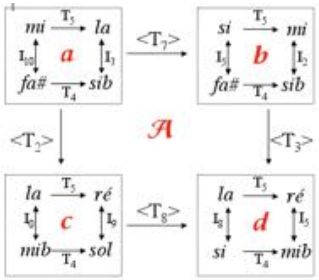
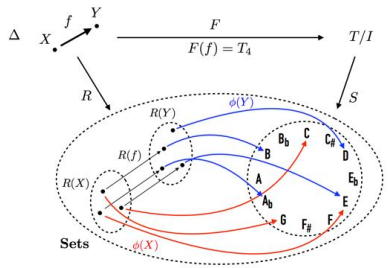


DIAGRAMS EDINBURGH 18



Diagrammatic approaches in computational musicology: some theoretical and philosophical aspects

Moreno Andreatta[1], Carlos Agon[2], Alexandre Popoff[3], Andrée Ehresmann[4]

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ircam Centre Pompidou

SORBONNE UNIVERSITÉ

cnrs

IRMA Institut de Recherche Mathématique Avancée

USIAS Université de Strasbourg Institut d'Études Avancées

UNIVERSITÉ de Picardie Jules Verne

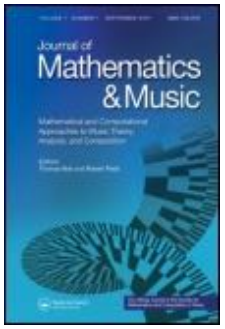
The *Society for Mathematics and Computation in Music*

Conferences:

- 2007 Technische Universität (Berlin, Allemagne)
- 2009 Yale University (New Haven, USA)
- 2011 Ircam (Paris, France)
- 2013 McGill University (Canada)
- 2015 Queen Mary University (Londres)
- 2017 UNAM (Mexico City)
- 2019 Universidad Politécnica de Madrid

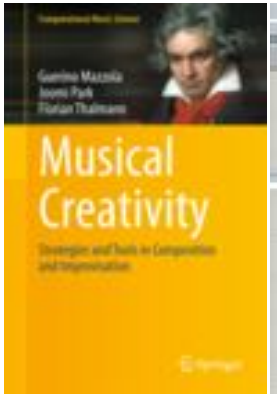
Official Journal:

- *Journal of Mathematics and Music*, Taylor & Francis (Editors : Th. Fiore & C. Callender),



Books Series:

- *Computational Music Sciences* Series, Springer (G. Mazzola & M. Andreatta eds. – 12 books published (since 2009)
- Collection *Musique/Sciences*, Ircam-Delatour France (J.-M. Bardez & M. Andreatta dir. – 16 books published (since 2006)

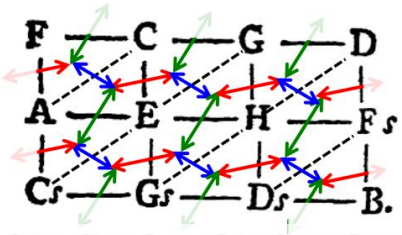


Mathematics Subject Classification : 00A65 Mathematics and music

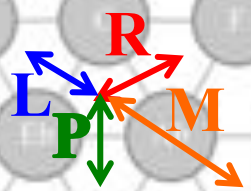
➔ **Towards a Diagrams and/or ICCS / MCM joint conference ?**

A diagrammatic listening experiment

"Take a bow" by Muse (*Black Holes and Revelations*, 2006)



Leonhard Euler's *Speculum Musicum*

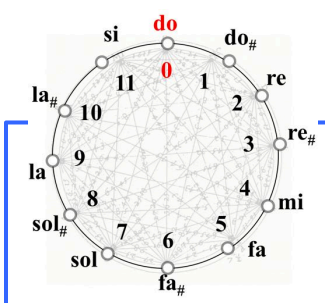


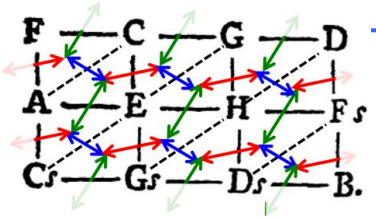
➔http://www.lacl.fr/~lbigo/hexachord

A focus on Category Theory and MM/FCA

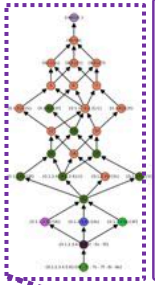
Signal-based Music Information Retrieval

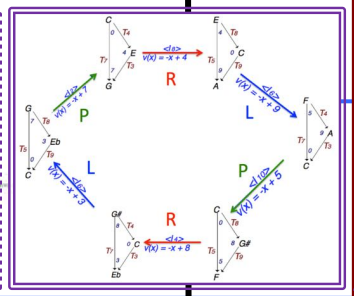
Algebraic models



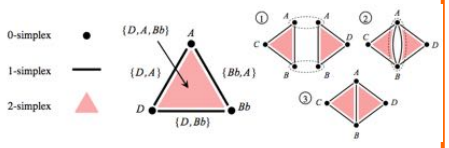


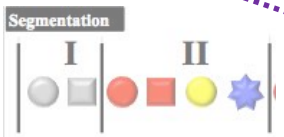
Topological models





Mathematical models

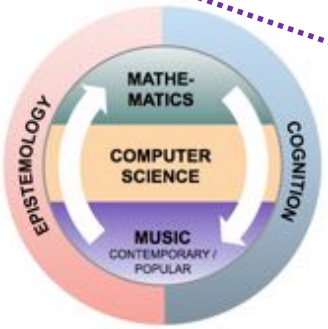




Semiotic models

Computational models

Cognitive models



DIAGRAMS 18
EDINBURGH

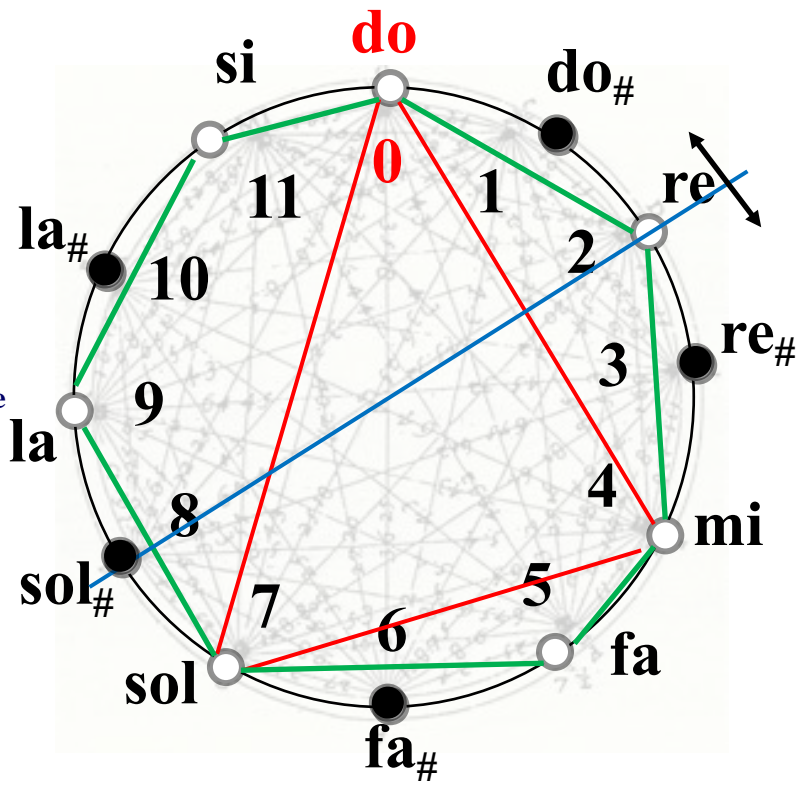
ICCS 18
EDINBURGH

11:00-12:30
Room: 2.D.05

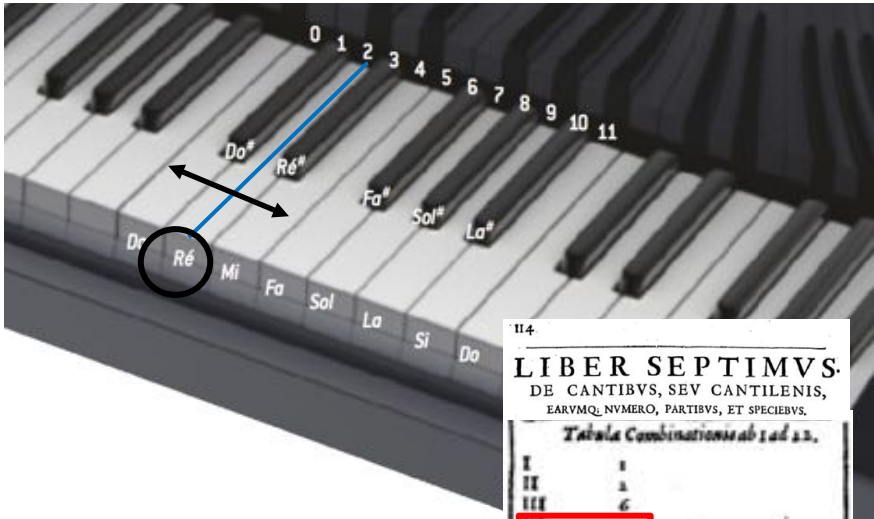
Symbolic Structural Music Information Research

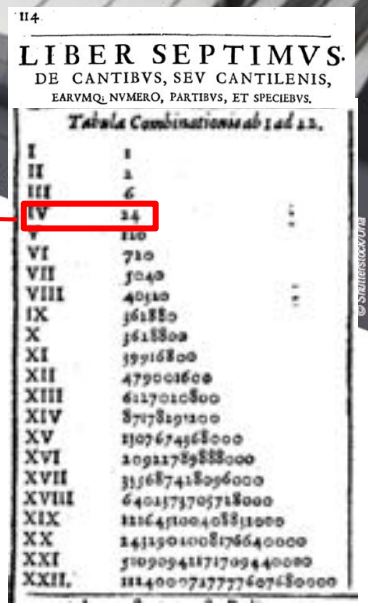
The circular representation of the pitch space

Marin Mersenne



Harmonicorum Libri XII, 1648




114.

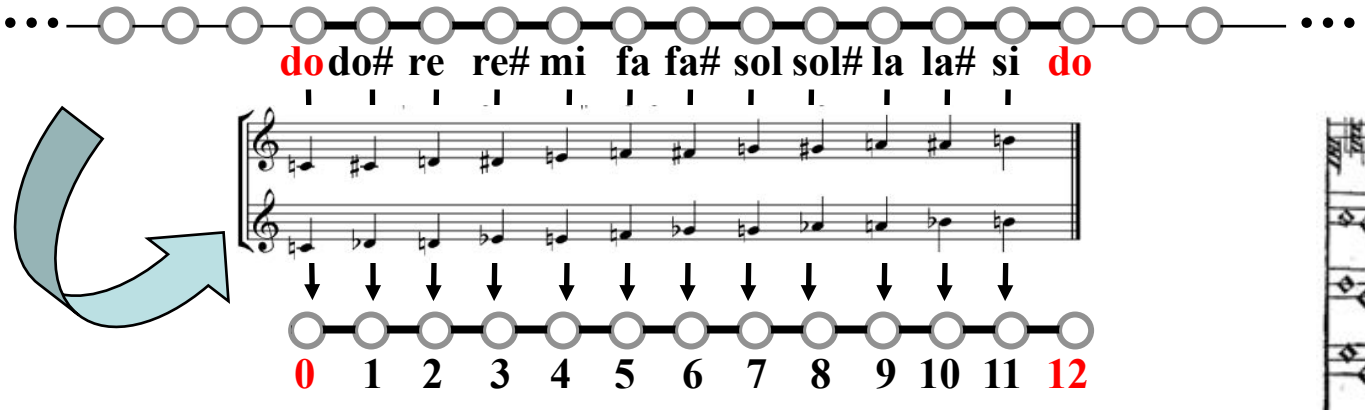
LIBER SEPTIMVS.

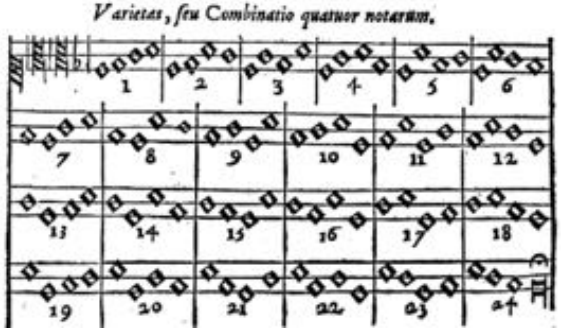
DE CANTIBVS, SEV CANTILENIS,

EARVMQ; NVMERO, PARTIBVS, ET SPECIEBVS.

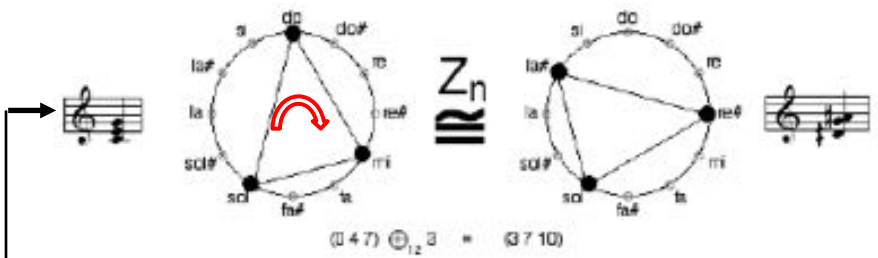
Tabula Combinationis ab 1 ad 12.

I	1
II	2
III	6
IV	24
V	120
VI	720
VII	5040
VIII	40320
IX	362880
X	3628800
XI	39916800
XII	479001600
XIII	6227020800
XIV	87178291200
XV	1307674368000
XVI	20922789888000
XVII	355687428096000
XVIII	6402373705728000
XIX	121645100408832000
XX	2432902008176640000
XXI	51090942171709440000
XXII.	1124000727777607680000





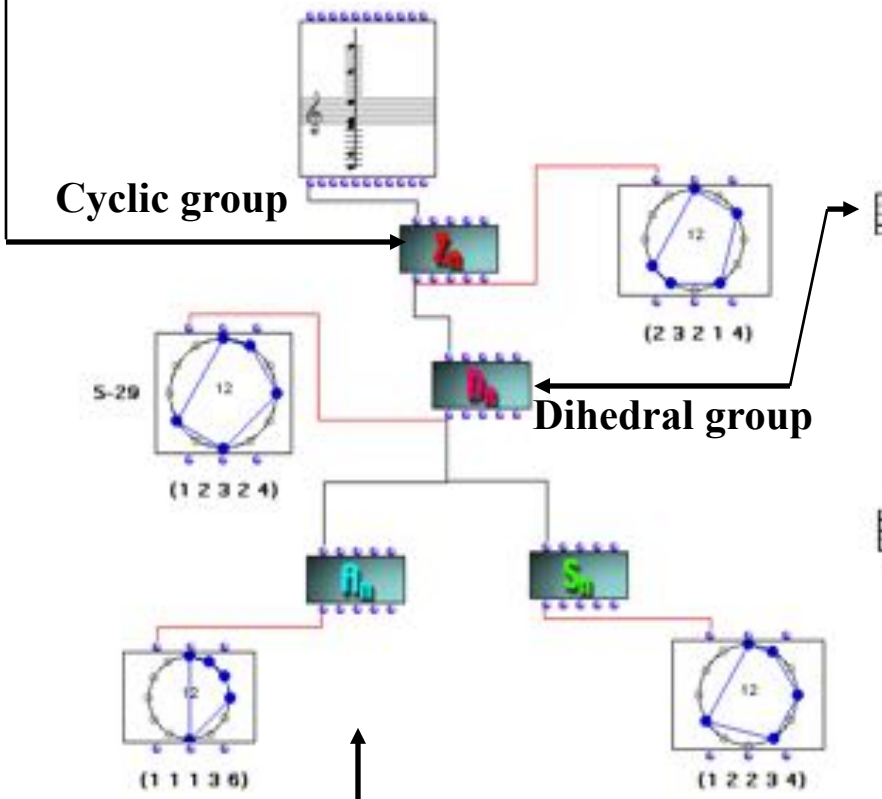
Equivalence classes of musical structures (up to a group action)

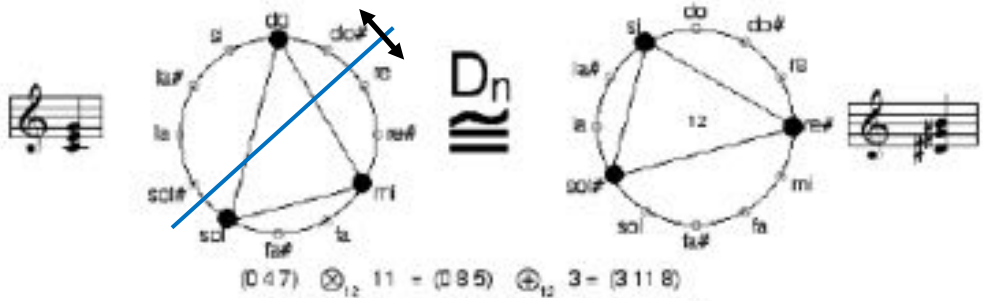


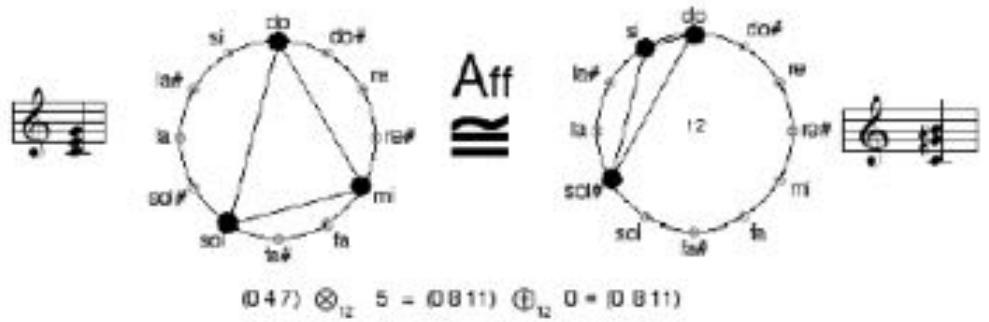
$\mathbf{Z_{12}} = < T_k \mid (T_k)^{12} = T_0 >$ where $T_k(x)=x+1$

$\mathbf{D_{12}} = < T_k , I \mid (T_k)^{12} = I^2 = T_0 , ITI=I(IT)^{-1} >$
where $I(x)=-x$

$\mathbf{Aff} = \{ f \mid f(x)=ax+b,\ a \in (\mathbf{Z_{12}})^*,\ b \in \mathbf{Z_{12}} \}$



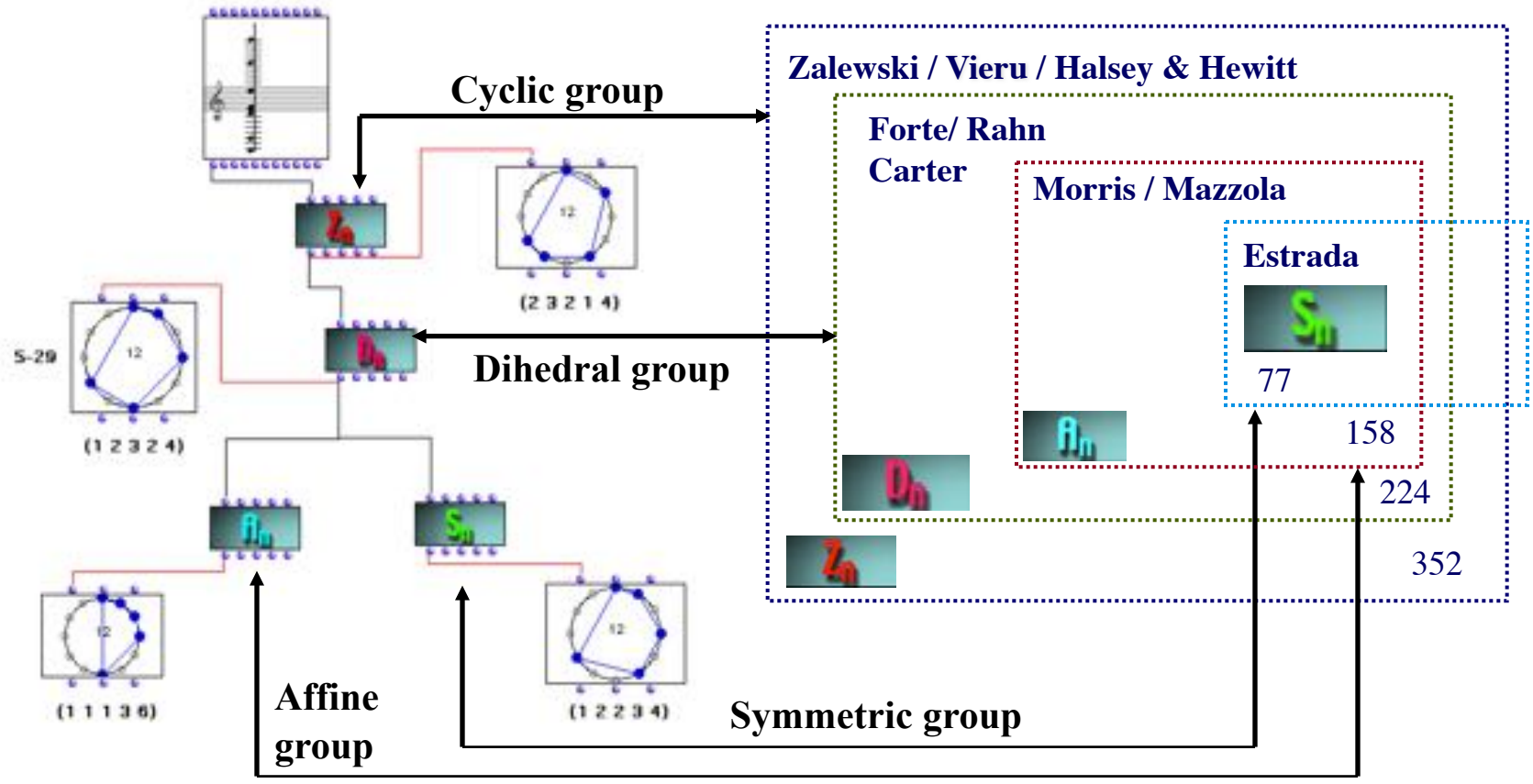




Paradigmatic architecture

Affine group

Group actions and the classification of musical structures




F. Klein

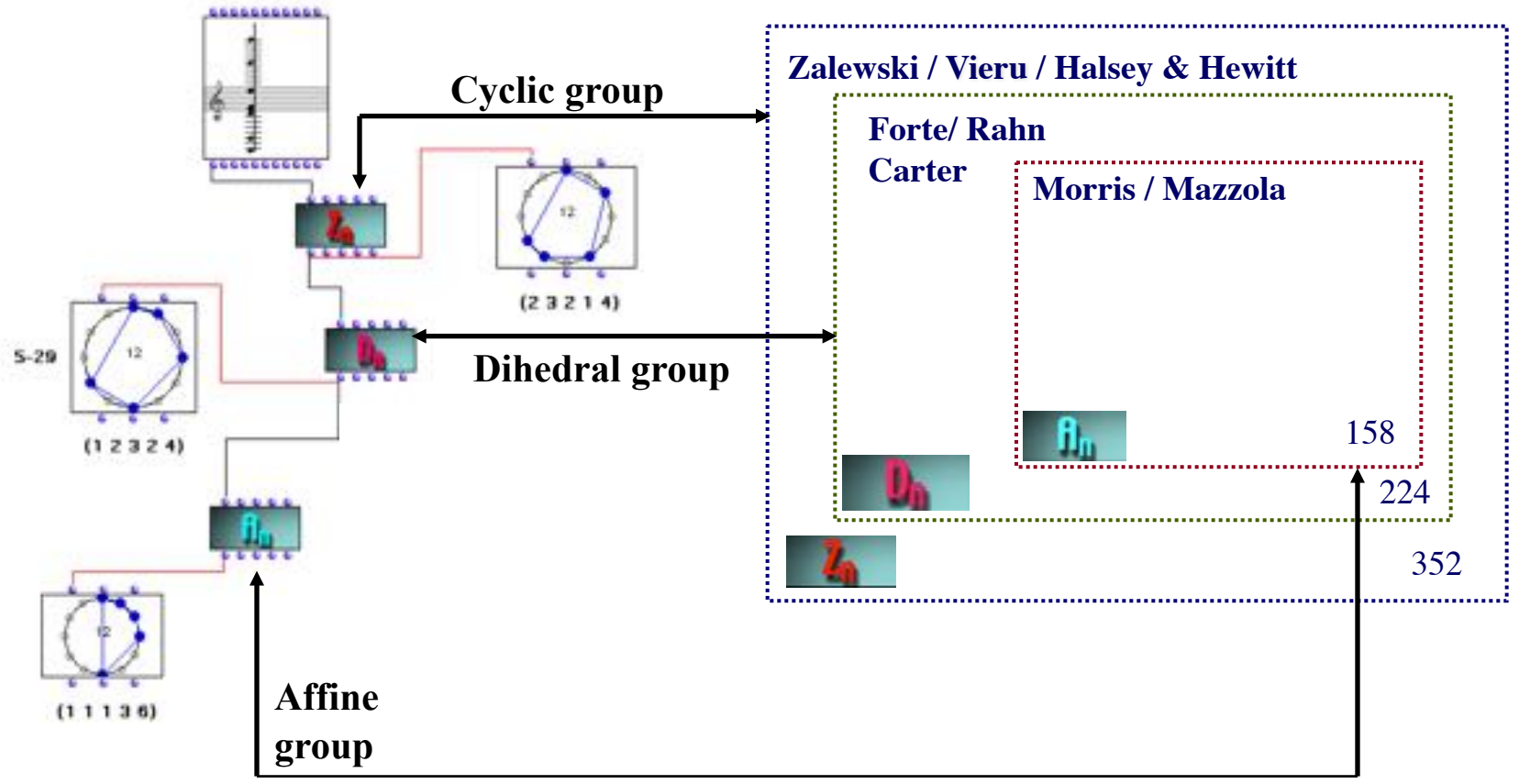

W. Burnside


G. Polya

	1	2	3	4	5	6	7	8	9	10	11	12
Z_n	1	6	19	43	66	80	66	43	19	6	1	1
D_n	1	6	12	29	38	50	38	29	12	6	1	1
A_n	1	5	9	21	25	34	25	21	9	5	1	1
S_n	*1*	*6*	*12*	*15*	*13*	*11*	*7*	*5*	*3*	*2*	*1*	*1*

ICCS EDINBURGH 18

Invariance and the classification of musical structures




F. Klein


W. Burnside

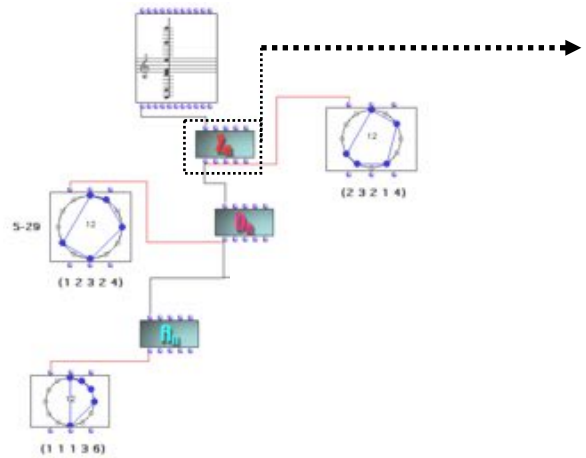

G. Polya

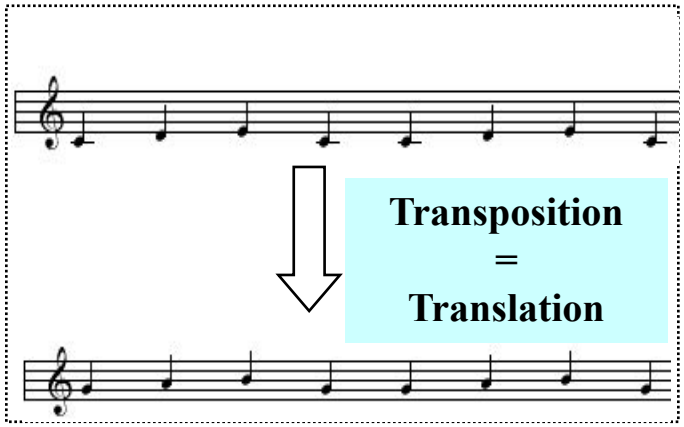
The nature of a given geometry is […] defined by the *reference* to a determinate group and the way in which spatial forms are related within that type of geometry. [*Cf. Felix Klein Erlangen Program - 1872*][…] We may raise the question whether there are any concepts and principles that are, although in different ways and different degrees of distinctness, necessary conditions for both the *constitution* of the perceptual world and the construction of the universe of geometrical thought. It seems to me that the **concept of group** and the **concept of invariance** are such principles.


E. Cassirer

E. Cassirer, "The concept of group and the theory of perception", 1944

The algebraic-geometric genealogy of structuralism






F. Klein


E. Cassirer


G.-G. Granger


J. Piaget

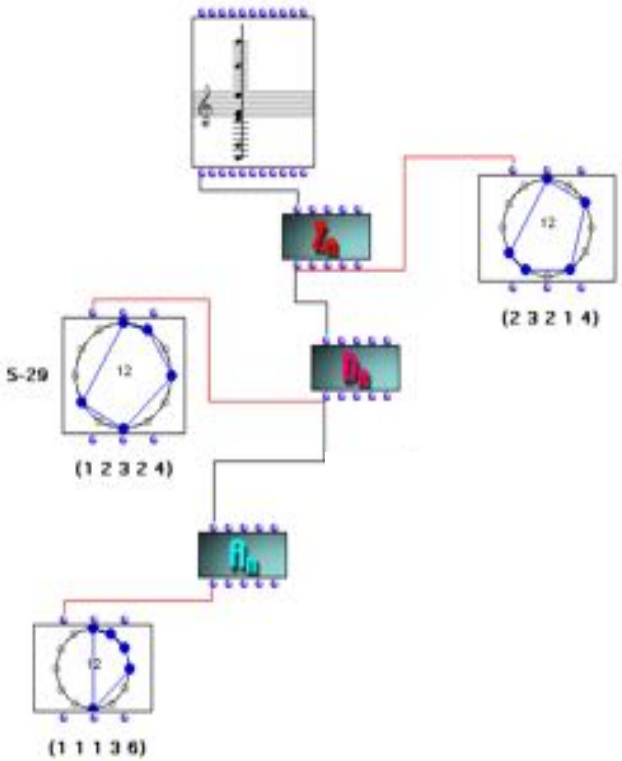
« [C'est la **notion de groupe** qui] donne un sens précis à l'idée de **structure** d'un ensemble [et] permet de déterminer les éléments efficaces des transformations en réduisant en quelque sorte à son **schéma opératoire** le domaine envisagé. […]L'objet véritable de la science est le **système des relations** et non pas les termes supposés qu'il relie. […] Intégrer les résultats - symbolisés - d'une expérience nouvelle revient […] à créer un canevas nouveau, un **groupe de transformations** plus complexe et plus compréhensif »

G.-G. Granger, "Pygmalion. Réflexions sur la pensée formelle", 1947

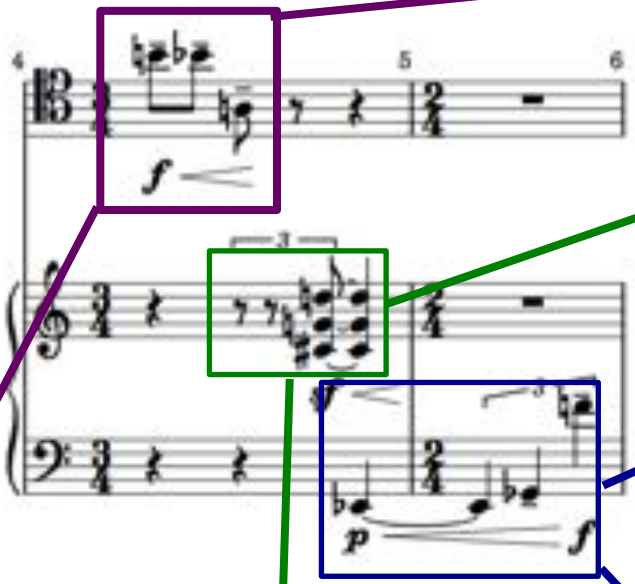
« La **théorie des catégories** est une théorie des constructions mathématiques, qui est macroscopique, et procède d'étage en étage. Elle est un bel exemple d'**abstraction réfléchissante**, cette dernière reprenant elle-même un principe constructeur présent dès le stade sensori-moteur. Le **style catégoriel** qui est ainsi à l'image d'un aspect important de la genèse des facultés cognitives, est un style adéquat à la description de cette genèse »

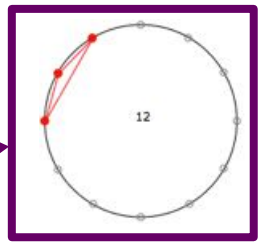
Jean Piaget, Gil Henriques et Edgar Ascher, *Morphismes et Catégories. Comparer et transformer*, 1990

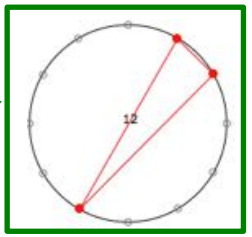
Some *analytical* limitations of a paradigmatic approach

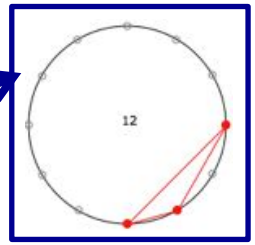


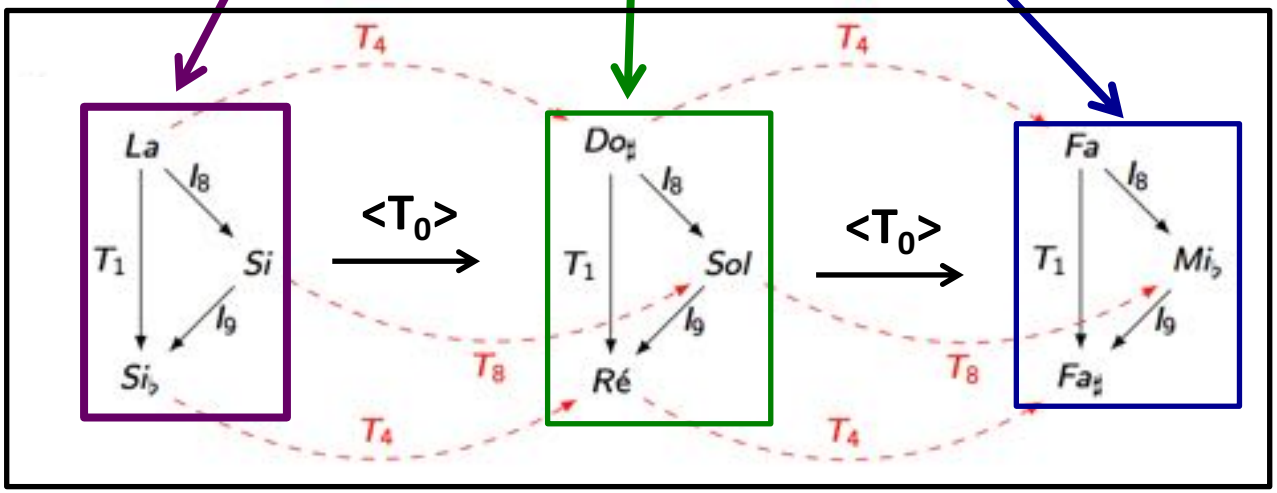
Anton Webern, *Drei Kleine Stücke*, Op. 11/2





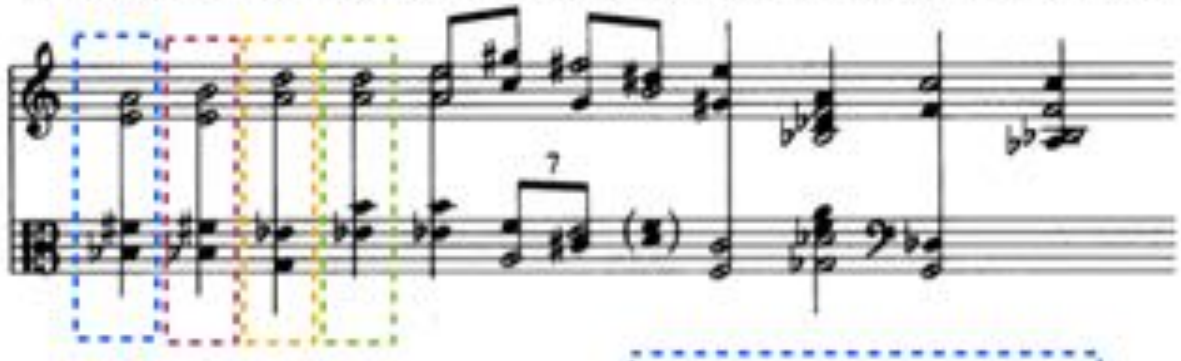


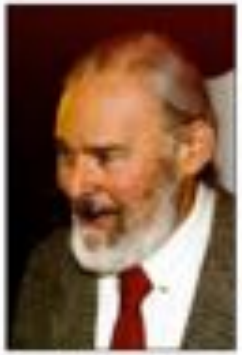




K-nets as a transformational construction

D. Lewin, "A Tutorial on K-nets using the Chorale in Schoenberg's Op.11, N°2 », *JMT*, 1994

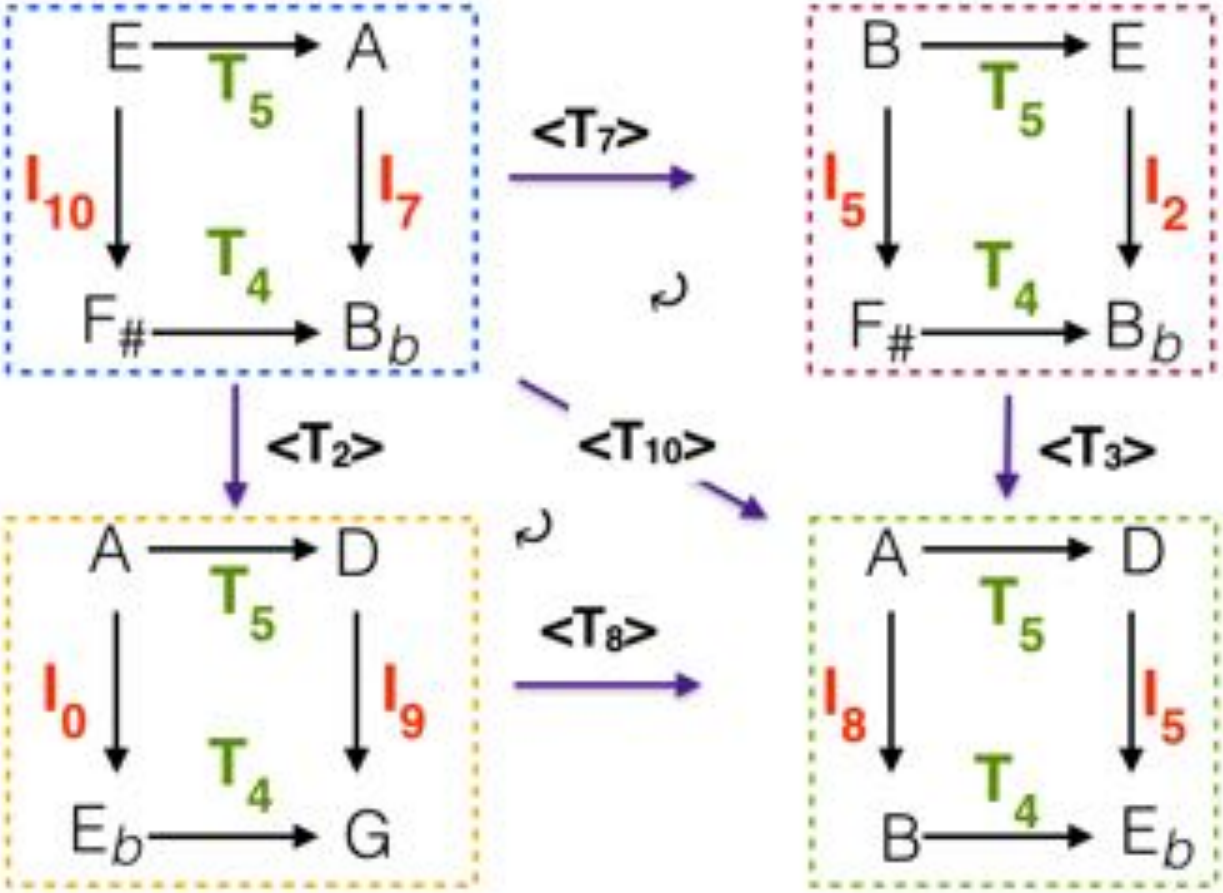




D. Lewin

H. Klumpenhouwer

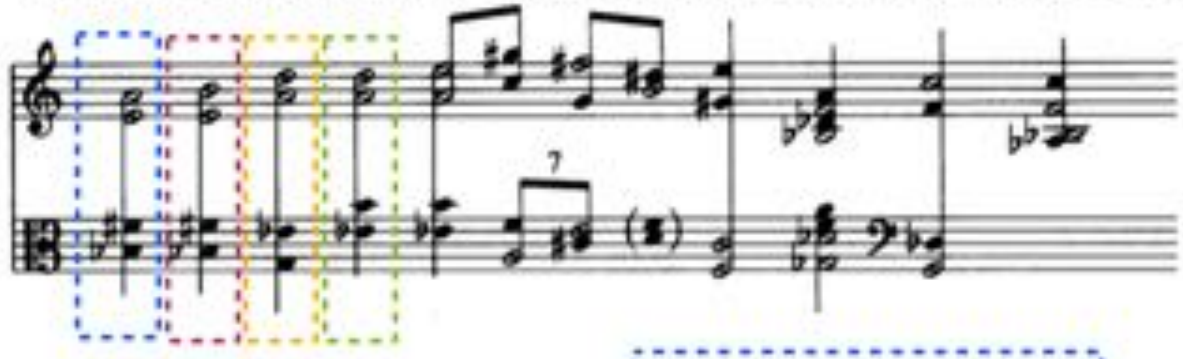
$\langle T_k \rangle : T_m \rightarrow T_m$
$I_m \rightarrow I_{k+m}$



$\langle T_k \rangle \cdot \langle T_m \rangle = \langle T_{k+m} \rangle$

K-nets as a transformational construction

D. Lewin, "A Tutorial on K-nets using the Chorale in Schoenberg's Op.11, N°2 », *JMT*, 1994



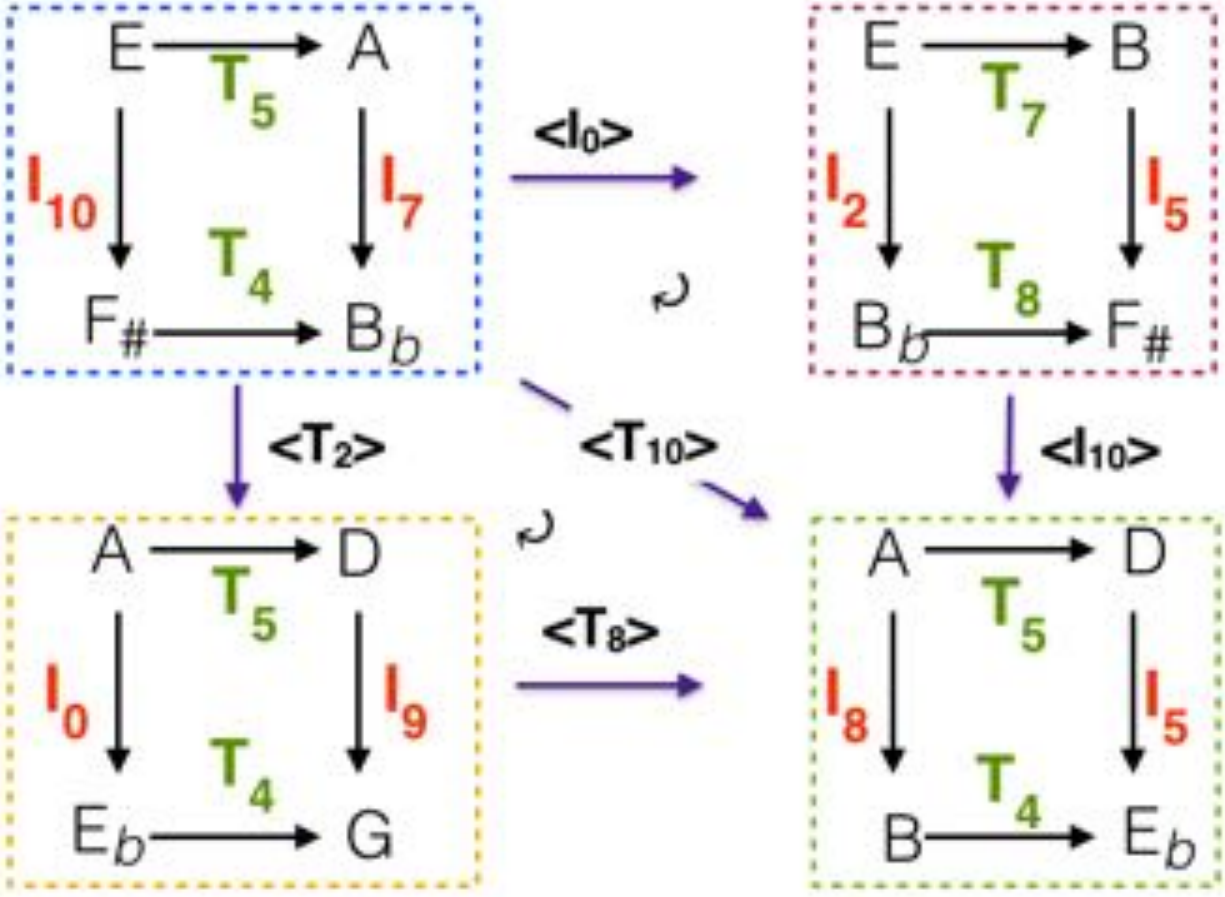
D. Lewin

H. Klumpenhouwer

$\langle T_k \rangle : T_m \rightarrow T_m$
$I_m \rightarrow I_{k+m}$

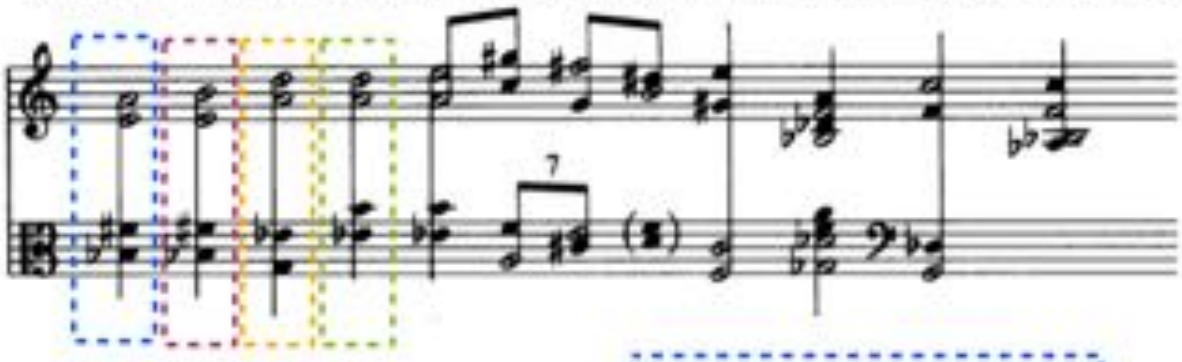
$\langle I_k \rangle : T_m \rightarrow T_{-m}$
$I_m \rightarrow I_{k-m}$

$\langle T_k \rangle \cdot \langle T_m \rangle = \langle T_{k+m} \rangle$
$\langle I_k \rangle \cdot \langle I_m \rangle = \langle T_{m-k} \rangle$



K-nets as a transformational construction

D. Lewin, "A Tutorial on K-nets using the Chorale in Schoenberg's Op.11, N°2 », *JMT*, 1994



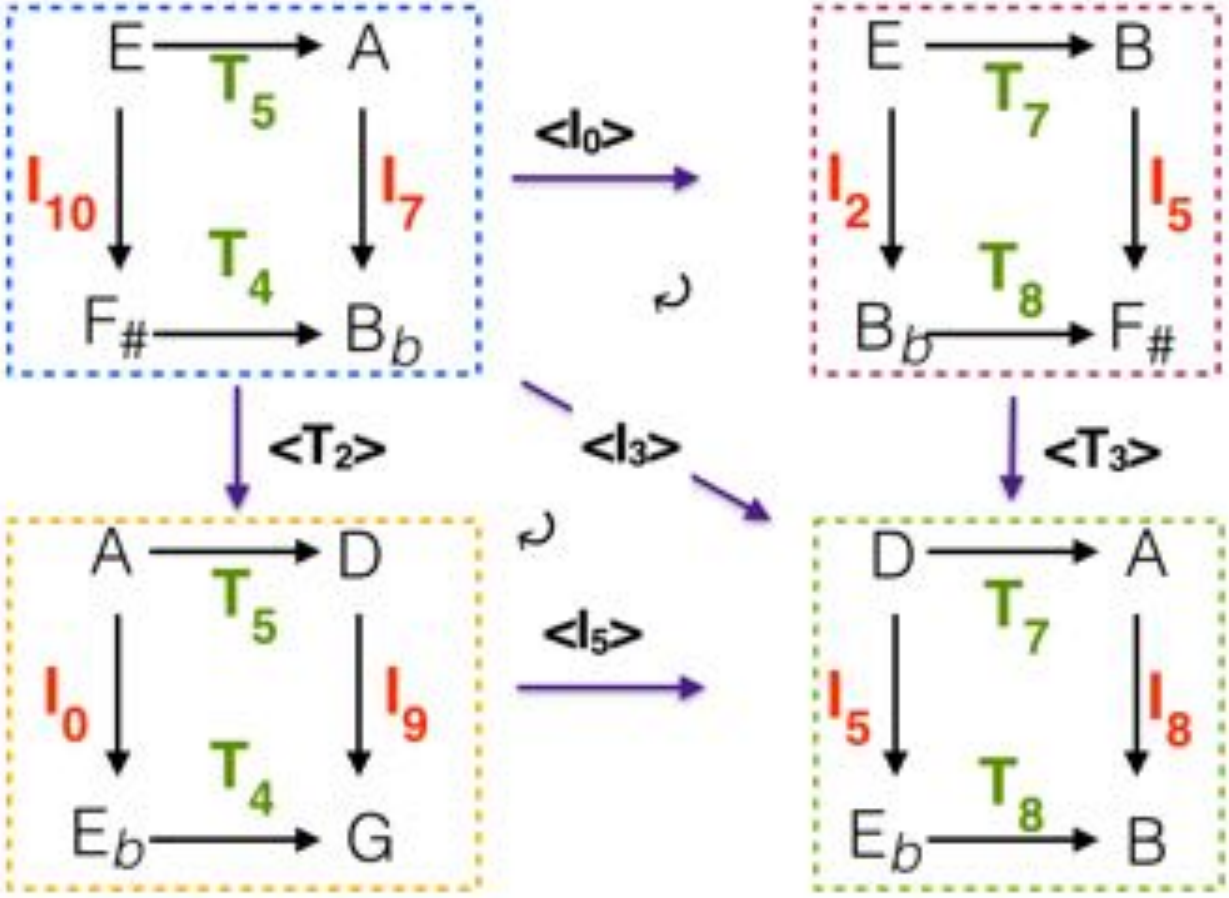
D. Lewin

H. Klumpenhouwer

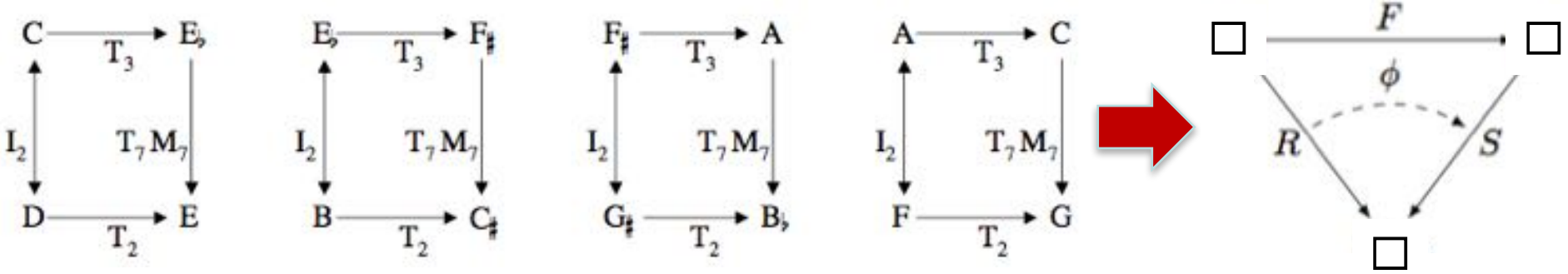
$\langle T_k \rangle : T_m \rightarrow T_m$
$I_m \rightarrow I_{k+m}$

$\langle I_k \rangle : T_m \rightarrow T_{-m}$
$I_m \rightarrow I_{k-m}$

$\langle T_k \rangle \cdot \langle T_m \rangle = \langle T_{k+m} \rangle$
$\langle T_k \rangle \cdot \langle I_m \rangle = \langle I_{m-k} \rangle$
$\langle I_m \rangle \cdot \langle T_k \rangle = \langle I_{k+m} \rangle$
$\langle I_k \rangle \cdot \langle I_m \rangle = \langle T_{m-k} \rangle$



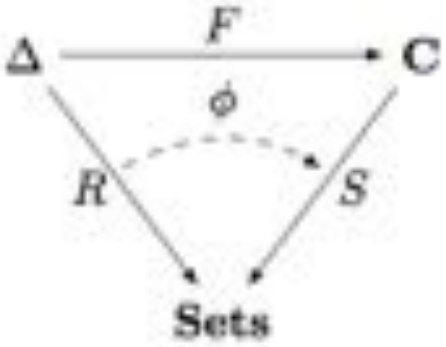
From K-Nets to category-based PK-Nets



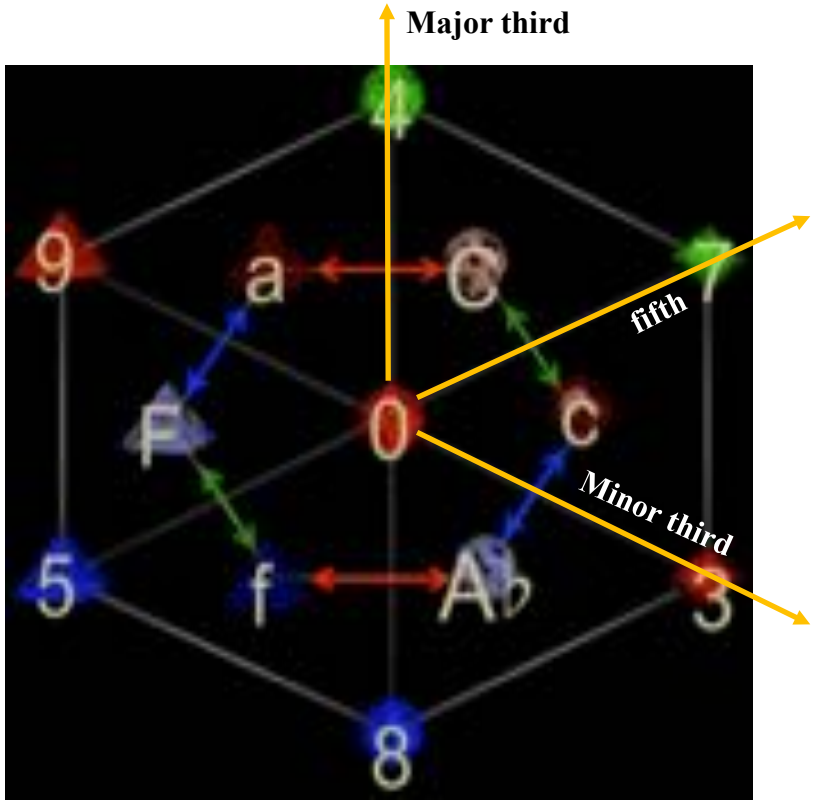
Definition 1 *Let* **C** *be a category, and* S *a functor from* **C** *to the category* **Sets**. *Let* Δ *be a small category and* R *a functor from* Δ *to* **Sets**. *A PK-net of form* R *and of support* S *is a 4-tuple* (R, S, F, ϕ), *in which*

- F *is a functor from* Δ *to* **C**,
- *and* ϕ *is a natural transformation from* R *to* SF.

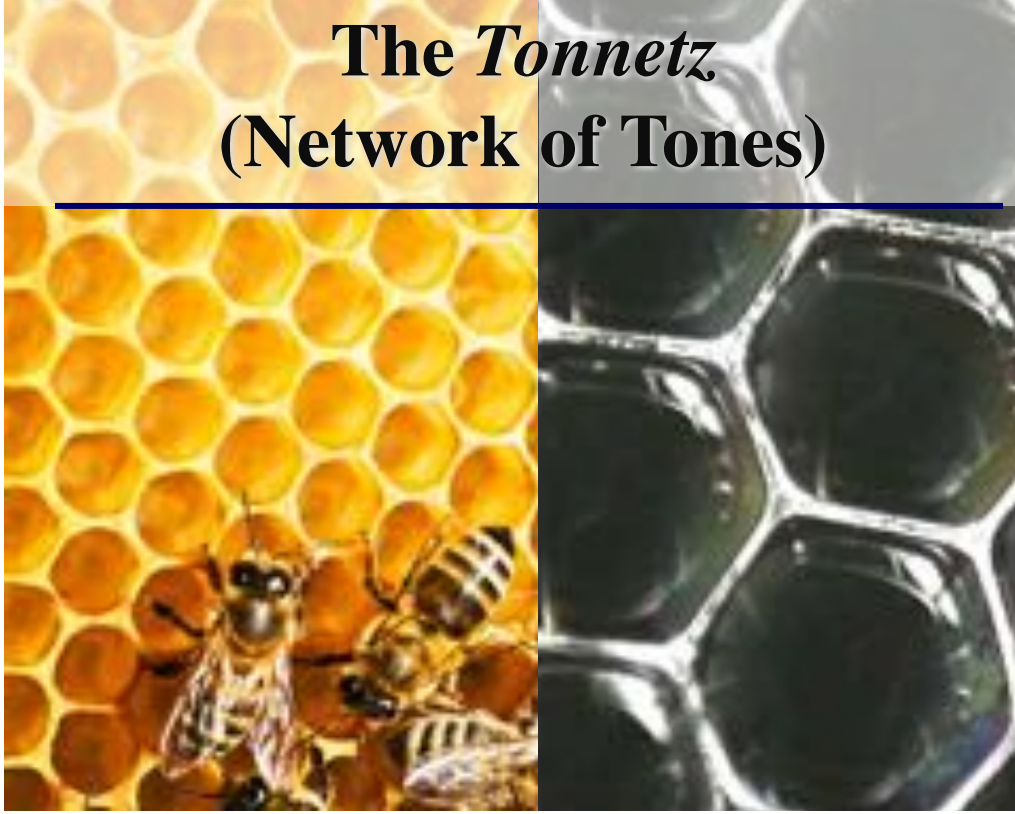
The definition of a PK-net is summed up by the following diagram:

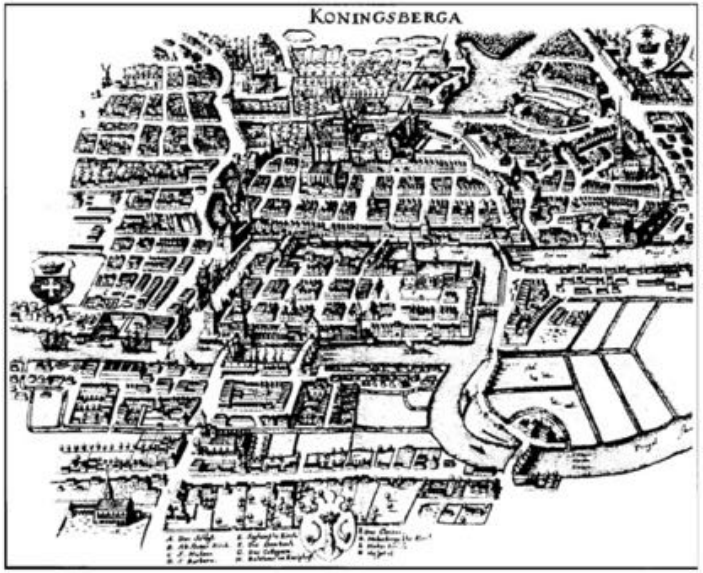


Popoff A., M. Andreatta, A. Ehresmann, « A Categorical Generalization of Klumpenhouwer Networks », MCM 2015, Queen Mary University, Springer, p. 303-314

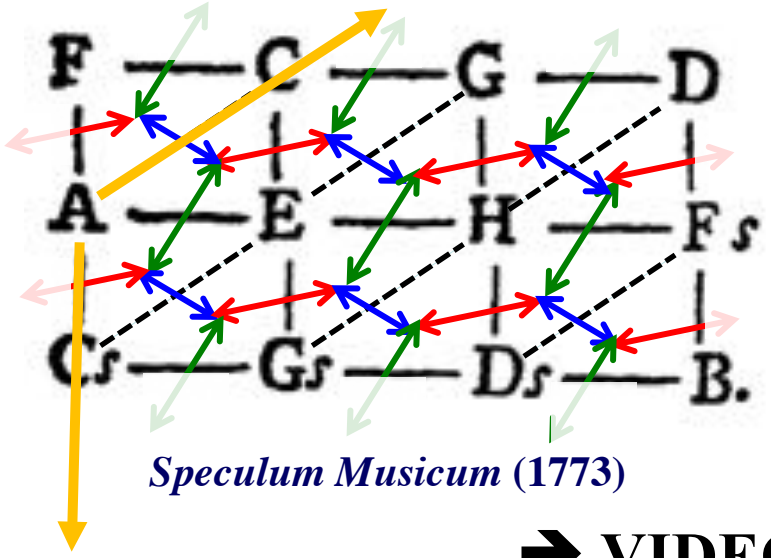


The *Tonnetz* (Network of Tones)





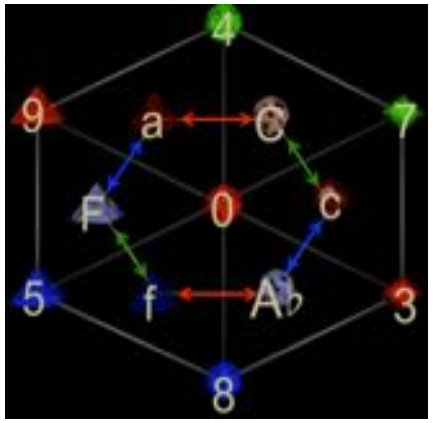
Leonhard Euler

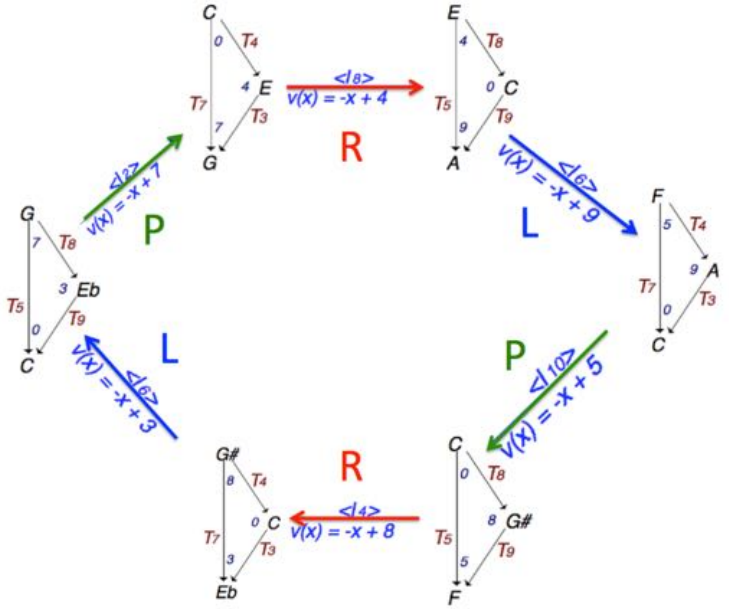


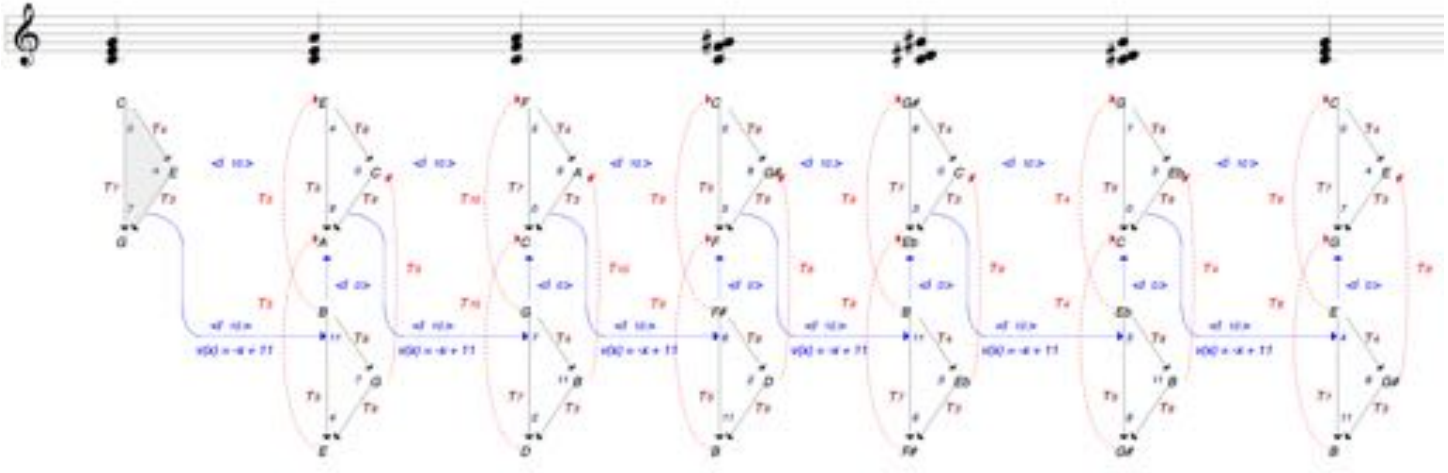
Speculum Musicum (1773)

➔ VIDEO

The *Tonnetz* as a categorical construction

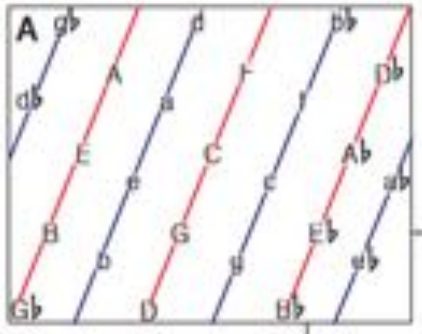






- **Popoff A., C. Agon, M. Andreatta, A. Ehresmann (2016), « From K-Nets to PK-Nets: A Categorical Approach », PNM, 54(1)**
- **Popoff A., M. Andreatta, A. Ehresmann, « Relational PK-Nets for Transformational Music Analysis » (forthcoming in the JMM)**

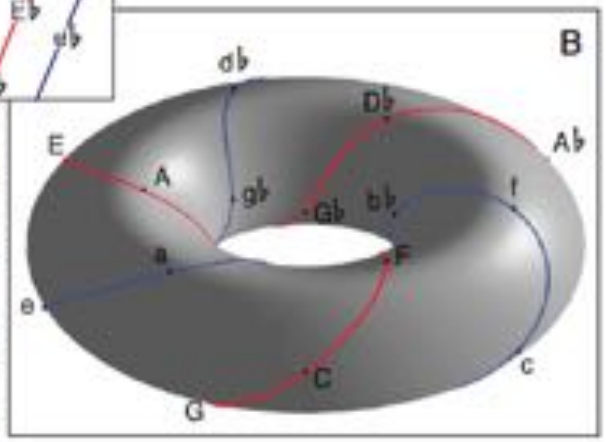
Neurosciences and *Tonnetz* representation



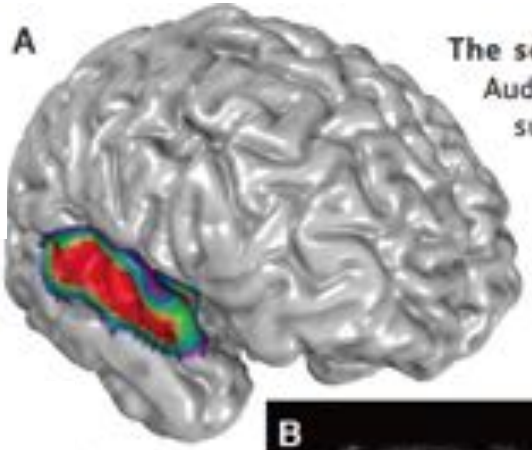
PERSPECTIVES: NEUROSCIENCE

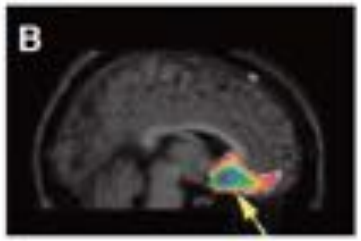
Mental Models and Musical Minds

Robert J. Zatorre and Carol L. Krumhansl

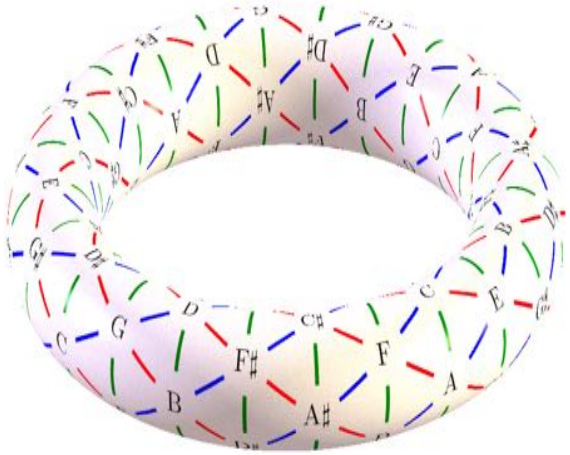


Mental key maps. (A) Unfolded version of the key map, with opposite edges to be considered matched. There is one circle of fifths for major keys (red) and one for minor keys (blue), each wrapping the torus three times. In this way, every major key is flanked by its relative minor on one side (for example, C major and a minor) and its parallel minor on the other (for example, C major and c minor). (B) Musical keys as points on the surface of a torus.





The sensation of music. (A) Auditory cortical areas in the superior temporal gyrus that respond to musical stimuli. Regions that are most strongly activated are shown in red. (B) Metabolic activity in the ventromedial region of the frontal lobe increases as a tonal stimulus becomes more consonant.

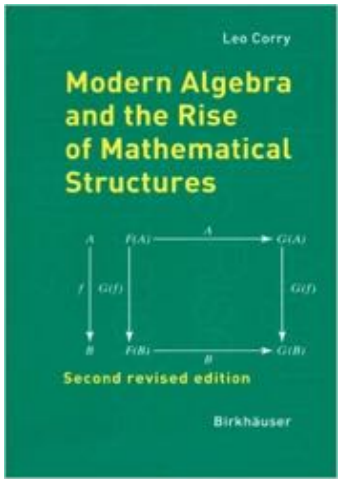


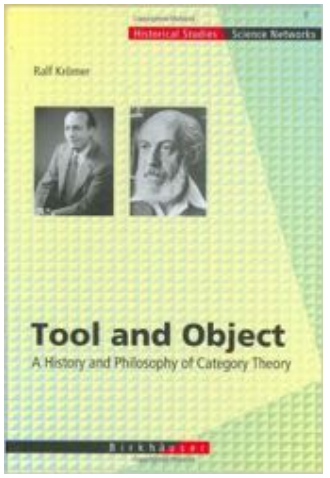
The *Tonnetz* (source: Wikipedia)

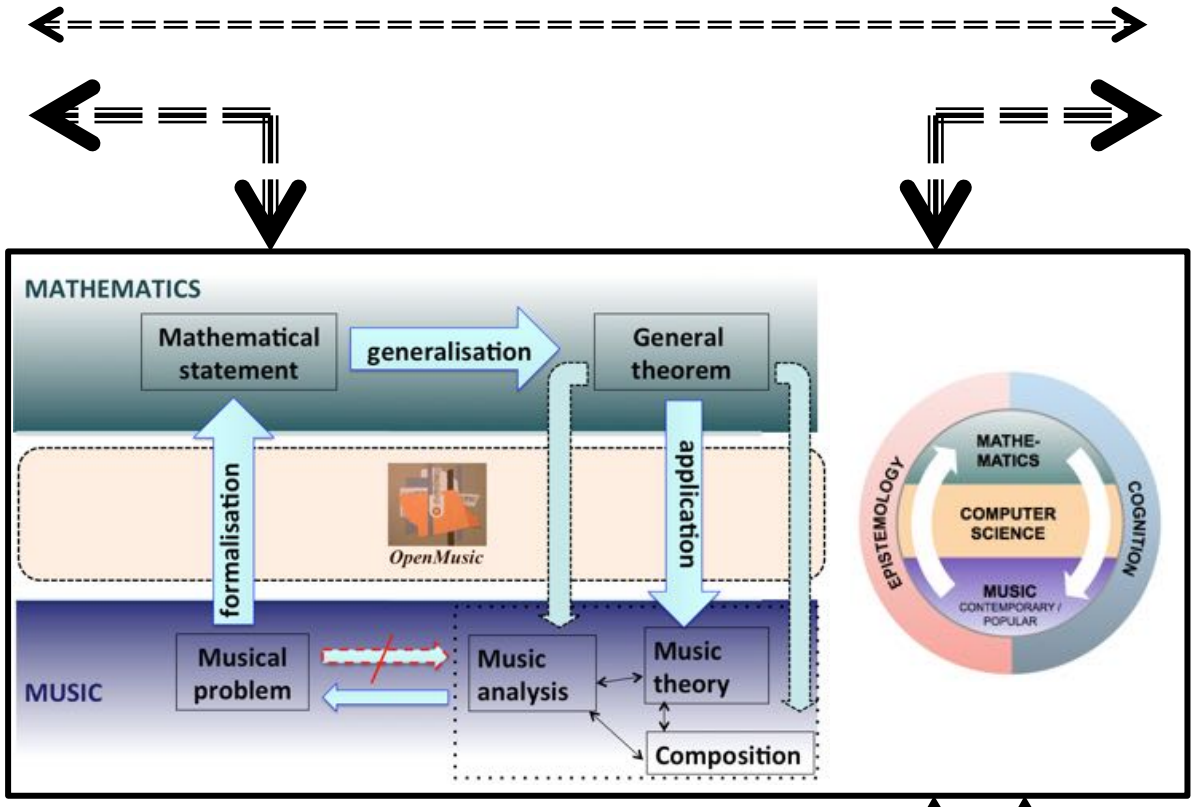
Acotto E. et M. Andreatta (2012), « Between Mind and Mathematics. Different Kinds of Computational Representations of Music », *Mathematics and Social Sciences*, n° 199, 2012(3), p. 9-26.

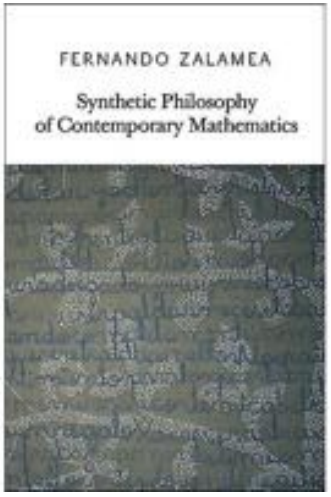
EUROPEAN MATHEMATICAL PSYCHOLOGY GROUP

Which type of philosophy for the *mathemusical* research?

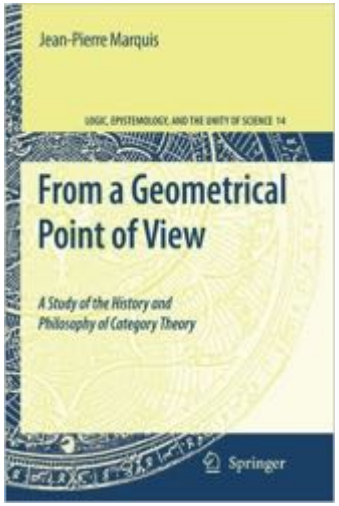








*A **synthetic** vision allows us to link together apparently distant strata of mathematics and culture, helping us to break down many artificial barriers. Not only can today's mathematics be appreciated through epistemic, ontic, phenomenological and aesthetic modes, but in turn, it should help to transform **philosophy.***



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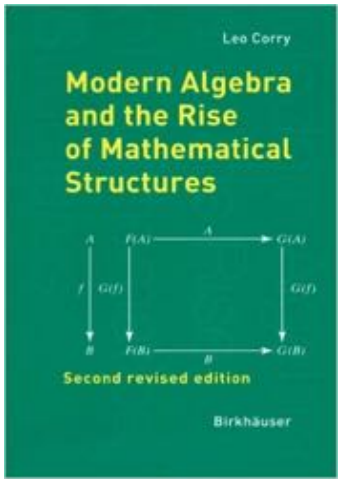

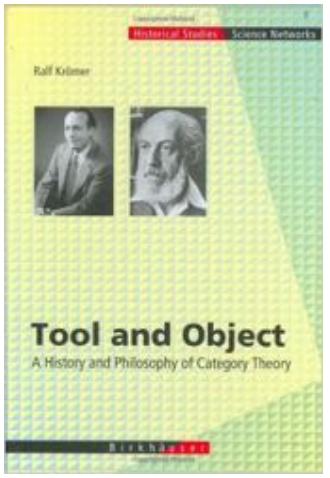

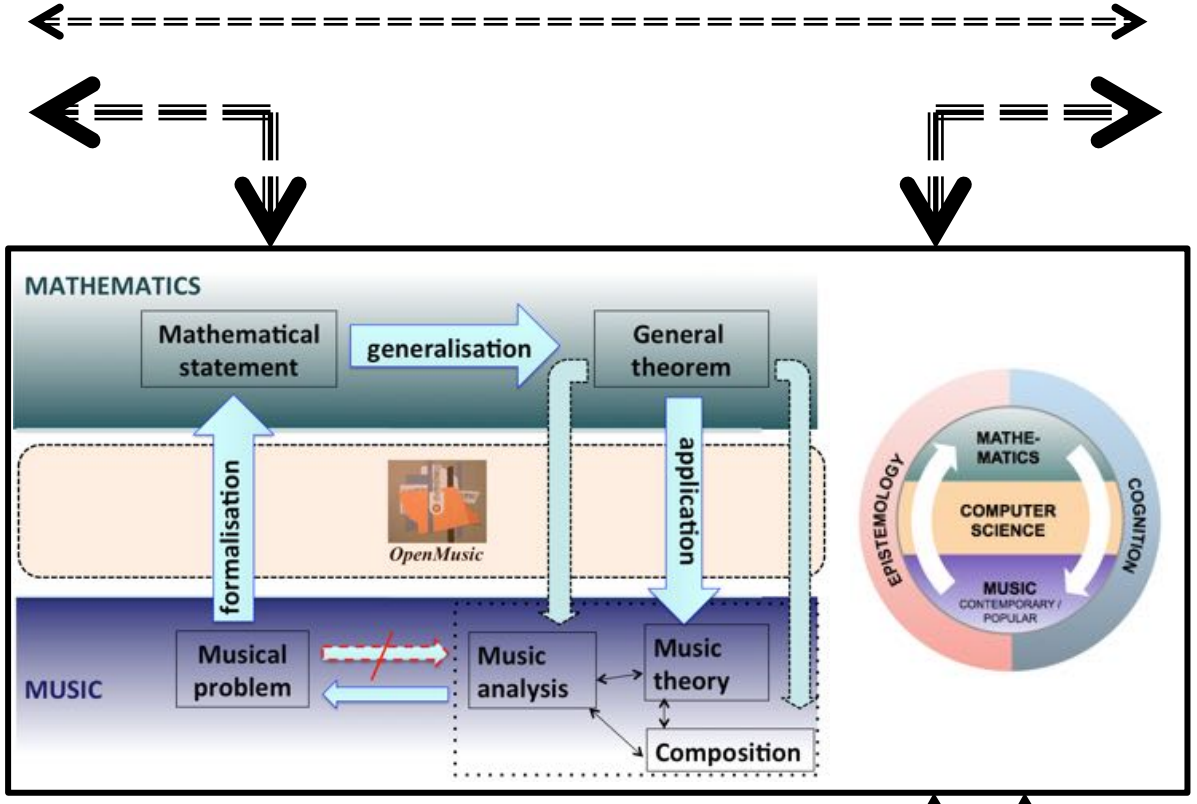

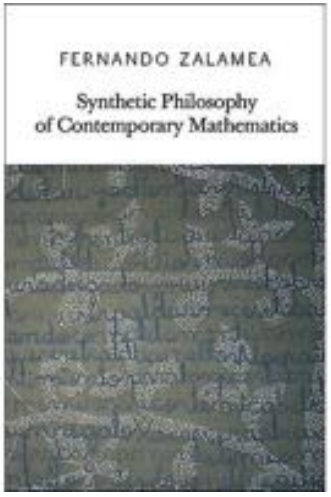

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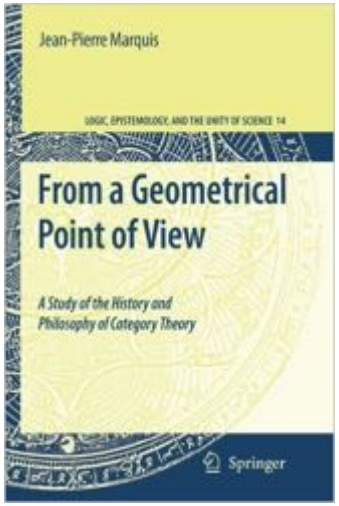

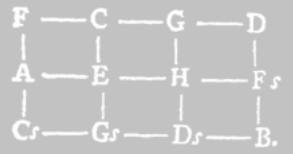

THANK YOU FOR YOUR ATTENTION