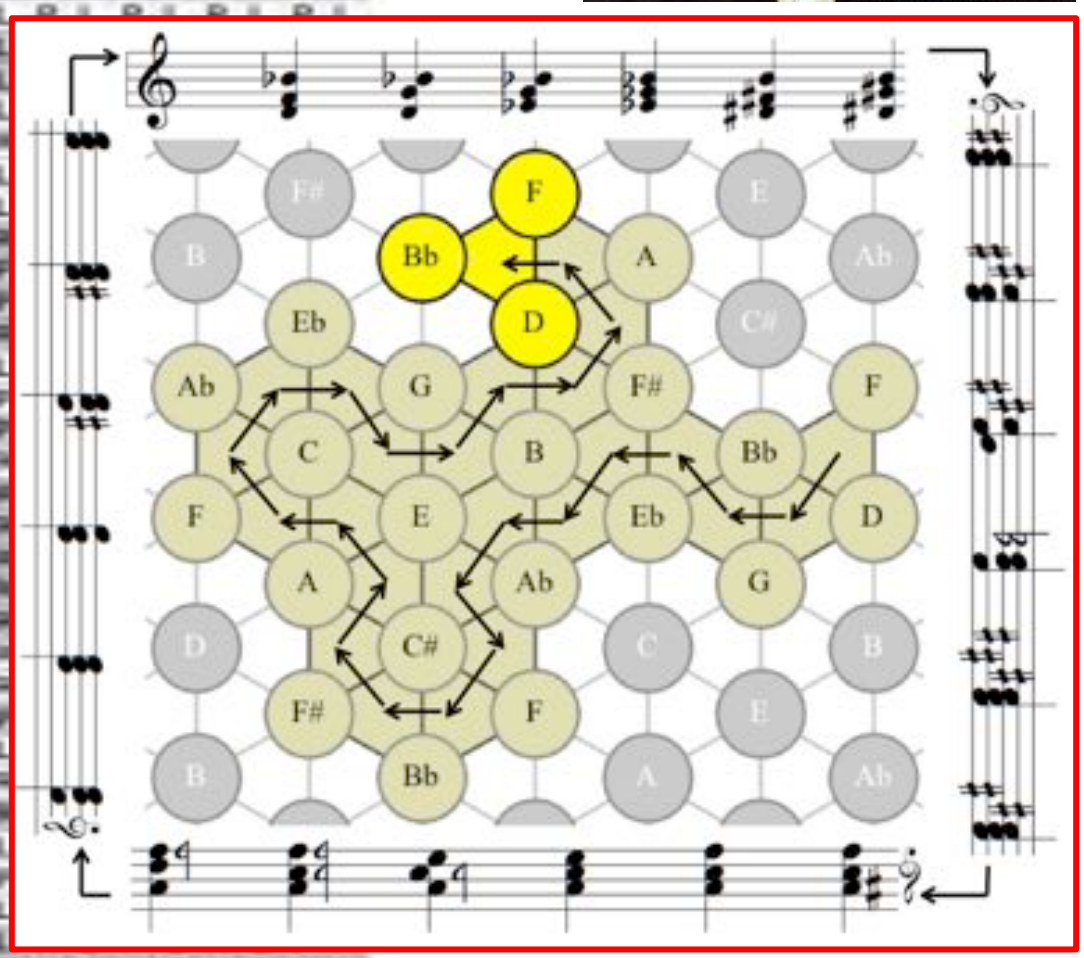
The collection of 124 Hamiltonian Cycles

ACTIONS Math'n'pop

Aprile (d'après Gabriele D'Annunzio)

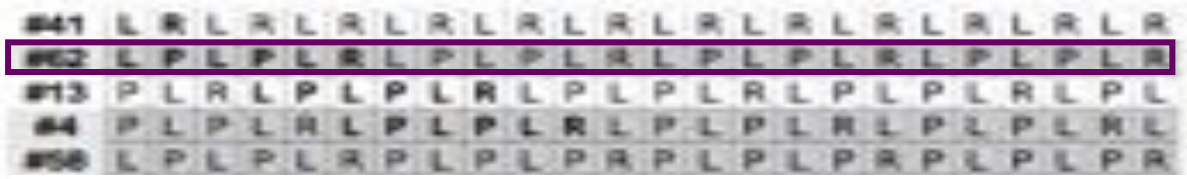
Tangente pour mieux comprendre le monde — L'aventure mathématique





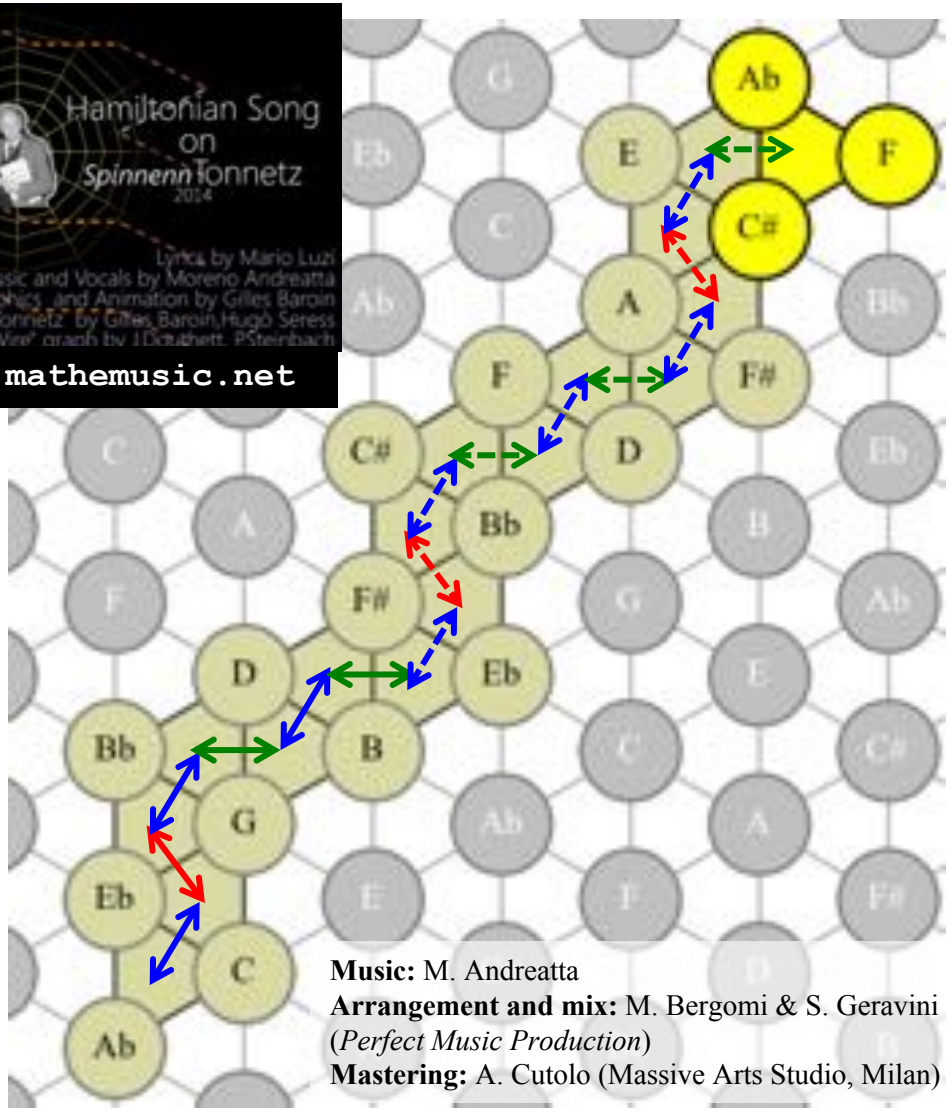
G. Albini & S. Antonini, University of Pavia, 2008

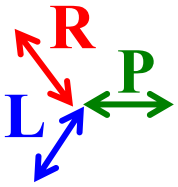
Hamiltonian Cycles with inner periodicities



L P L P L R …
P L P L R L …
L P L R L P …
P L R L P L …
L R L P L P …
R L P L P L …

min. 1'02"





Music: M. Andreatta
Arrangement and mix: M. Bergomi & S. Geravini (*Perfect Music Production*)
Mastering: A. Cutolo (Massive Arts Studio, Milan)

La sera non è più la tua canzone
(Mario Luzi, 1945, in *Poesie sparse*)

La sera non è più la tua canzone,
è questa roccia d'ombra traforata
dai lumi e dalle voci senza fine,
la quiete d'una cosa già pensata.

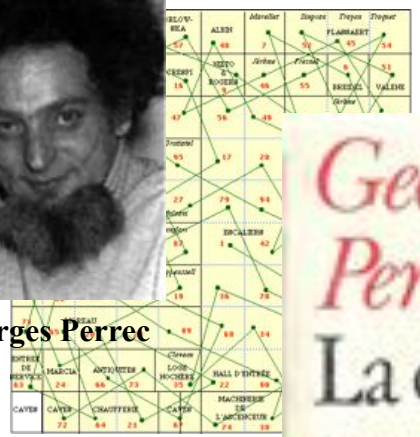
Ah questa luce viva e chiara viene
solo da te, sei tu così vicina
al vero d'una cosa conosciuta,
per nome hai una parola ch'è passata
nell'intimo del cuore e s'è perduta.

Caduto è più che un segno della vita,
riposi, dal viaggio sei tornata
dentro di te, sei scesa in questa pura
sostanza così tua, così romita
nel silenzio dell'essere, (compiuta).

L'aria tace ed il tempo dietro a te
si leva come un'arida montagna
dove vaga il tuo spirito e si perde,
un vento raro scivola e ristagna.

The use of constraints in arts

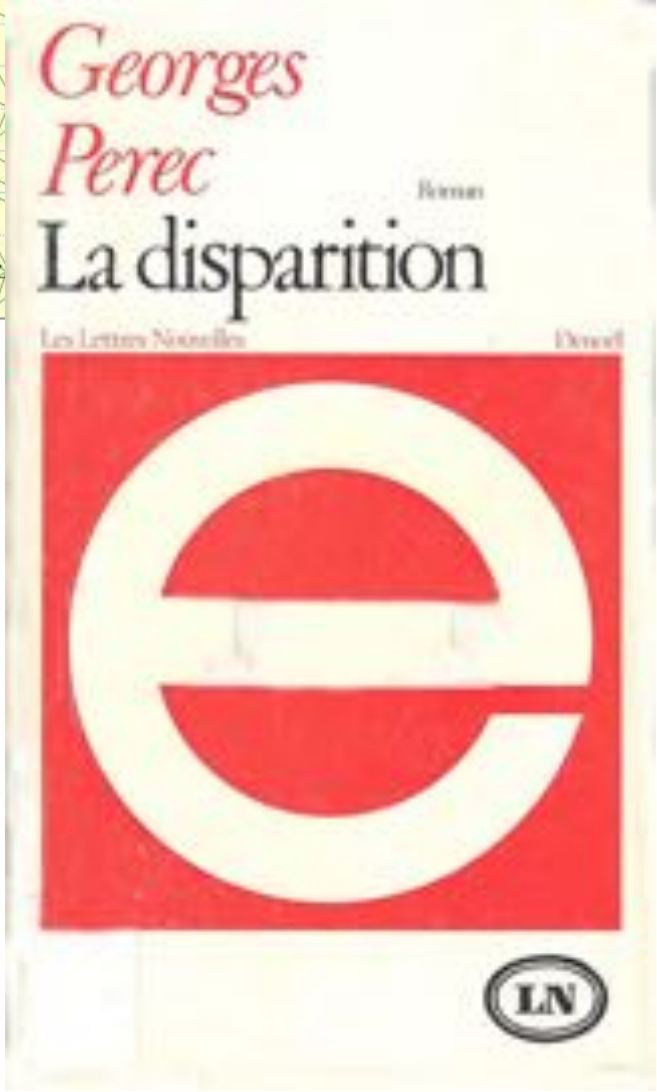
Cent mille milliards de poèmes, 1961



Georges Perrec

La vie mode d'emploi,

OuLiPo (Ouvroir de Littérature Potentielle)

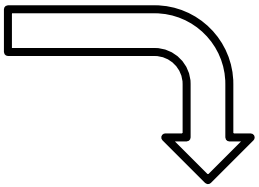


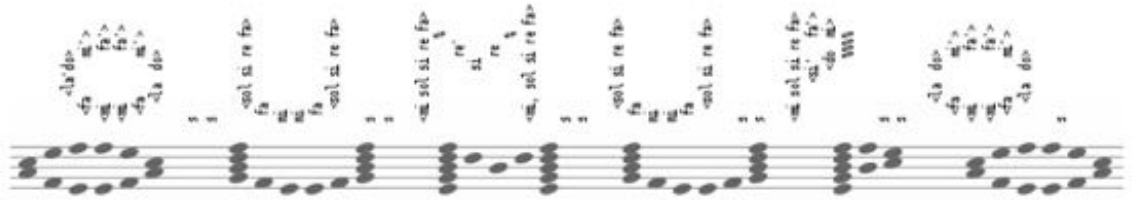
Raymond Queneau

Italo Calvino

Il castello dei destini incrociati, 1969

From the OuLiPo to the OuMuPo (Ouvroir de Musique Potentielle)





Valentin Villenave

Mike Solomon

Jean-François Piette

Martin Granger

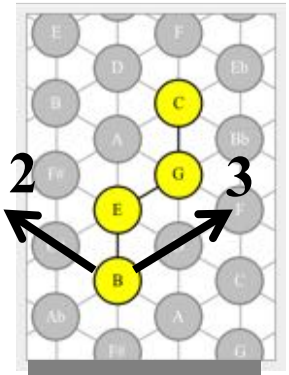
Joseph Boisseau

Moreno Andreatta

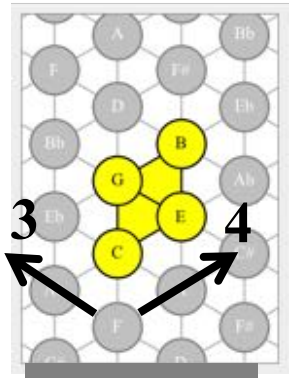
Tom Johnson

➔ http://oumupo.org/

The geometric character of musical logic



T[2,3,7]



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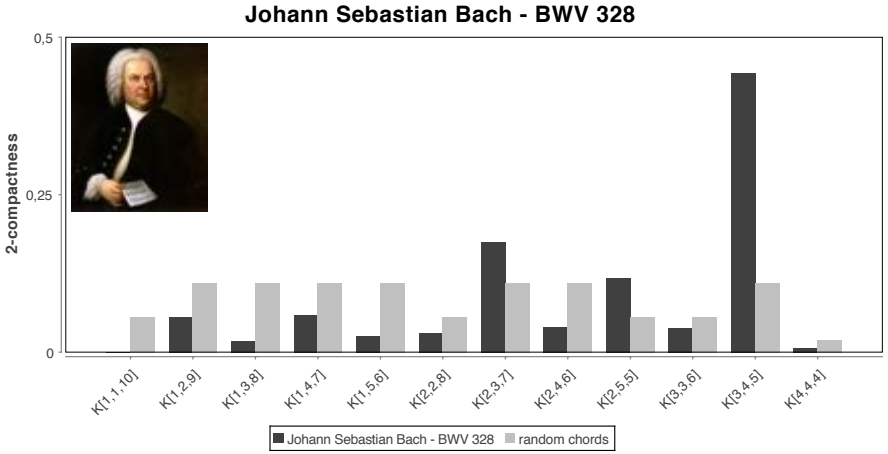
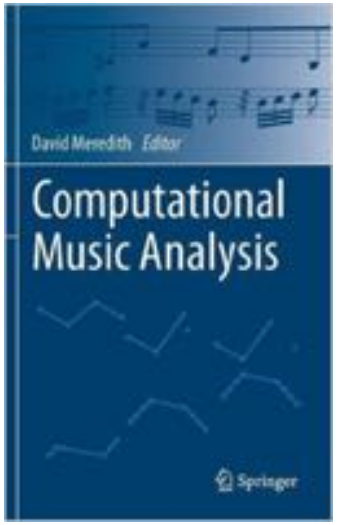
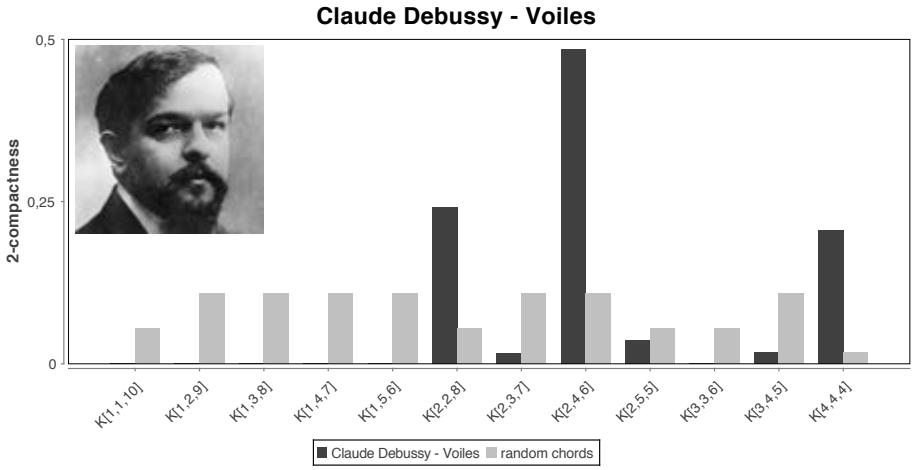
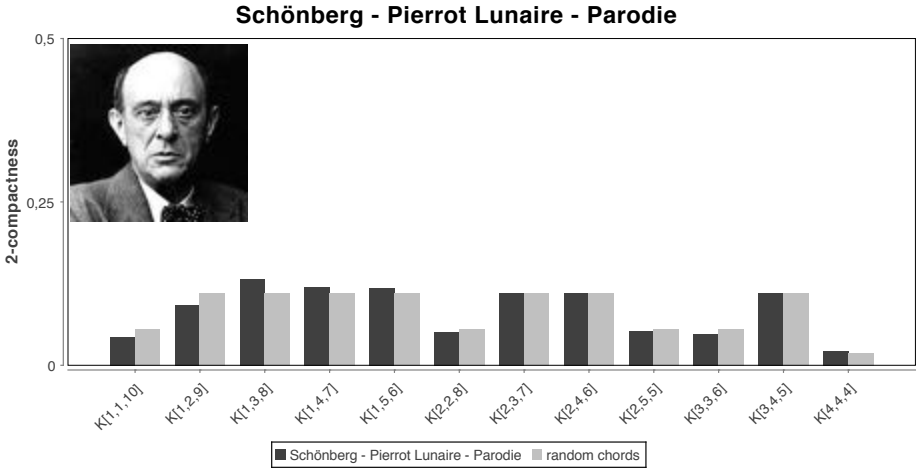




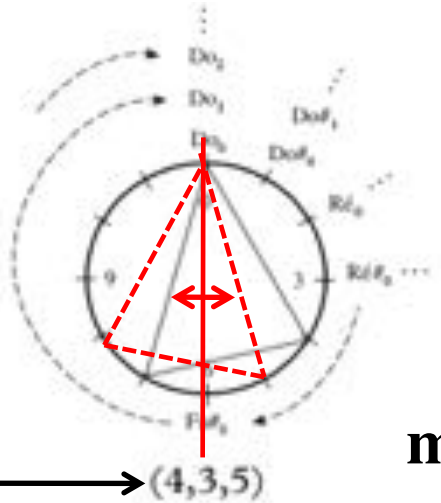




The *Tonnetz* as a simplicial complex

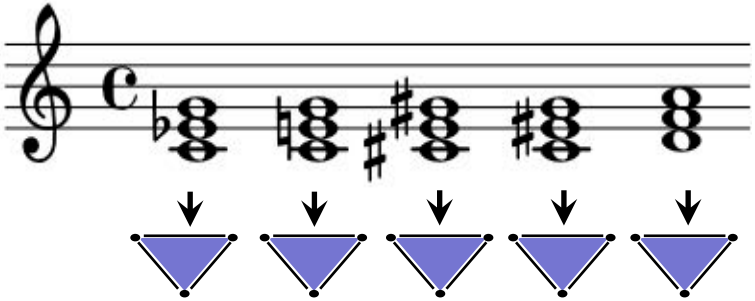
L. Bigo, *Représentation symboliques musicales et calcul spatial*, PhD, Ircam / LACL, 2013

- Assembling chords related by some equivalence relation
  - Transposition/inversion: Dihedral group action on $P(Z_n)$

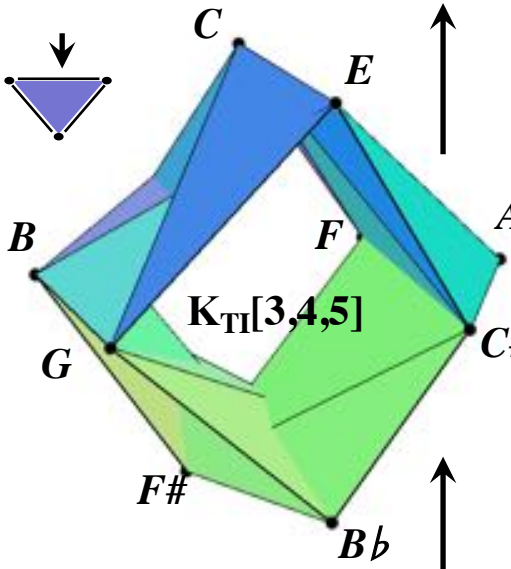


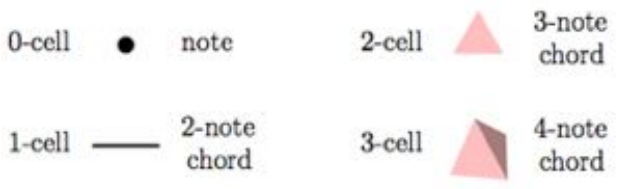
Intervallic structure

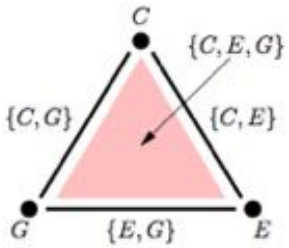
(4,3,5)

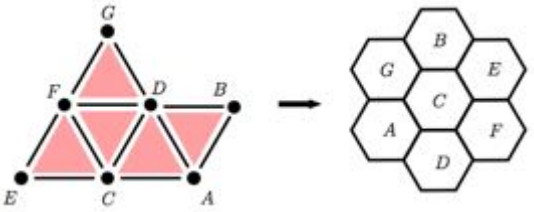


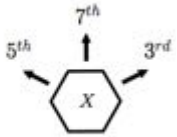
major/minor triads



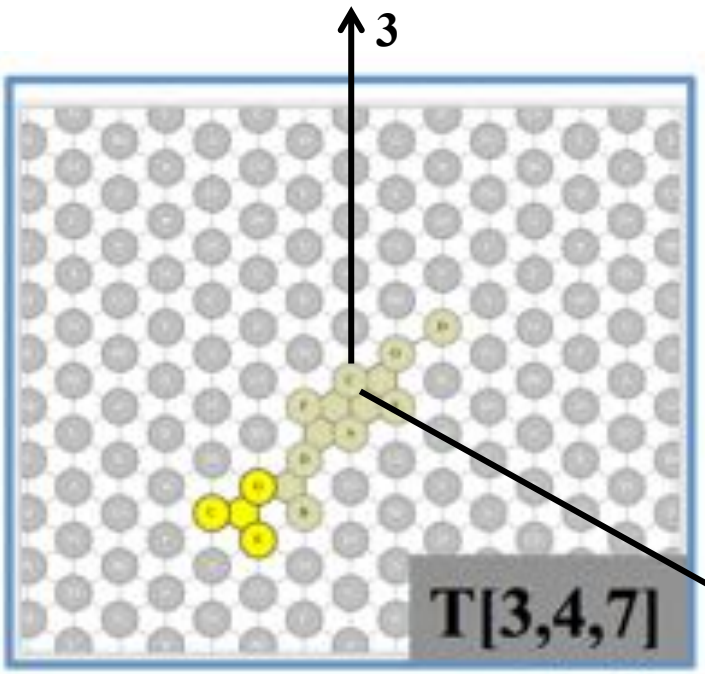


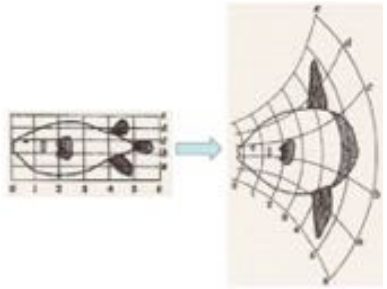


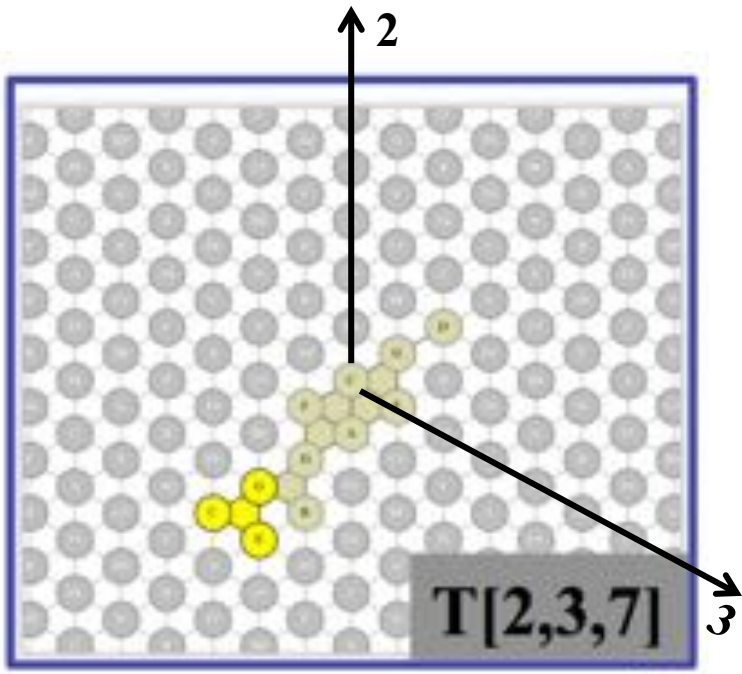


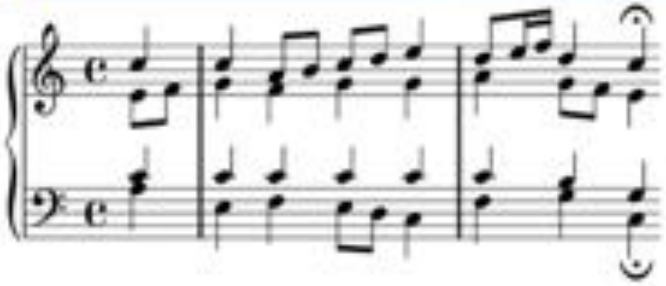


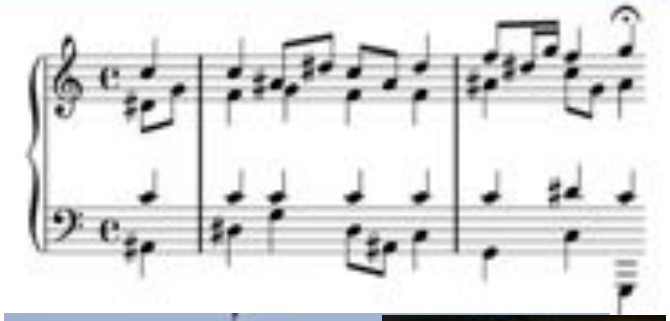
The musical style...is the space!



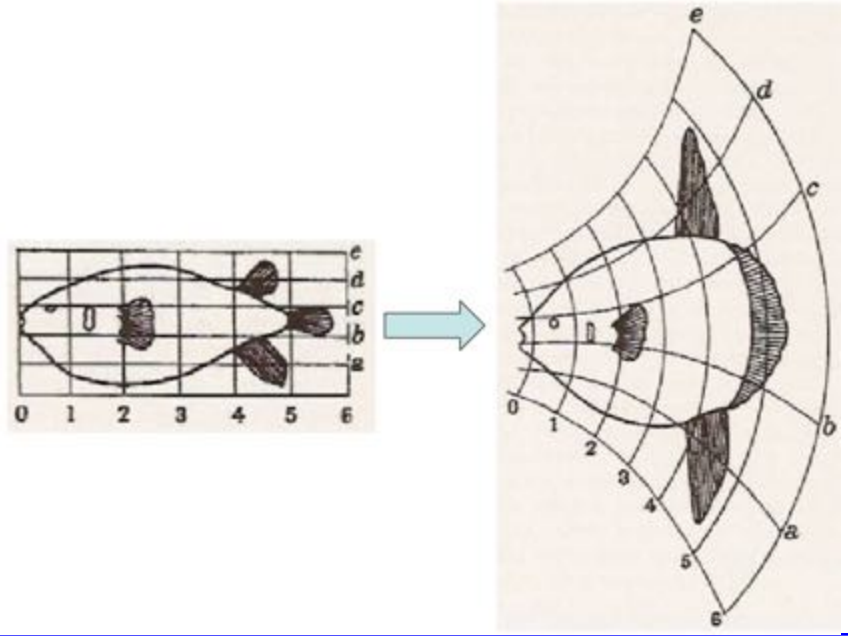




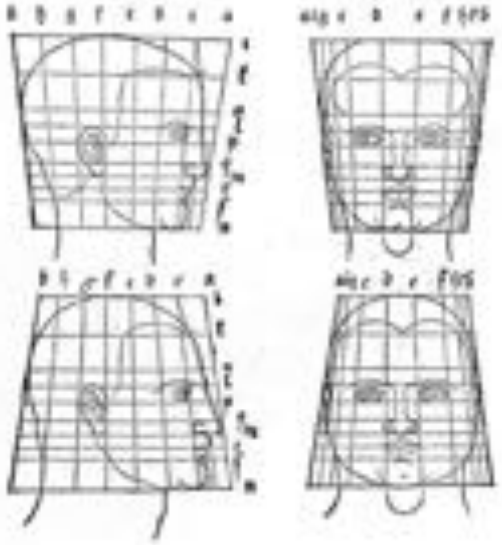




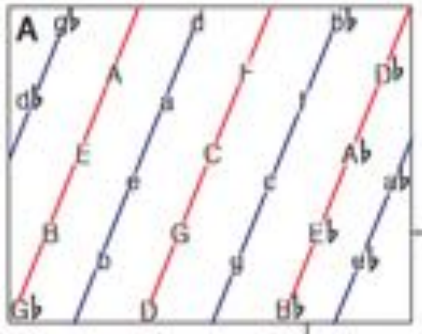
The morphological *vs* the mathematical genealogy of the structuralism



"[The notion of **transformation**] comes from a work which played for me a very important role and which I have read during the war in the United States : *On Growth and Form*, in two volumes, by **D'Arcy Wentworth Thompson**, originally published in 1917. The author (...) proposes an interpretation of the visible transformations between the species (animals and vegetables) within a same gender. This was fascinating, in particular because I was quickly realizing that this perspective had a long tradition: behind Thompson, there was **Goethe's** botany and behind Goethe, **Albert Dŭrer** with his *Treatise of human proportions*" (Lévi-Strauss, conversation with Eribon, 1988).



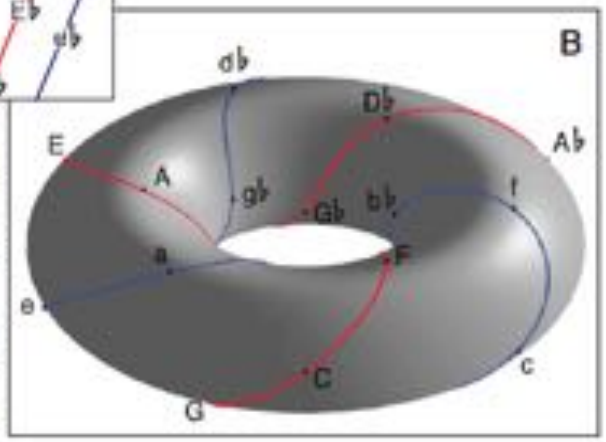
Neurosciences and Tonnetz



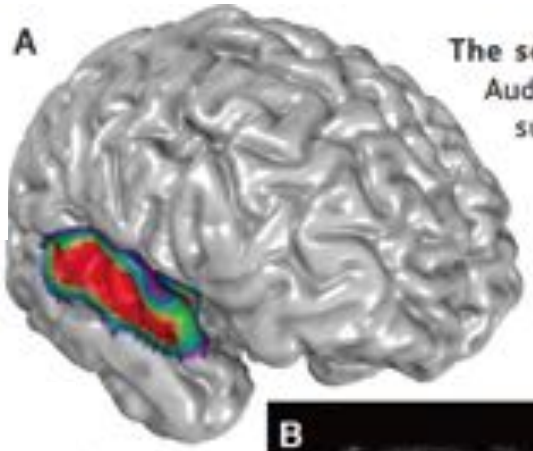
PERSPECTIVES: NEUROSCIENCE

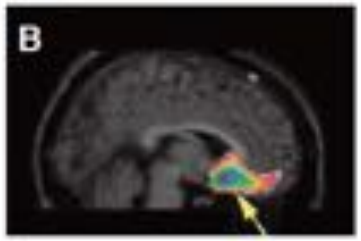
Mental Models and Musical Minds

Robert J. Zatorre and Carol L. Krumhansl

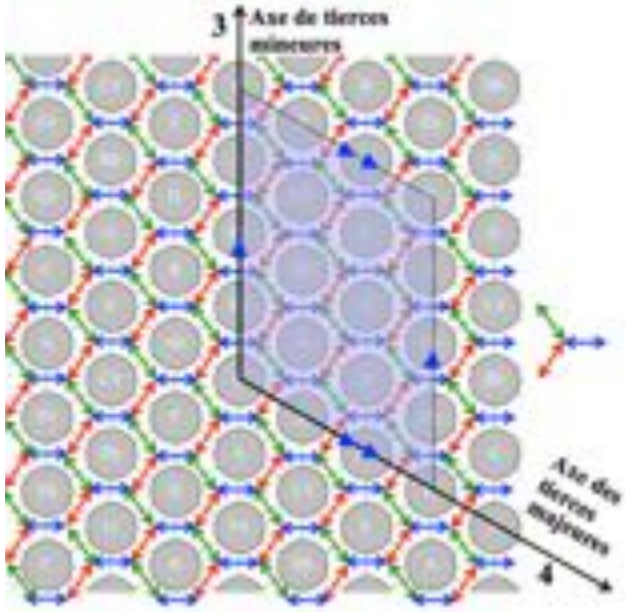


Mental key maps. (A) Unfolded version of the key map, with opposite edges to be considered matched. There is one circle of fifths for major keys (red) and one for minor keys (blue), each wrapping the torus three times. In this way, every major key is flanked by its relative minor on one side (for example, C major and a minor) and its parallel minor on the other (for example, C major and c minor). (B) Musical keys as points on the surface of a torus.





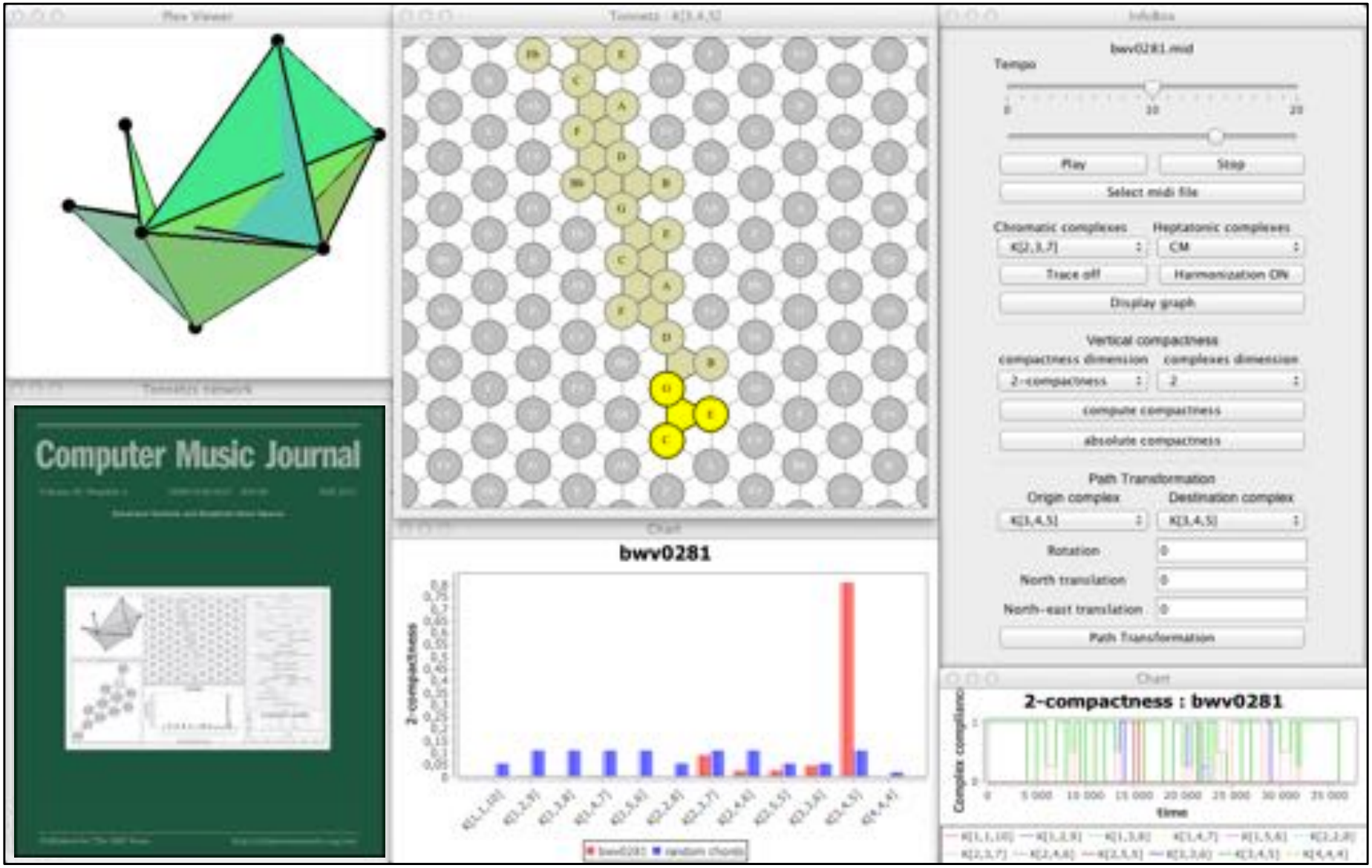
The sensation of music. (A) Auditory cortical areas in the superior temporal gyrus that respond to musical stimuli. Regions that are most strongly activated are shown in red. (B) Metabolic activity in the ventromedial region of the frontal lobe increases as a tonal stimulus becomes more consonant.



Acotto E. et M. Andreatta (2012), « Between Mind and Mathematics. Different Kinds of Computational Representations of Music », *Mathematics and Social Sciences*, n° 199, 2012(3), p. 9-26.

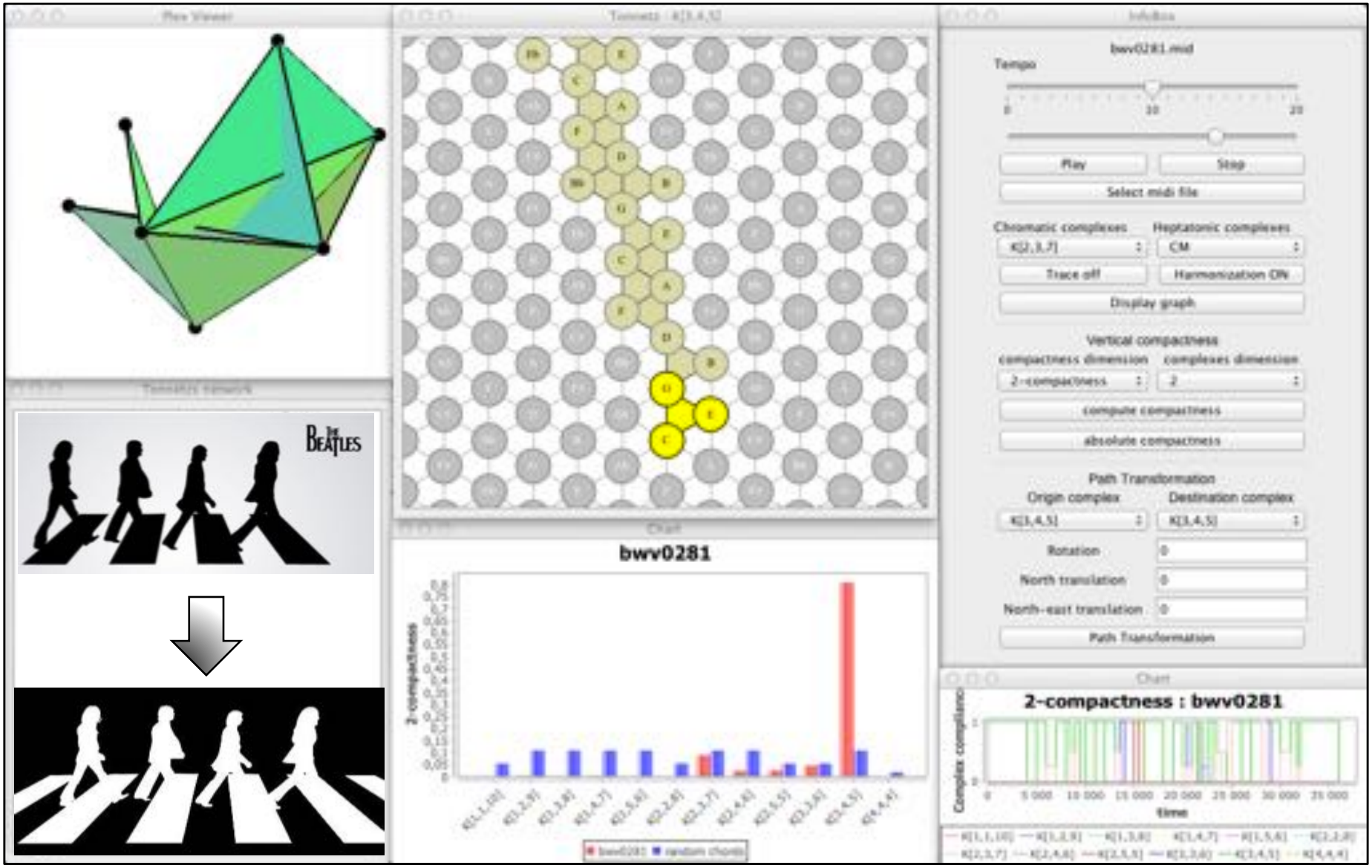
EMPG European Mathematical Psychology Group

Spatial music analysis via Hexachord



➔**http://www.lacl.fr/~lbigo/hexachord**

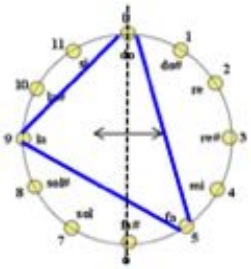
Keeping the space...but changing the trajectory!



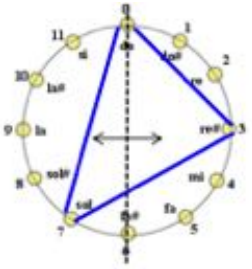
➔http://www.lacl.fr/~lbigo/hexachord

Keeping the space...but changing the trajectory!

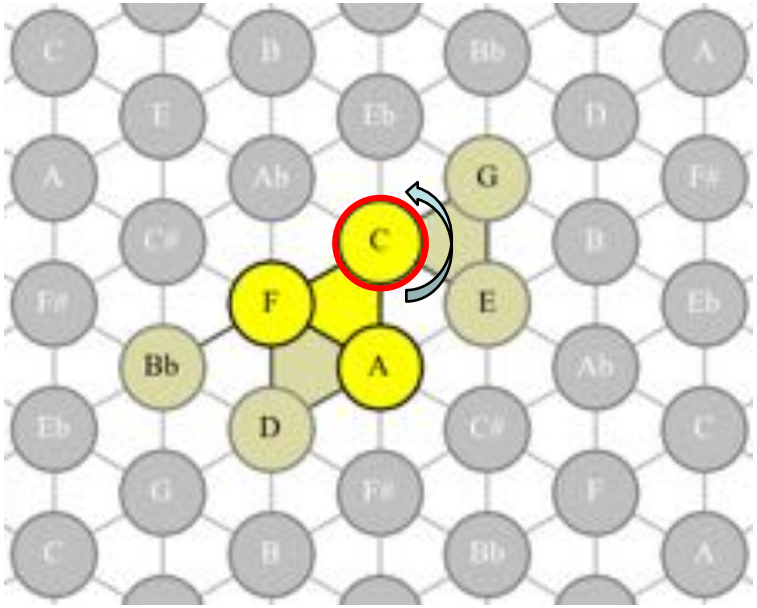
F major chord



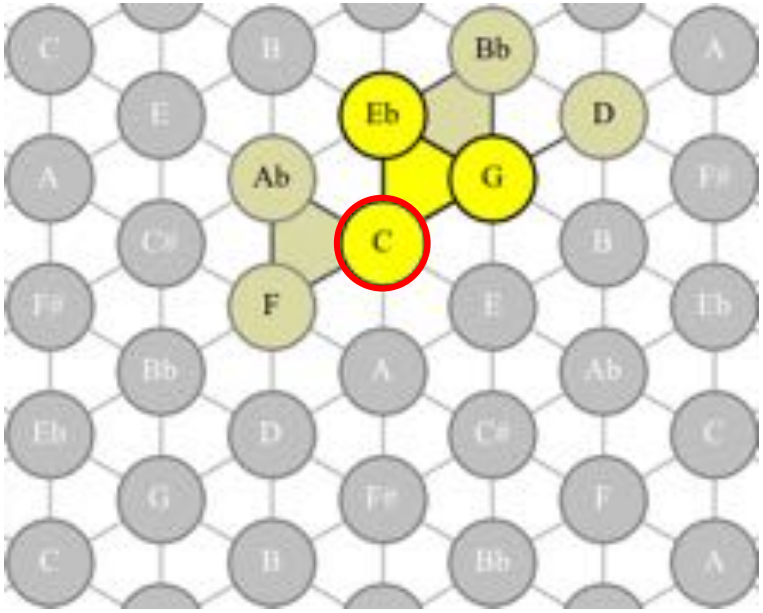
inversion



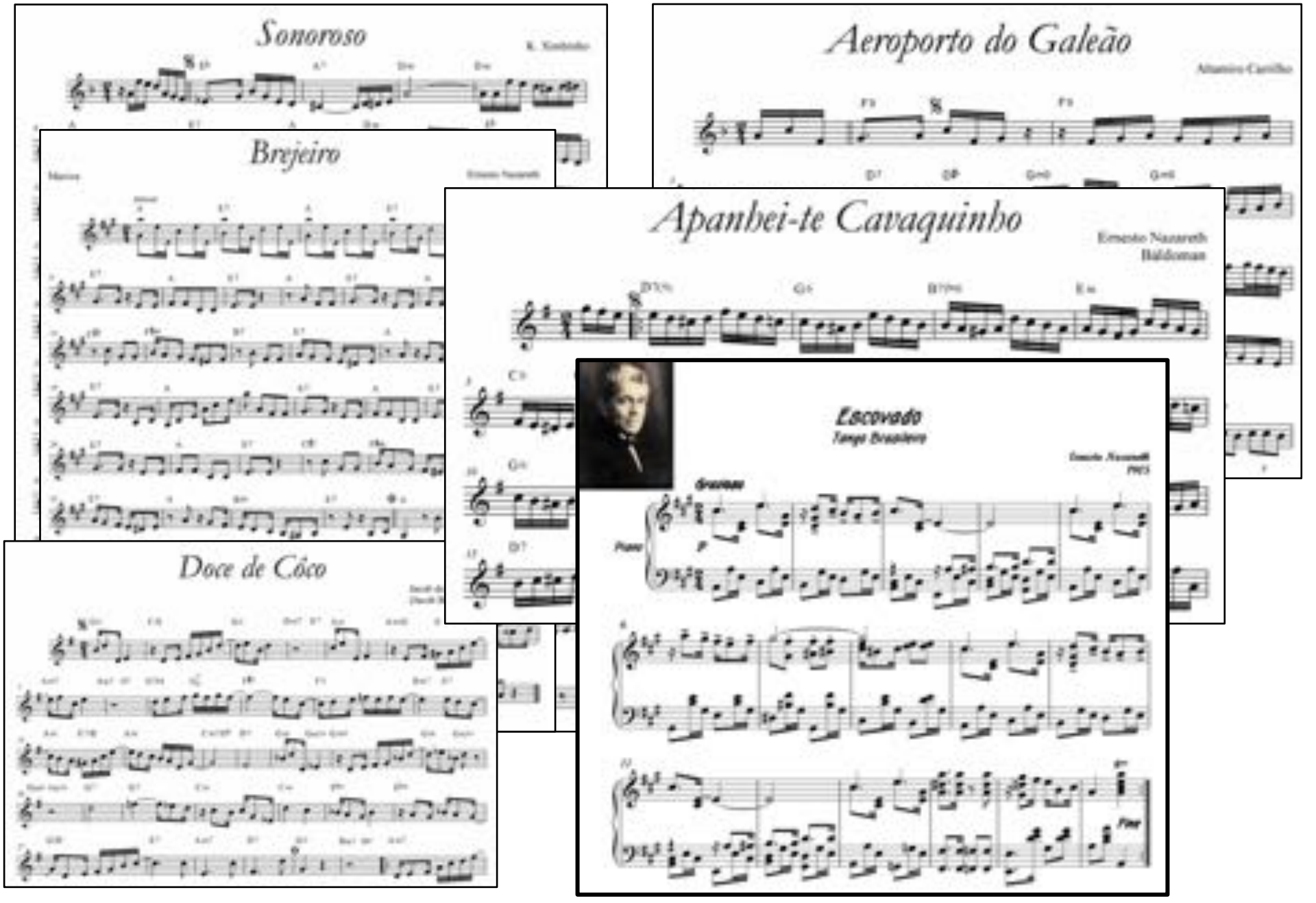
C minor chord



Rotation
(autour du *do*)



Rotational symmetry applied to traditional Brazilian music




Thank you for your attention!