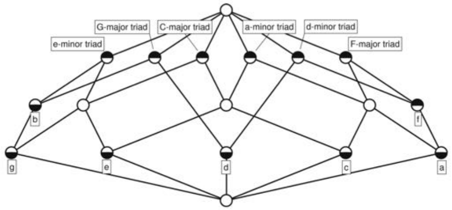

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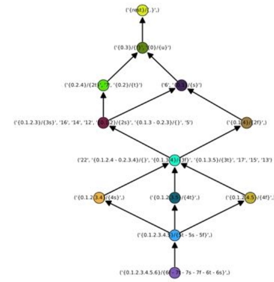

Musical Descriptions based on Formal Concept Analysis and Mathematical Morphology

Carlos Agon[1], Moreno Andreatta[1,2], Jamal Atif[3], Isabelle Bloch[4], and Pierre Mascarade[3]

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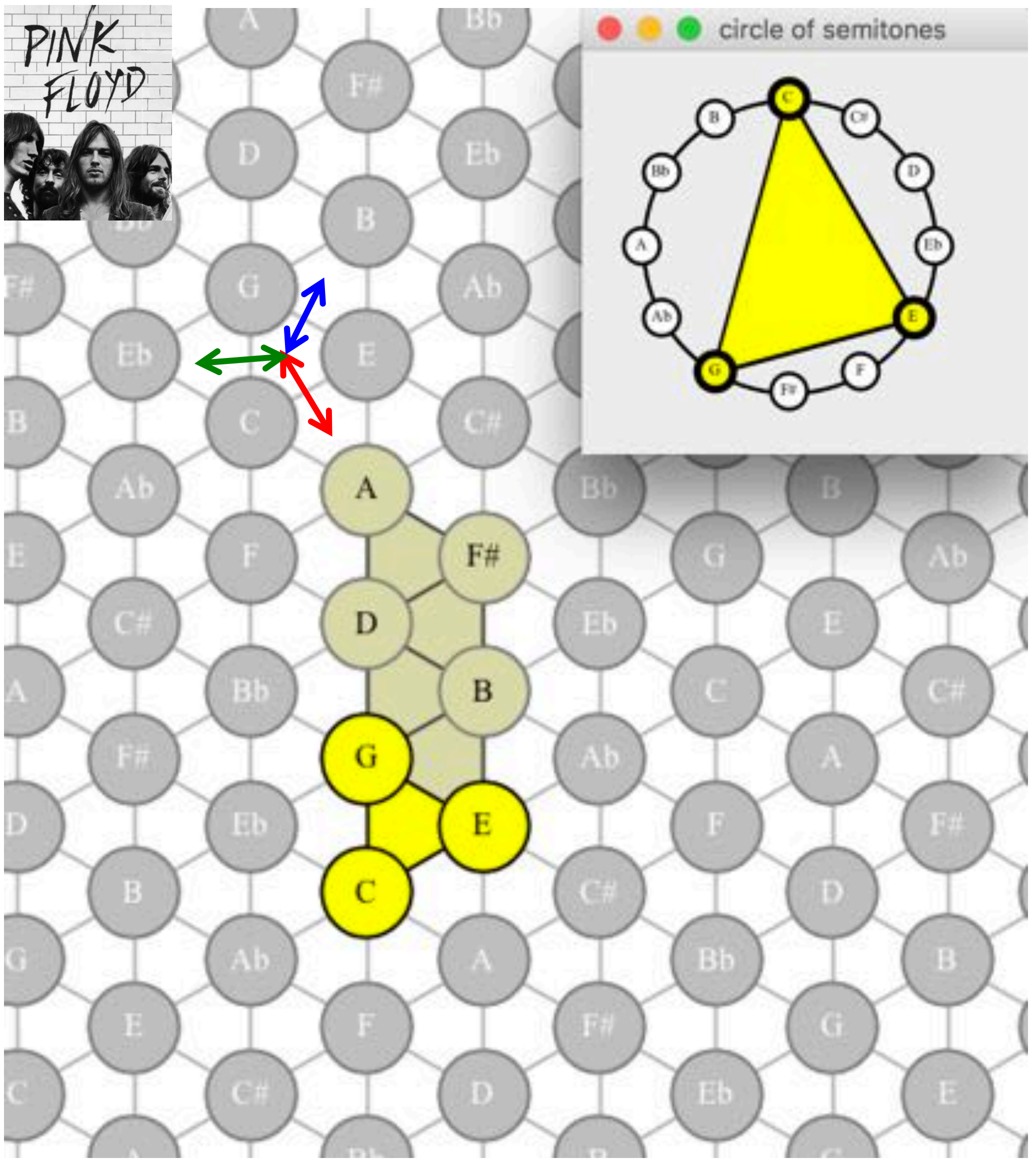
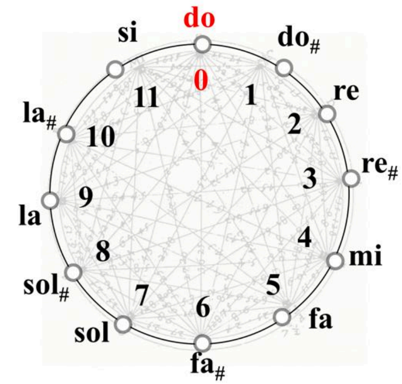
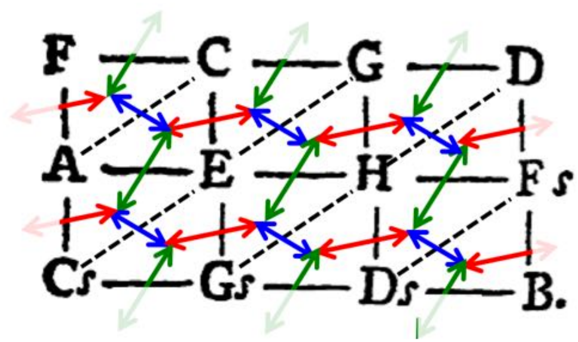
[4] LTCI, Télécom ParisTech, Université Paris-Saclay, Paris, France, isabelle.bloch@telecom-paristech.fr

A starting example: the Gunner's Dream by Pink-Floyd





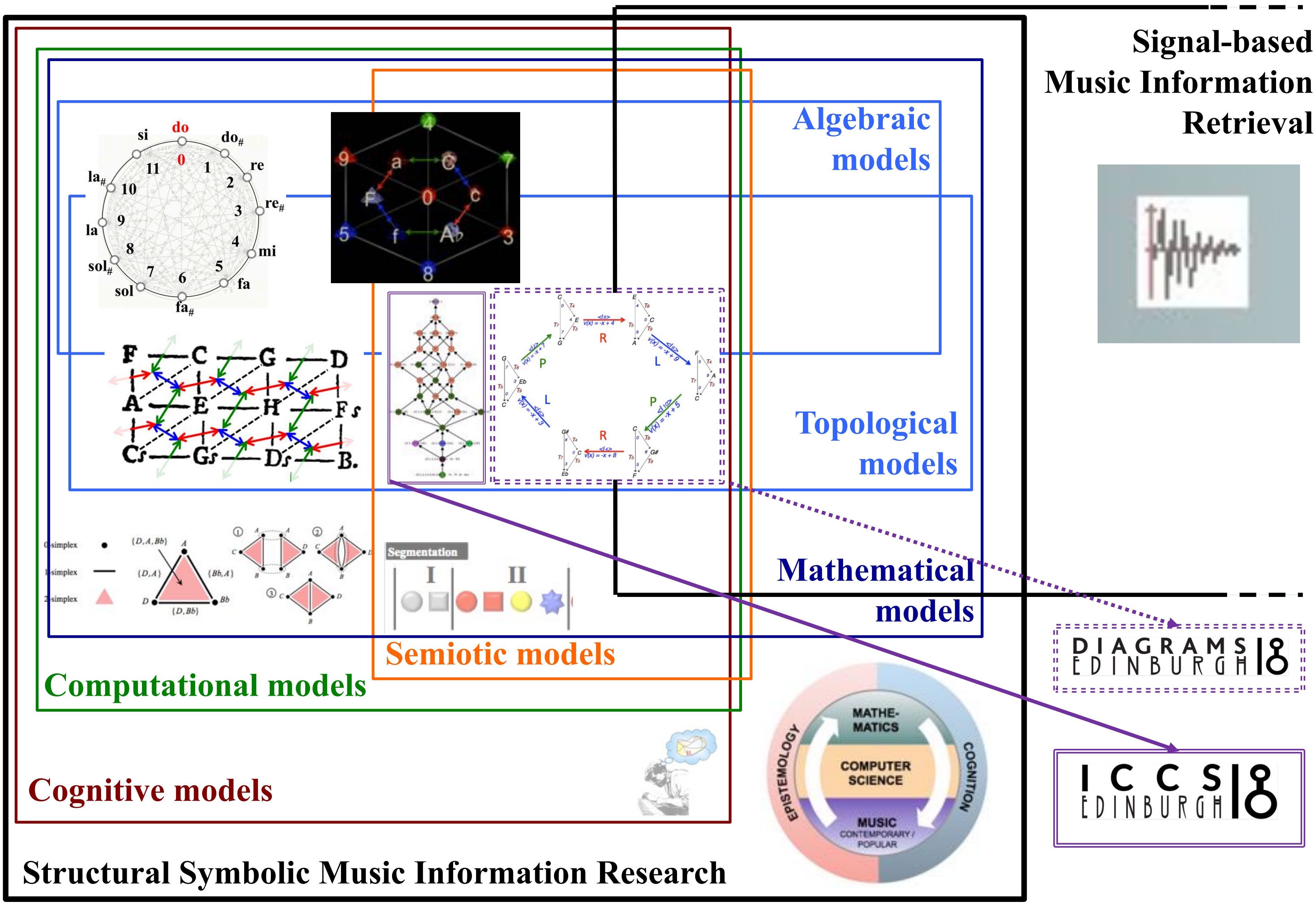
Marin Mersenne



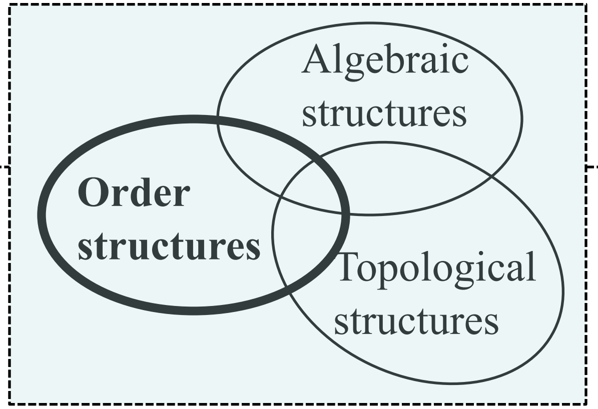
Leonhard Euler's *Speculum Musicum*

→`http://www.lacl.fr/~lbigo/hexachord`

Lattices and diagrams in Structural Symbolic MIR



Formal Concept Analysis: the double history



M. Barbut

Centre d'analyse et de mathématique sociales

- M. Barbut, « Note sur l'algèbre des techniques d'analyse hiérarchique », in B. Matalon (éd.), L'analyse hiérarchique, Paris, Gauthier-Villars, 1965.
- M. Barbut, B. Monjardet, *Ordre et Classification. Algèbre et Combinatoire*, en deux tomes, 1970
- M. Barbut, L. Frey, « Techniques ordinales en analyse des données », Tome I, *Algèbre et Combinatoire des Méthodes Mathématiques en Sciences de l'Homme*, Paris, Hachette, 1971.
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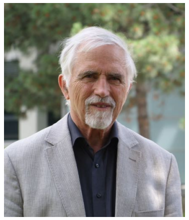
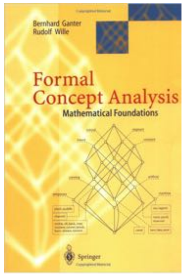
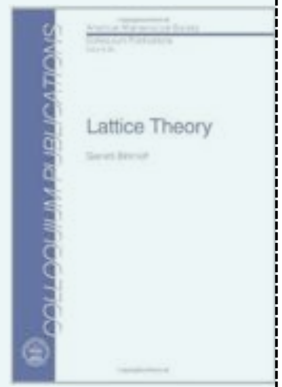


R. Wille



- R. Wille, « Mathematische Sprache in der Musiktheorie », in B. Fuchssteiner, U. Kulisch, D. Laugwitz, R. Liedl (Hrsg.): Jahrbuch Überblicke Mathematik. B.I.-Wissenschaftsverlag, Mannheim, 1980, p. 167-184.
- R. Wille, « Restructuring Lattice Theory: An approach based on Hierarchies of Concepts », I. Rival (ed.), *Ordered Sets,* 1982
- R. Wille, « Sur la fusion des contextes individuals », Mathématiques et sciences humaines, tome 85, 1984.
- B. Ganter & R. Wille, *Formal Concept Analysis: Mathematical Foundations*, Springer, Berlin, 1998

Formal Concept Analysis: the common root

Garrett Birkhoff (1911-1996)

M. Barbut

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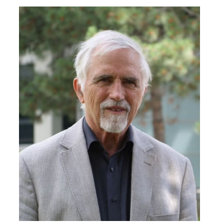

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Formal Concept Analysis in a nutshell

- G = set of objects.
- M = set of attributes or properties.
- $I \subseteq G \times M$.
- (X, Y) $(X \in \mathcal{P}(G), Y \in \mathcal{P}(M))$ = formal concept if (X, Y) is maximal for $X \times Y \subseteq I$.
- Partial ordering: $(X_1, Y_1) \preceq (X_2, Y_2) \Leftrightarrow X_1 \subseteq X_2 (\Leftrightarrow Y_2 \subseteq Y_1)$.
- $\Rightarrow$ Lattice structure $\mathbb{C}$ and
  $\bigwedge_{t \in T}(X_t, Y_t) = \left(\bigcap_{t \in T} X_t, \alpha(\beta(\bigcup_{t \in T} Y_t))\right)$,
  $\bigvee_{t \in T}(X_t, Y_t) = \left(\beta(\alpha(\bigcup_{t \in T} X_t)), \bigcap_{t \in T} Y_t\right)$.
- Derivation operators:
  $\alpha(X) = \{m \in M \mid \forall g \in X, (g, m) \in I\}$,
  $\beta(Y) = \{g \in G \mid \forall m \in Y, (g, m) \in I\}$.
- (X, Y) formal concept $\Leftrightarrow \alpha(X) = Y$ and $\beta(Y) = X$.

Formal Concept Analysis and Mathematical Morphology

Garrett Birkhoff
(1911-1996)

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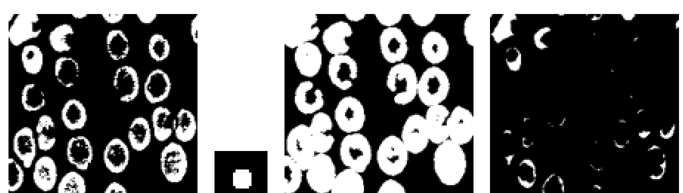
I. Bloch

• **Bloch, I.**, Heijmans,H., Ronse, C.: Mathematical Morphology. In: Aiello, M., Pratt- Hartman, I., van Benthem, J. (eds.) *Handbook of Spatial Logics*, chap. 13, pp. 857–947. Springer (2007)

• **Atif, J., Bloch,** I., Distel, F., Hudelot, C.: Mathematical morphology operators over concept lattices. In: International Conference on Formal Concept Analysis. vol. LNAI 7880, pp. 28–43. Dresden, Germany (May 2013)

• **Atif, J.**, **Bloch, I.**, Hudelot, C.: Some relationships between fuzzy sets, mathematical morphology, rough sets, F-transforms, and formal concept analysis. International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems 24(S2), 1–32 (2016)



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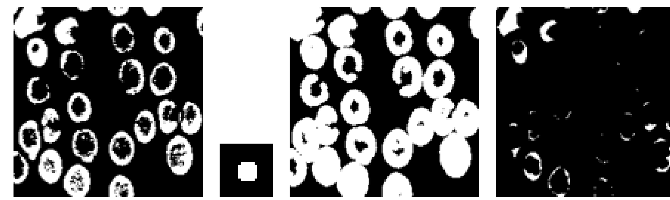
J. Atif

The two main MM operations: dilation & erosion

Dilation: operation in complete lattices that commutes with the supremum.
Erosion: operation in complete lattices that commutes with the infimum.

Complete lattices $(\mathcal{T}, \leq)$, $(\mathcal{T}', \leq')$



Algebraic dilation: $\delta : \mathcal{T} \rightarrow \mathcal{T}'$ such that

$$\forall (x_i) \in \mathcal{T},\ \delta(\vee_i x_i) = \vee'_i \delta(x_i)$$

Algebraic erosion: $\varepsilon : \mathcal{T}' \rightarrow \mathcal{T}$ such that

$$\forall (x_i) \in \mathcal{T}',\ \varepsilon(\wedge'_i x_i) = \wedge_i \varepsilon(x_i)$$

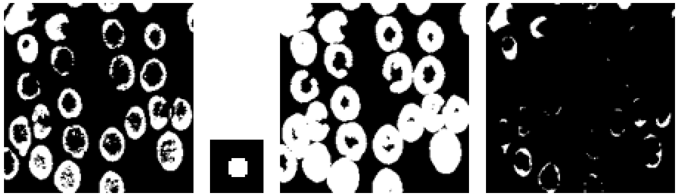
$\delta : \mathcal{T} \rightarrow \mathcal{T}'$, $\varepsilon : \mathcal{T}' \rightarrow \mathcal{T}$, (ε, δ) adjunction if:

$$\forall x \in \mathcal{T}, \forall y \in \mathcal{T}',\ \delta(x) \leq' y \Leftrightarrow x \leq \varepsilon(y)$$

The two main MM operations: dilation & erosion

Dilation: operation in complete lattices that commutes with the supremum.
Erosion: operation in complete lattices that commutes with the infimum.

Complete lattices $(\mathcal{T}, \leq)$, $(\mathcal{T}', \leq')$



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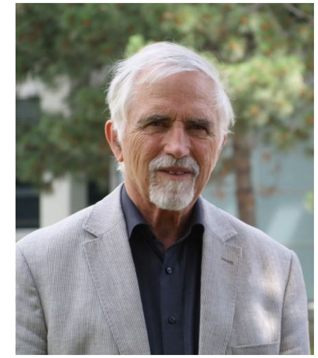
Equivalent concepts by reversing the order on one space.

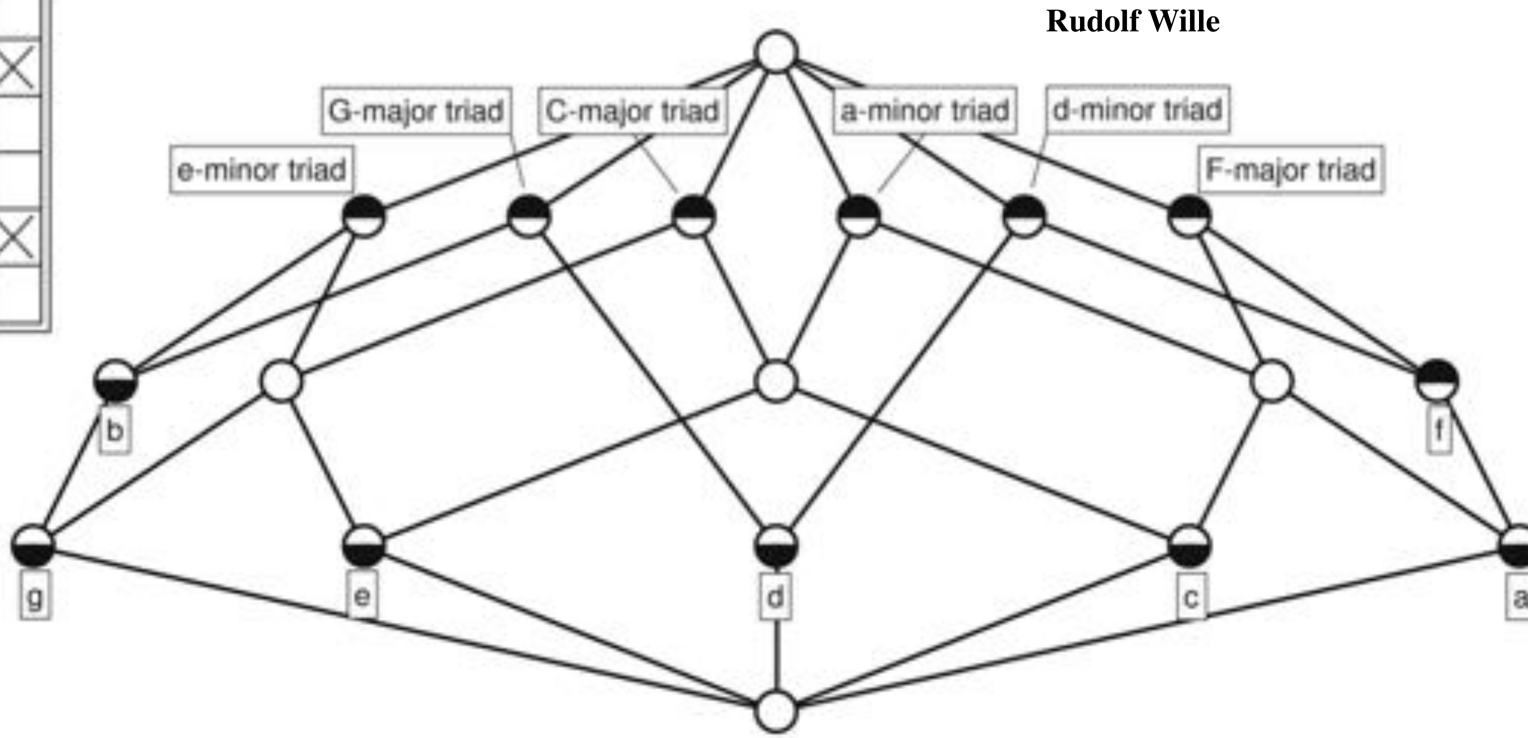
$$\delta : A \to B,\ \varepsilon : B \to A \qquad \alpha : B \to A, \beta : A \to B$$
$$\delta(a) \leq_B b \Leftrightarrow a \leq_A \varepsilon(b) \qquad a \leq_A \alpha(b) \Leftrightarrow b \leq_B \beta(a)$$

A concept lattice for musical structures

	C-major triad	d-minor triad	e-minor triad	F-major triad	G-major triad	a-minor triad
c	X			X		X
d		X			X	
e	X		X			X
f		X		X		
g	X		X		X	
a		X		X		X
b			X		X	



Rudolf Wille



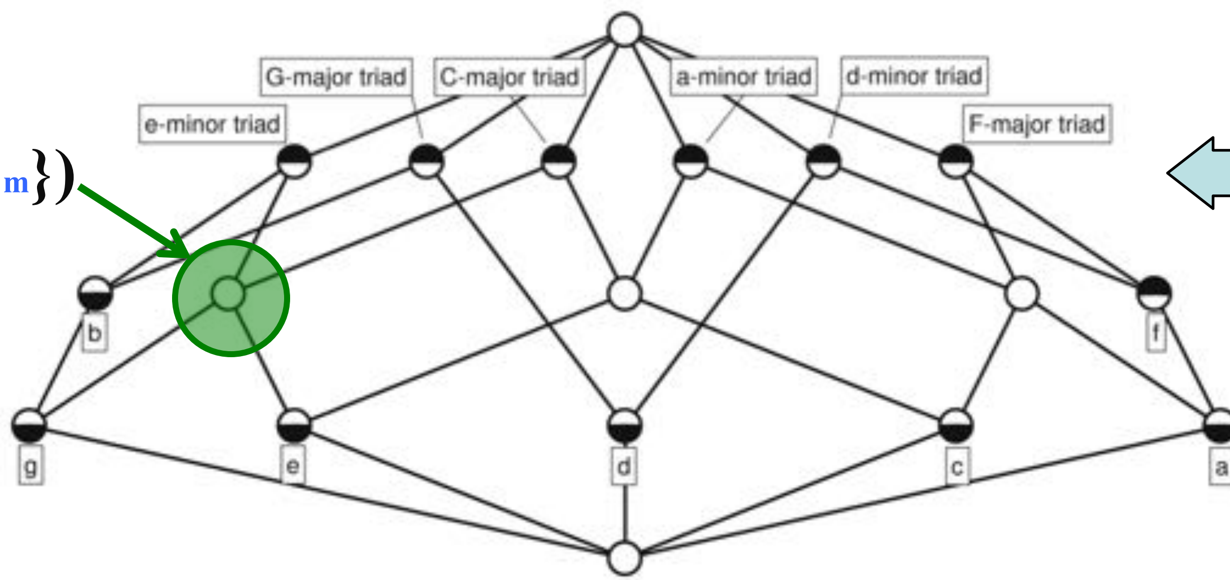
[R. Wille & R. Wille-Henning, « Towards a Semantology of Music », ICCS 2007, Springer, 2007]

A concept lattice for the diatonic scale

	C-major triad	d-minor triad	e-minor triad	F-major triad	G-major triad	a-minor triad
c	X			X		X
d		X			X	
e	X		X			X
f		X		X		
g	X		X		X	
a		X		X		X
b			X		X	

	CM	Em				
mi	X	X				
sol	X	X				

({***mi, sol***},{**CM, Em**})



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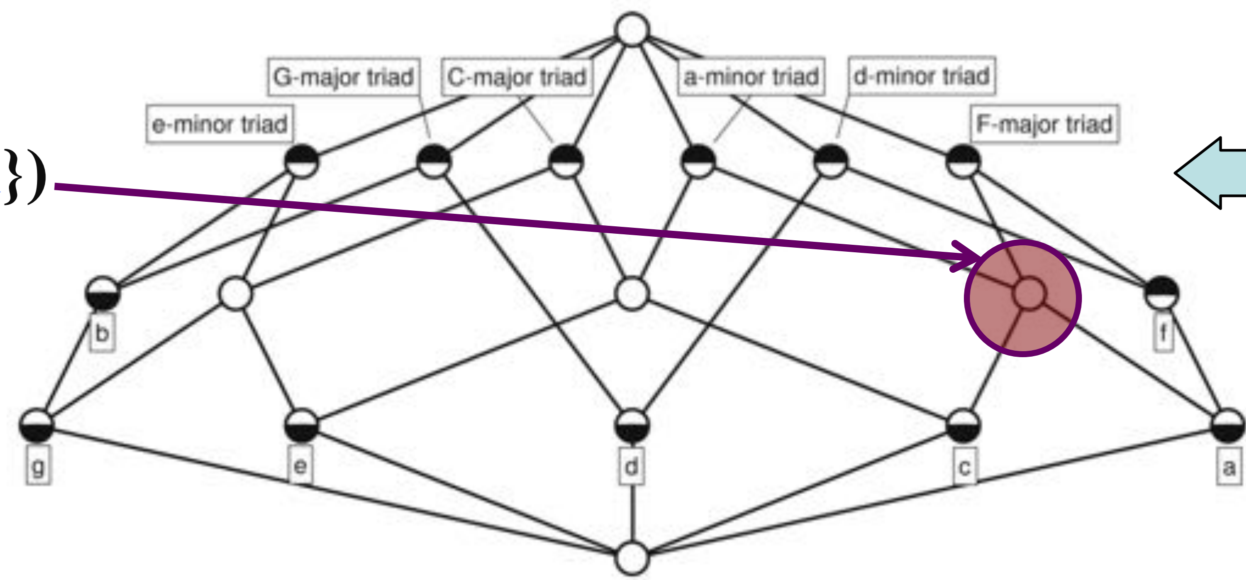
e	X		X			X
f		X		X		
g	X		X		X	
a		X		X		X
b			X		X	

	C_M	E_m	A_m	F_M		
mi	X	X				
sol	X	X				
la			X	X		
do			X	X		

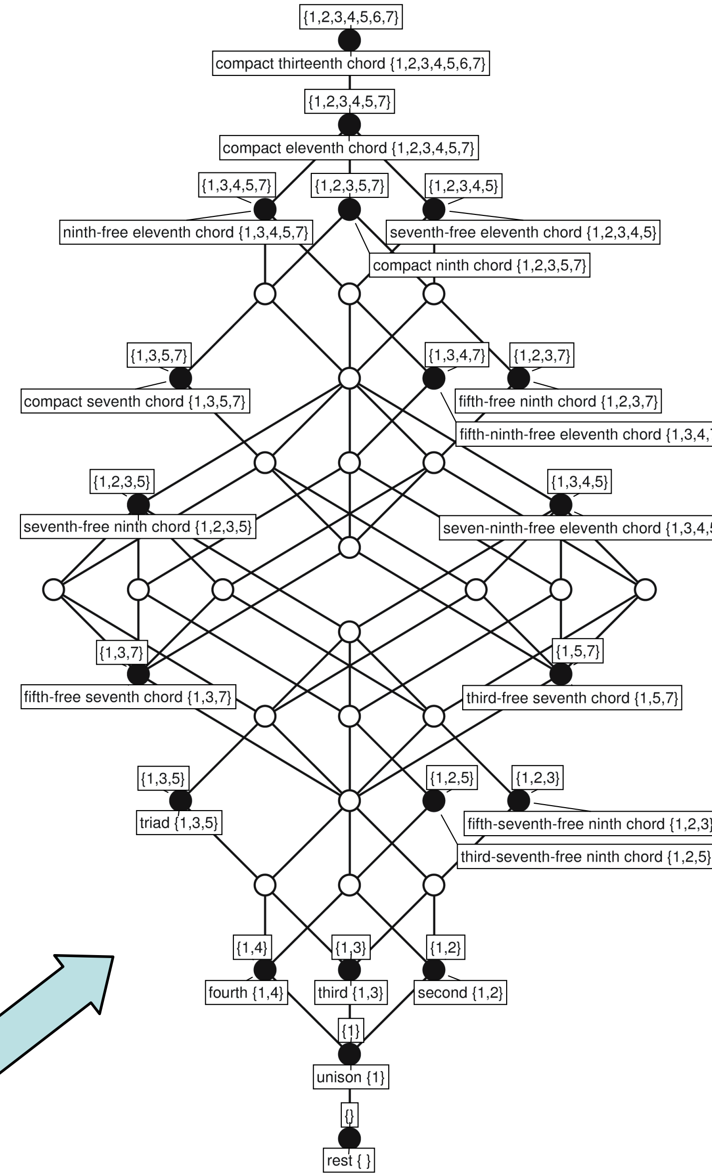
({*la*, *do*},{F_M, A_m})



[R. Wille & R. Wille-Henning, « Towards a Semantology of Music », ICCS 2007, Springer, 2007]

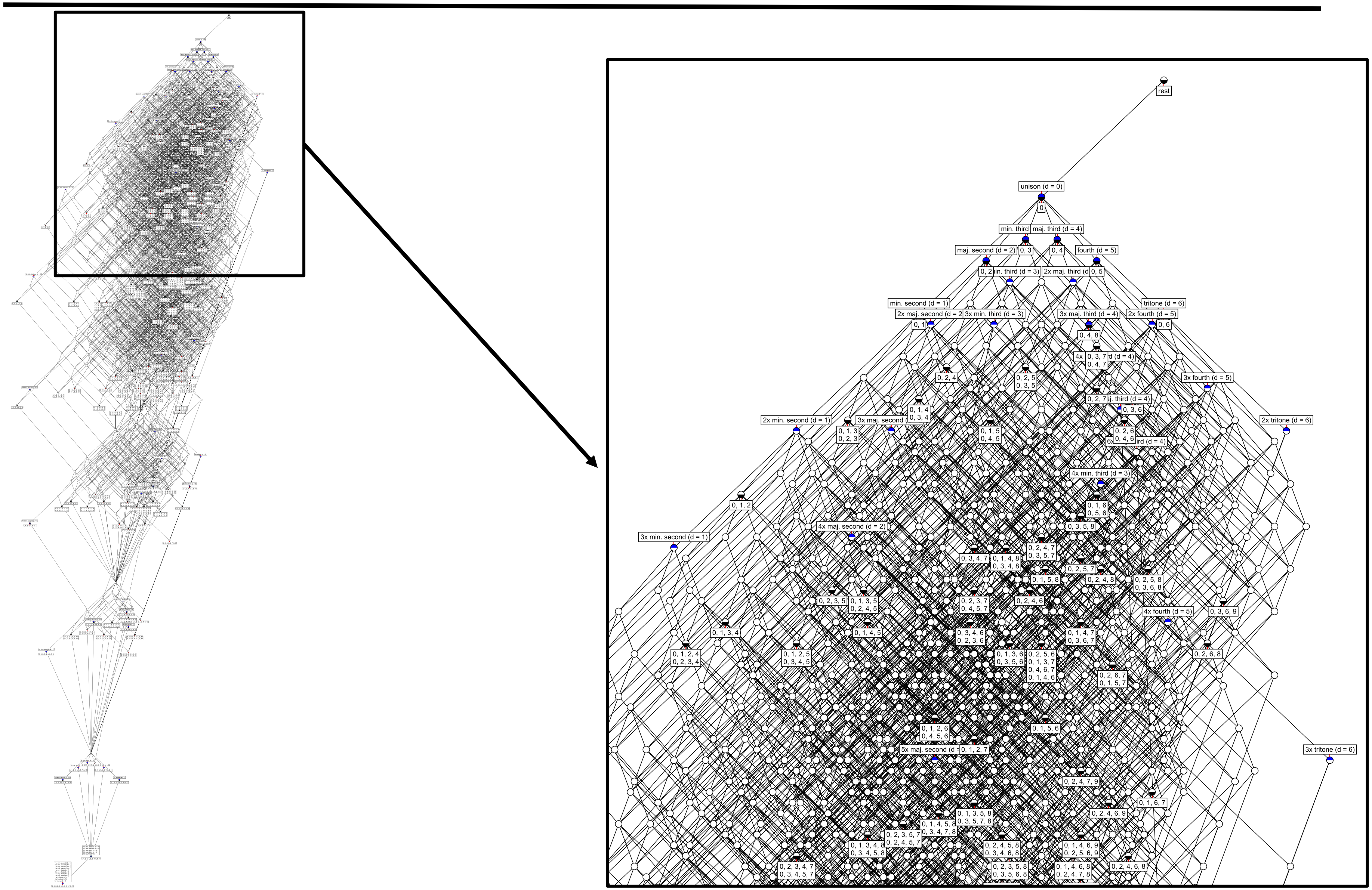
A different concept lattice for the diatonic scale

	{}	{1}	{1,2}	{1,3}	{1,4}	{1,2,3}	{1,3,7}	{1,2,5}	{1,5,7}	{1,3,5}	{1,3,5,7}	{1,2,3,5}	{1,3,4,7}	{1,3,4,5}	{1,2,3,7}	{1,3,4,5,7}	{1,2,3,5,7}	{1,2,3,4,5}	{1,2,3,4,5,7}	{1,2,3,4,5,6,7}
rest { }	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×
unison {1}		×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×
second {1,2}			×			×	×	×	×		×	×	×	×	×	×	×	×	×	×
third {1,3}				×		×	×		×	×	×	×	×	×	×	×	×	×	×	×
fourth {1,4}					×		×	×	×	×	×	×	×	×	×	×	×	×	×	×
fifth-seventh-free ninth chord {1,2,3}						×						×		×	×	×	×	×	×	×
fifth-free seventh chord {1,3,7}							×				×	×	×		×	×	×	×	×	×
third-seventh-free ninth chord {1,2,5}								×				×	×	×		×	×	×	×	×
third-free seventh chord {1,5,7}									×		×		×	×	×	×	×	×	×	×
triad {1,3,5}										×	×	×		×		×	×	×	×	×
compact seventh chord {1,3,5,7}											×					×	×		×	×
seventh-free ninth chord {1,2,3,5}												×				×	×	×	×	×
fifth-ninth-free eleventh chord {1,3,4,7}													×			×		×	×	×
seventh-ninth-free eleventh chord {1,3,4,5}														×		×	×	×	×	×
fifth-free ninth chord {1,2,3,7}															×		×	×	×	×
ninth-free eleventh chord {1,3,4,5,7}																×			×	×
compact ninth chord {1,2,3,5,7}																	×		×	×
seventh-free eleventh chord {1,2,3,4,5}																		×	×	×
compact eleventh chord {1,2,3,4,5,7}																			×	×
compact thirteenth chord {1,2,3,4,5,6,7}																				×



[R. Wille & R. Wille-Henning, « Towards a Semantology of Music », ICCS 2007, Springer, 2007]

How to reduce the combinatorial explosion?



• T. Schlemmer, S. E. Schmidt, « A formal concept analysis of harmonic forms and interval structures », *Annals of Mathematics and Artificial Intelligence* 59(2), 241– 256 (2010)

A lattice structure based on musical intervals

Core idea:

- Harmonic forms = objects
- Intervals = attributes

Harmonic system: $\mathbb{T} = (T, \Delta, \mathrm{I})$, with T = set of tones, I = musical intervals, and $\Delta : T \times T \to I$ s.t.

$$\forall (t_1, t_2, t_3), \Delta(t_1, t_2) + \Delta(t_2, t_3) = \Delta(t_1, t_3) \text{ and } \Delta(t_1, t_2) = 0 \text{ iff } t_1 = t_2$$

Here: $\mathbb{T}_n = (\mathbb{Z}_n, \Delta_n, \mathbb{Z}_n)$, where $n \in \mathbb{Z}_+$ represents an octave, $\mathbb{Z}_n = \mathbb{Z}/n\mathbb{Z}$, and Δ_n is the difference modulo n.

Harmonic forms $\mathcal{H}(\mathbb{T}_n)$: equivalence classes of Ψ:

$$\forall H_1 \subseteq \mathbb{Z}_n, \forall H_2 \subseteq \mathbb{Z}_n, \; H_1 \Psi H_2 \text{ iff } \exists i \text{ s.t. } H_1 = H_2 + i$$

where $H + i = \{t + i \mid t \in H\}$ if $t + i$ exists for all $t \in H$.

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Group action

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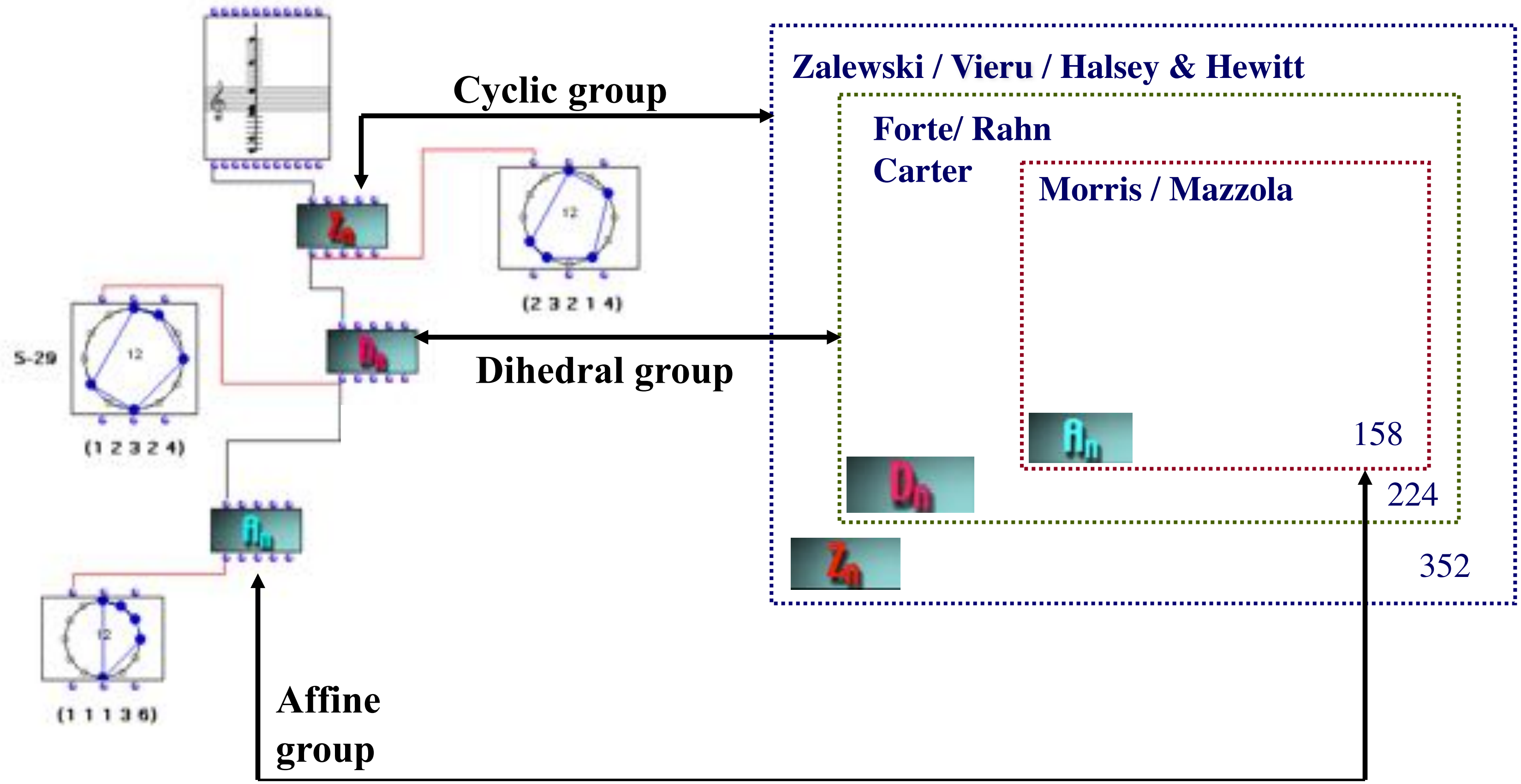
Group actions and the definition of harmonic forms



F. Klein

W. Burnside

G. Polya

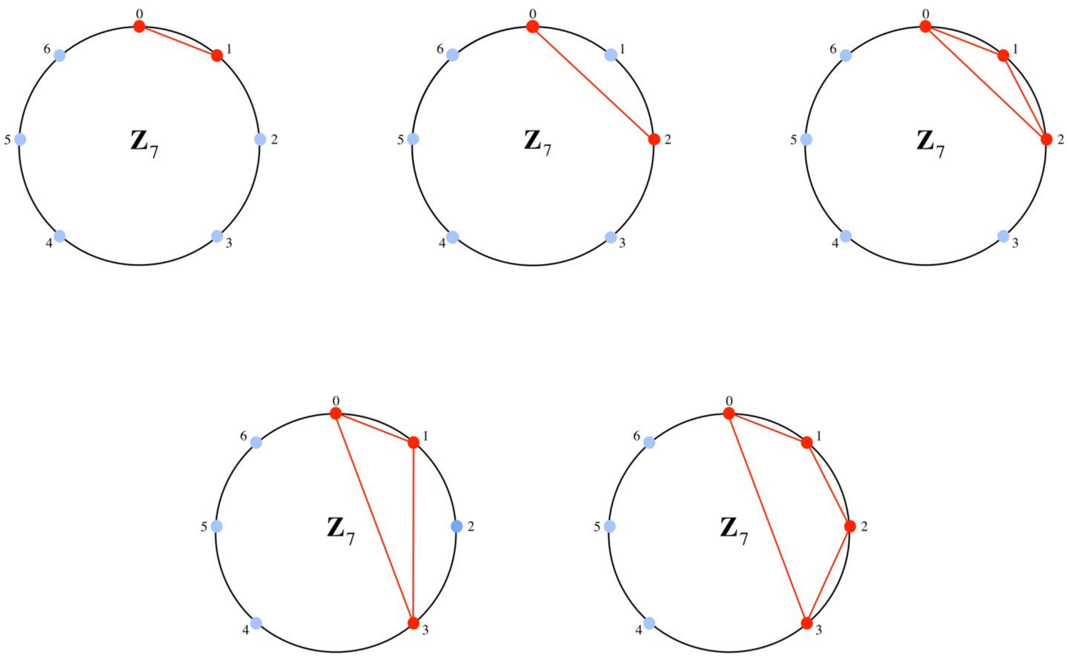
	1	2	3	4	5	6	7	8	9	10	11	12
Z_n	1	6	19	43	66	80	66	43	19	6	1	1
D_n	1	6	12	29	38	50	38	29	12	6	1	1
A_n	1	5	9	21	25	34	25	21	9	5	1	1

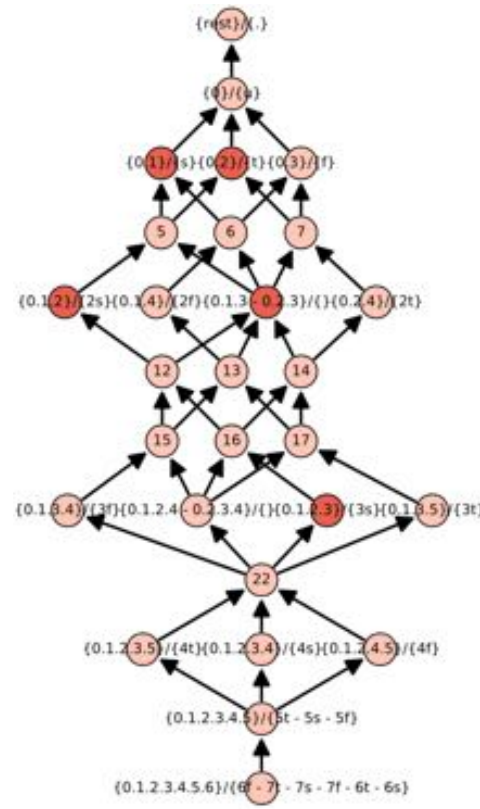
Reducing a concept lattice using congruences

Congruence: equivalence relation θ on a lattice $\mathcal{L}$, compatible with join and meet, i.e. $(\theta(a,b)$ and $\theta(c,d)) \Rightarrow (\theta(a \vee c, b \vee d)$ and $\theta(a \wedge c, b \wedge d))$, for all $a,b,c,d \in \mathcal{L}$.

Quotient lattice: $\mathcal{L}/\theta$

Example: congruence grouping the most common harmonic forms in a same equivalence class.





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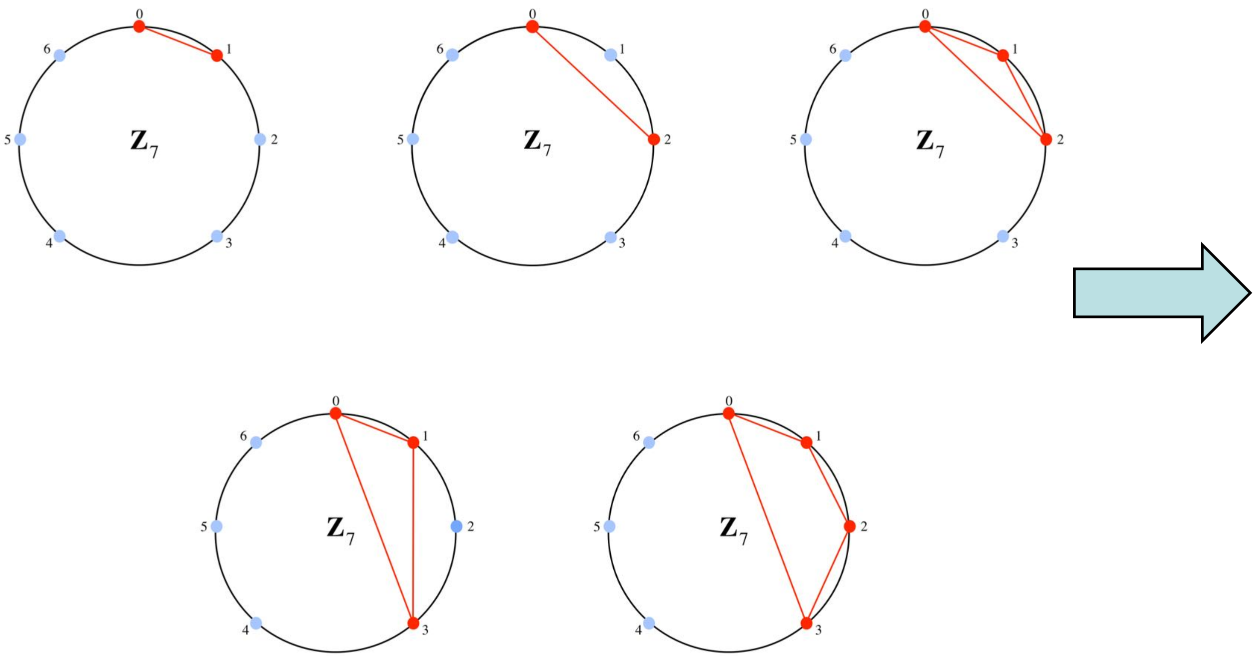

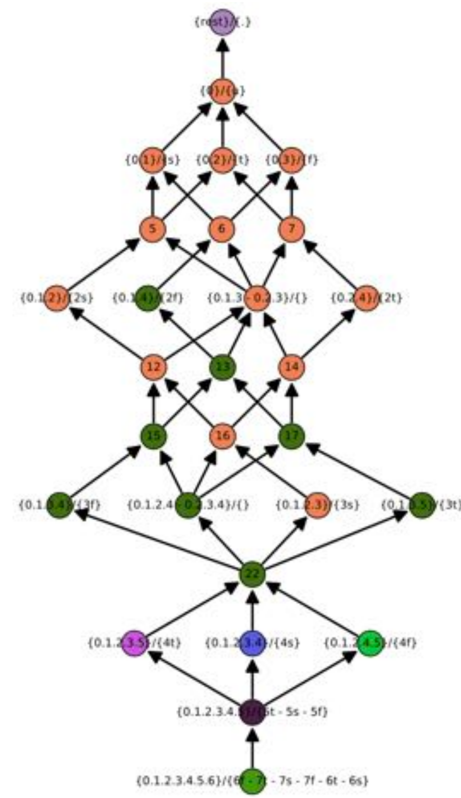

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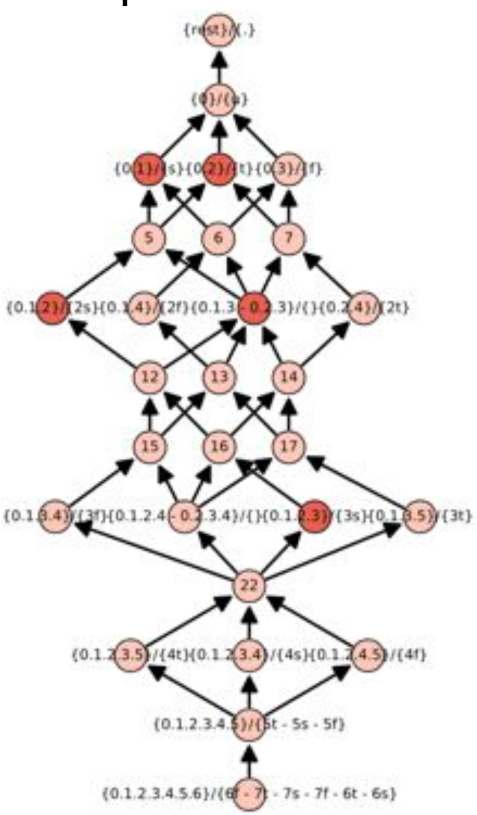
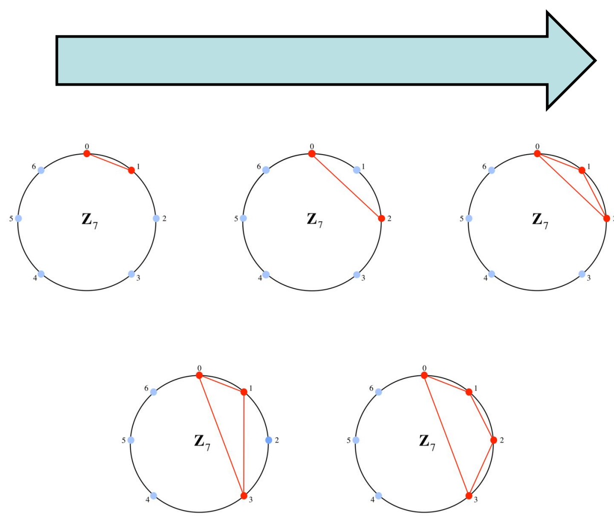
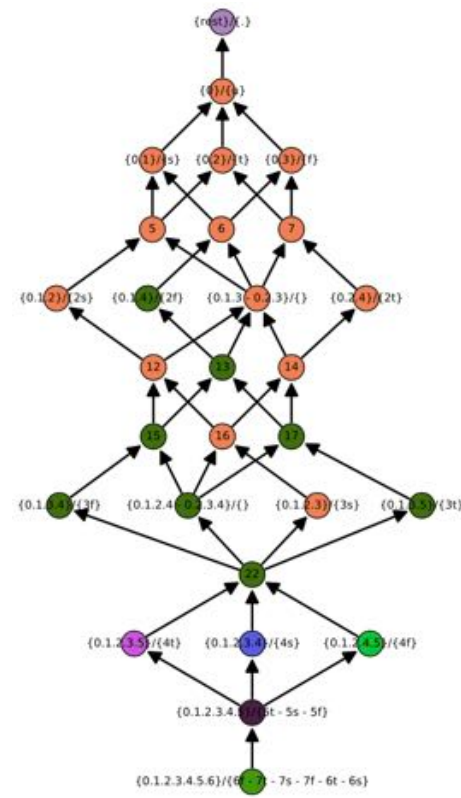




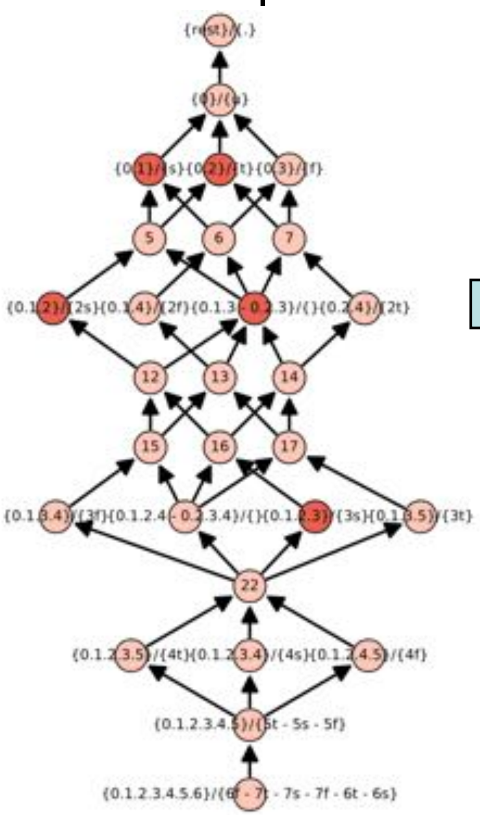


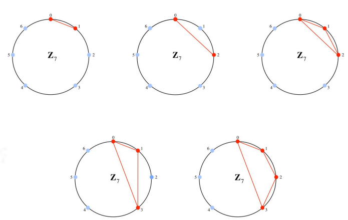
Reducing a concept lattice using congruences

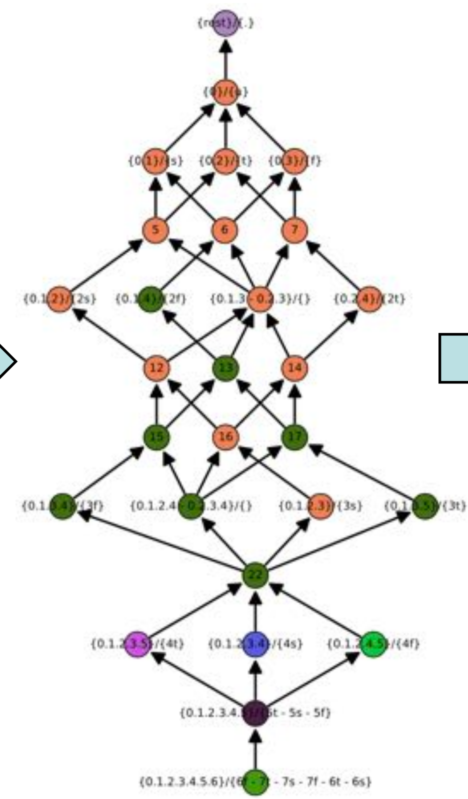
Congruence: equivalence relation θ on a lattice $\mathcal{L}$, compatible with join and meet, i.e. $(\theta(a,b)$ and $\theta(c,d)) \Rightarrow (\theta(a \vee c, b \vee d)$ and $\theta(a \wedge c, b \wedge d))$, for all $a,b,c,d \in \mathcal{L}$.

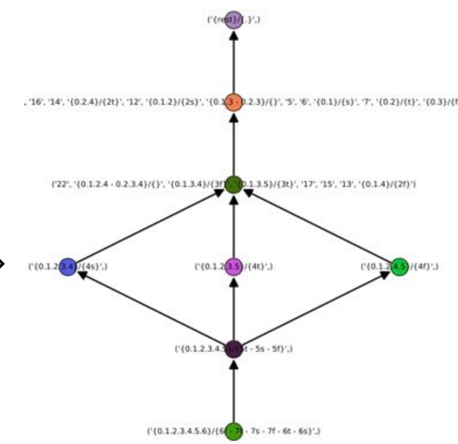
Quotient lattice: $\mathcal{L}/\theta$

Example: congruence grouping the most common harmonic forms in a same equivalence class.







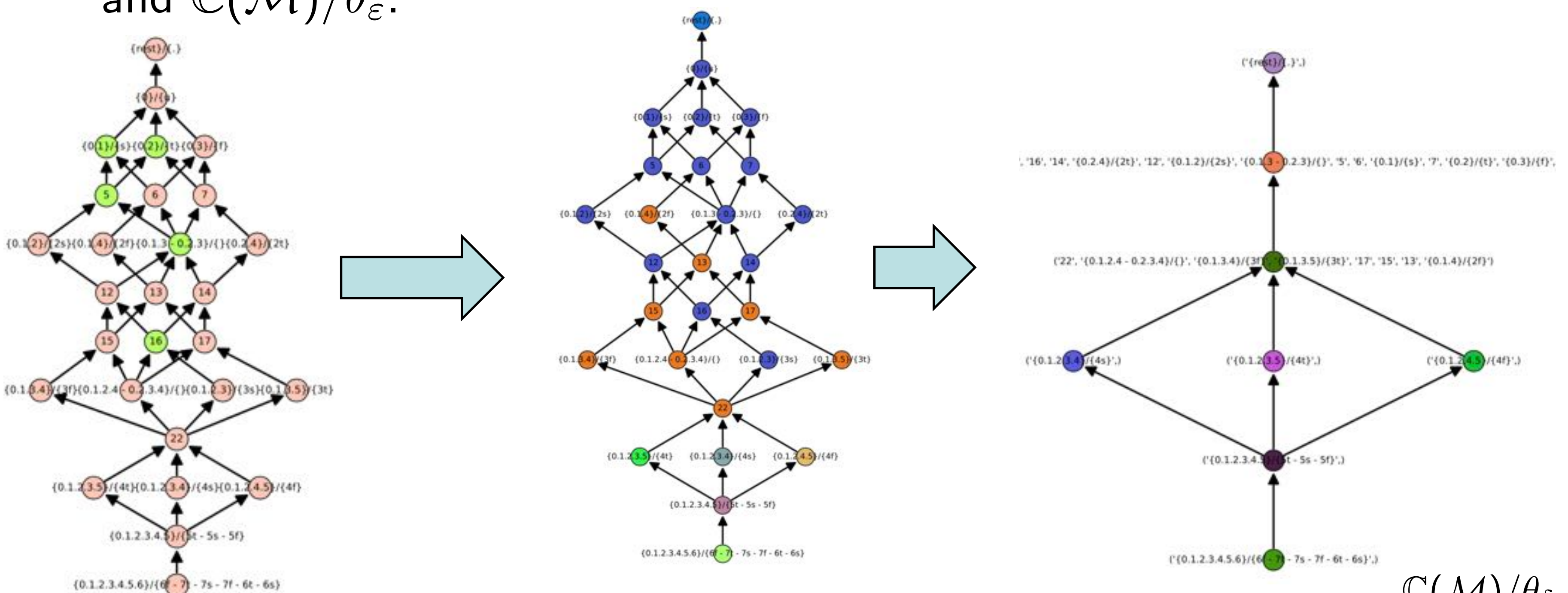


$\mathbb{C}(\mathcal{M})/\theta$

Reducing a concept lattice using congruences

Harmonico-morphological descriptors:

- Musical piece $\mathcal{M}$, harmonic system $\mathbb{T}_{\mathcal{M}}$, concept lattice $\mathbb{C}(\mathcal{M})$
- $H_{\mathbb{C}}^{\mathcal{M}}$: formal concepts corresponding to the harmonic forms in $\mathcal{M}$
  - θ grouping all formal concepts in $H_{\mathbb{C}}^{\mathcal{M}}$ into one same class;
  - θ_δ grouping all formal concepts in $\delta(H_{\mathbb{C}}^{\mathcal{M}})$ into one same class;
  - θ_ε grouping all formal concepts in $\varepsilon(H_{\mathbb{C}}^{\mathcal{M}})$ into one same class.
- Proposed harmonic descriptors: quotient lattices $\mathbb{C}(\mathcal{M})/\theta$, $\mathbb{C}(\mathcal{M})/\theta_\delta$, and $\mathbb{C}(\mathcal{M})/\theta_\varepsilon$.



$\mathbb{C}(\mathcal{M})/\theta_\delta$

Reducing a concept lattice using congruences

Harmonico-morphological descriptors:

- Musical piece $\mathcal{M}$, harmonic system $\mathbb{T}_{\mathcal{M}}$, concept lattice $\mathbb{C}(\mathcal{M})$
- $H_{\mathbb{C}}^{\mathcal{M}}$: formal concepts corresponding to the harmonic forms in $\mathcal{M}$
  - θ grouping all formal concepts in $H_{\mathbb{C}}^{\mathcal{M}}$ into one same class;
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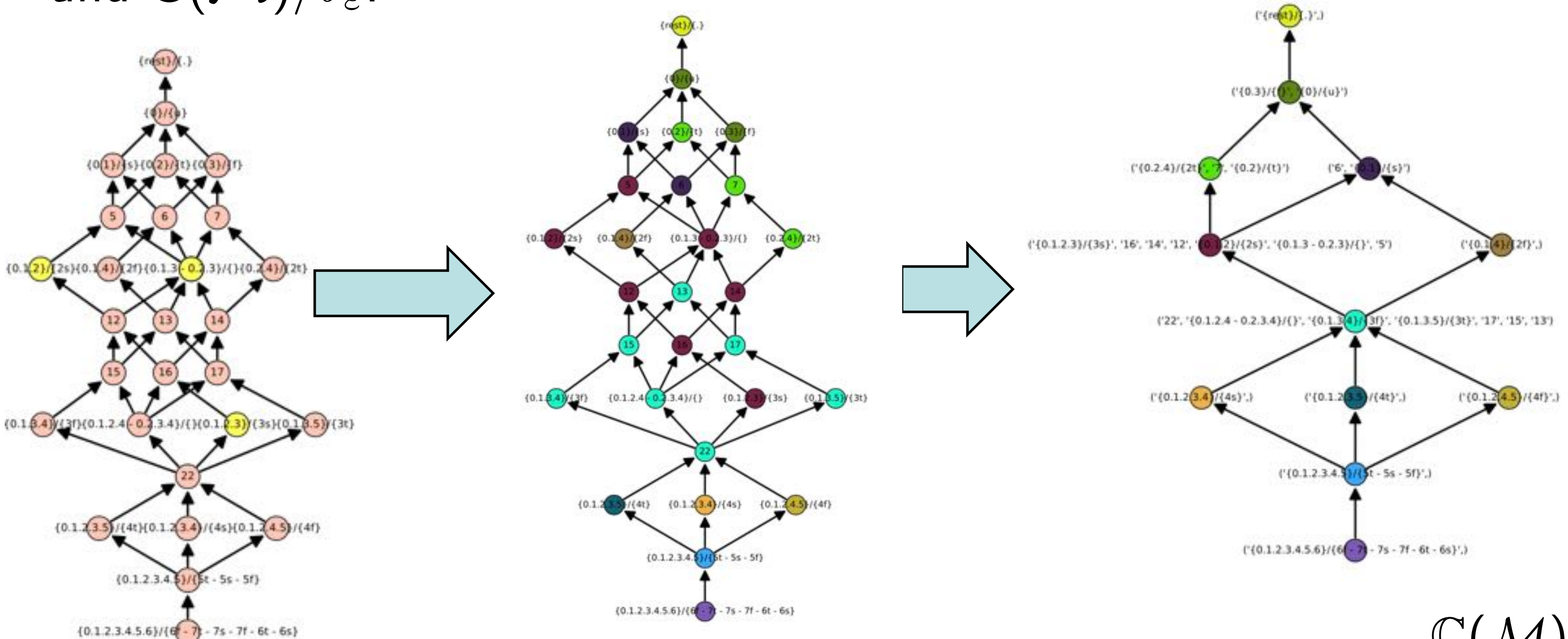
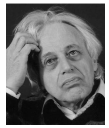
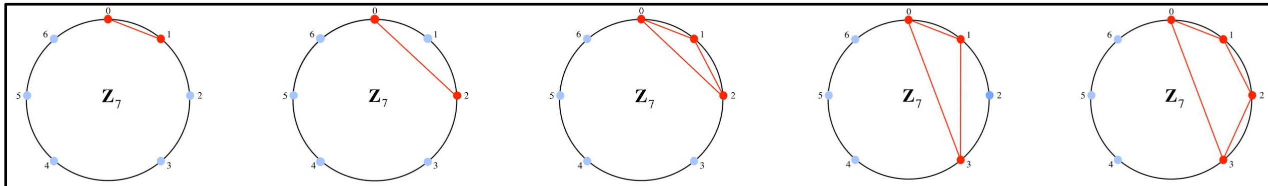


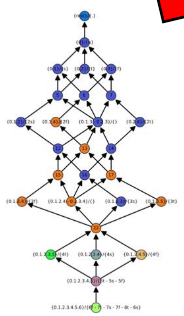
$\mathbb{C}(\mathcal{M})/\theta_\varepsilon$

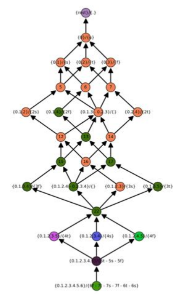
Summary of the lattice-reduction process

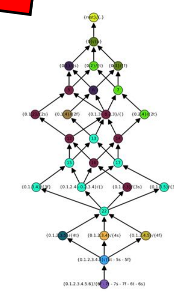


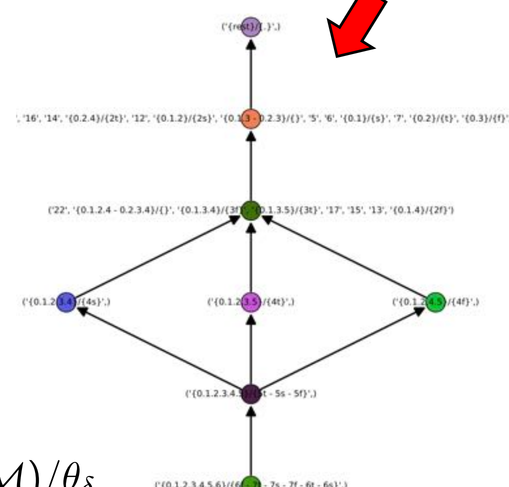


harmonic forms

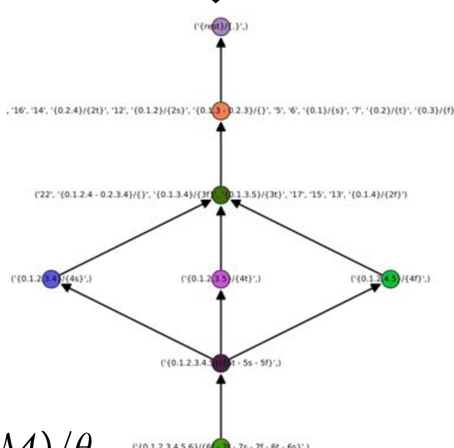




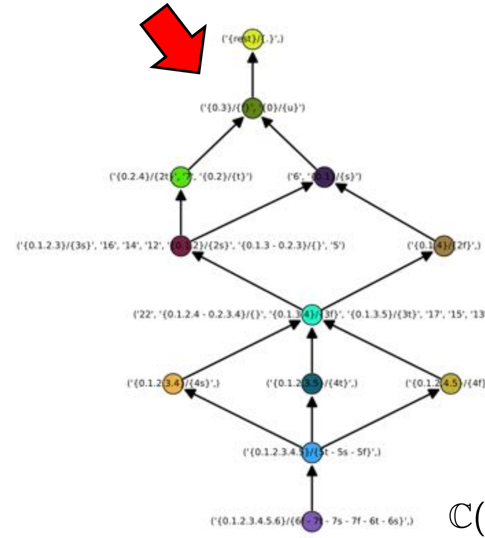




$\mathbb{C}(\mathcal{M})/\theta_\delta$

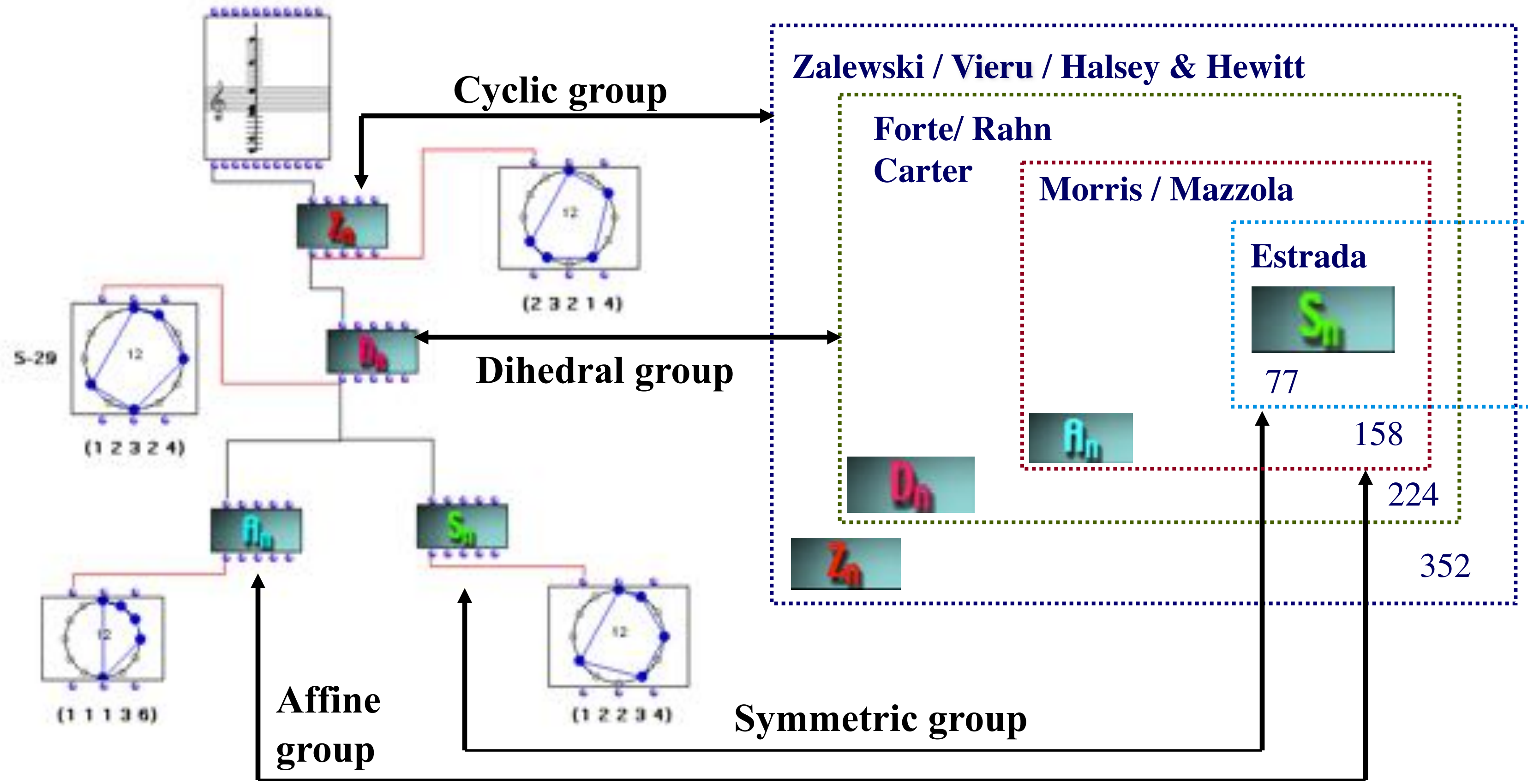


$\mathbb{C}(\mathcal{M})/\theta$

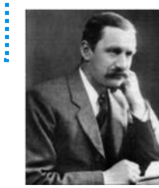


$\mathbb{C}(\mathcal{M})/\theta_\varepsilon$

The paradigmatic music classification approach




F. Klein


W. Burnside


G. Polya

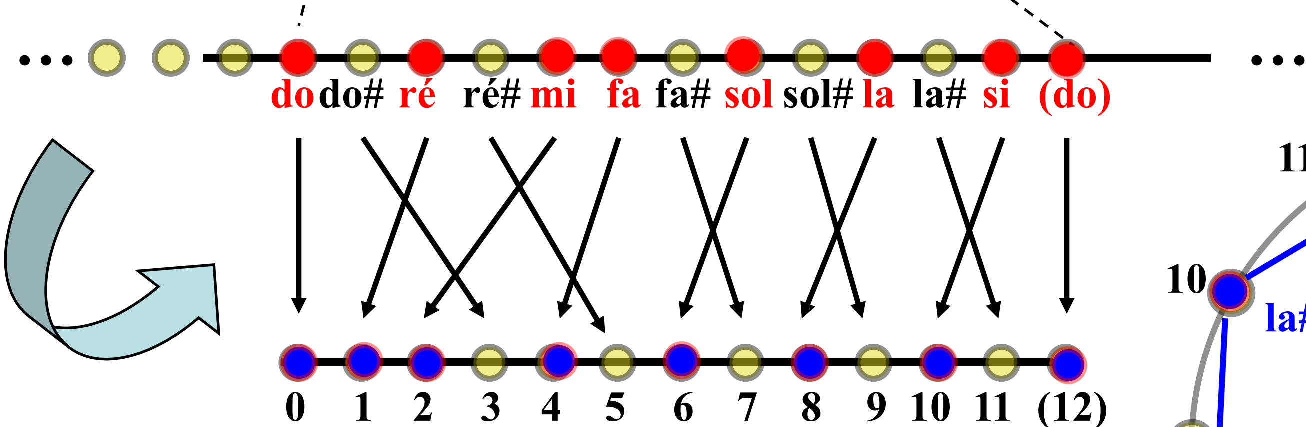
	1	2	3	4	5	6	7	8	9	10	11	12
Z_n	1	6	19	43	66	80	66	43	19	6	1	1
D_n	1	6	12	29	38	50	38	29	12	6	1	1
A_n	1	5	9	21	25	34	25	21	9	5	1	1
S_n	*1*	*6*	*12*	*15*	*12*	*11*	*7*	*5*	*3*	*2*	*1*	*1*

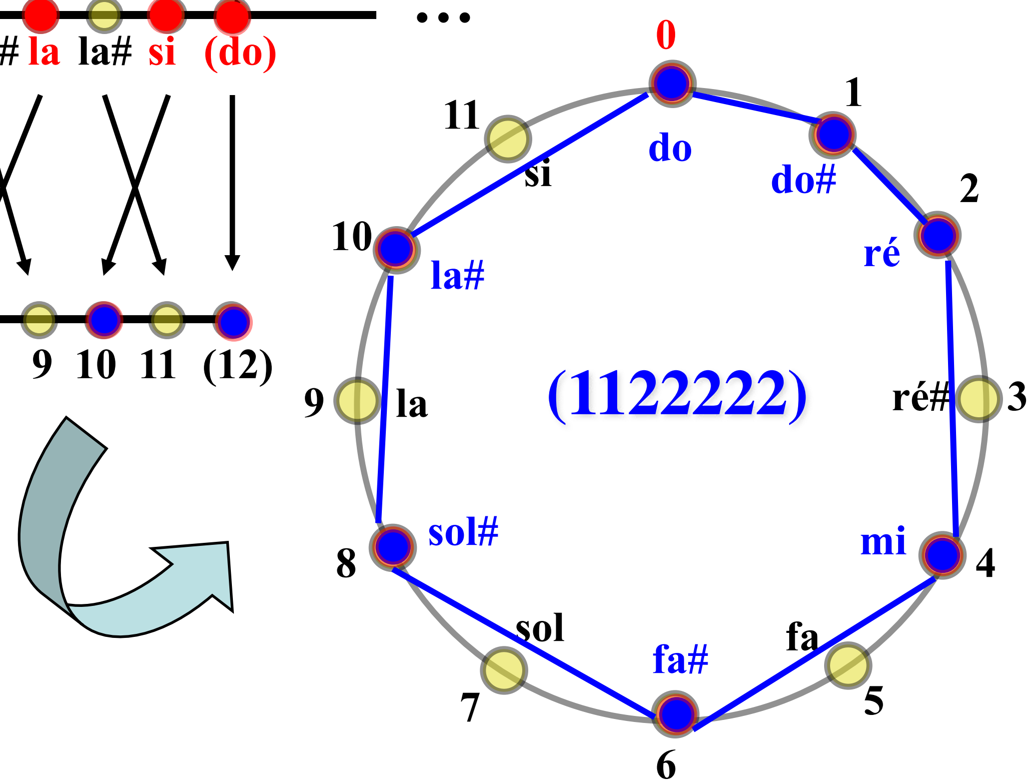
S_n acting on the intervallic structures

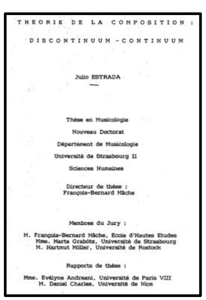
Permutations on the intervallic structures

$DIA = (2,2,1,2,2,2,1)$

$DIA_E = (1,1,2,2,2,2,2)$







THEORIE DE LA COMPOSITION :

DISCONTINUUM - CONTINUUM

Julio ESTRADA

Thèse en Musicologie

Nouveau Doctorat

Département de Musicologie

Université de Strasbourg II

Sciences Humaines

Directeur de thèse :
François-Bernard Mâche

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M. François-Bernard Mâche, Ecole d'Hautes Etudes
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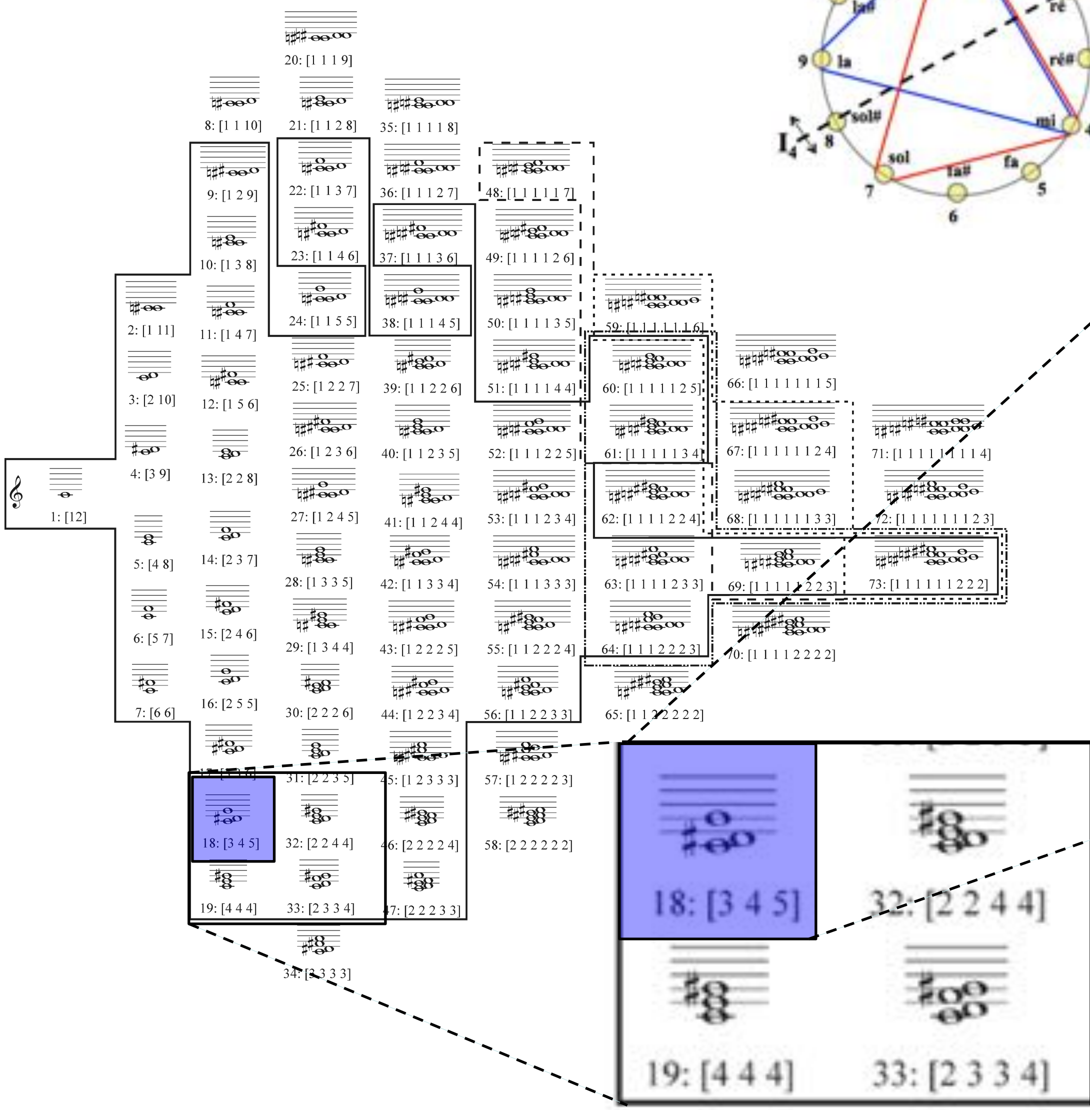
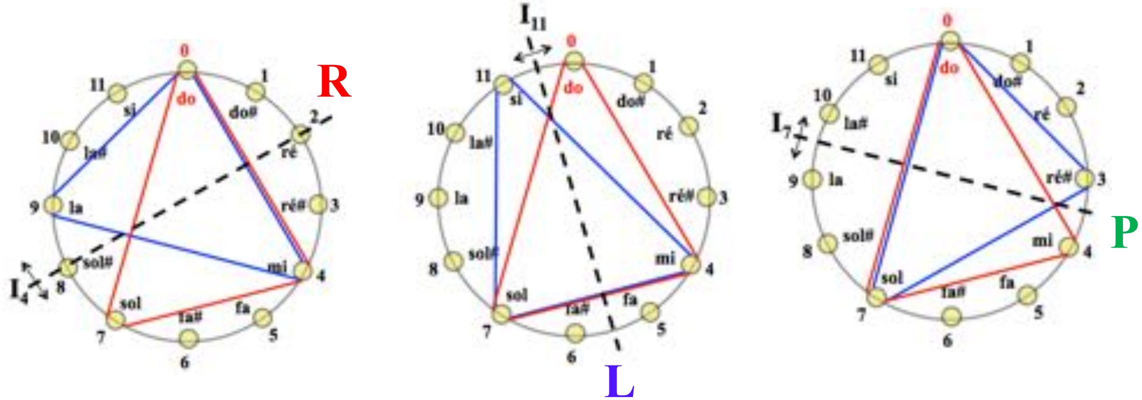
Rapports de thèse :

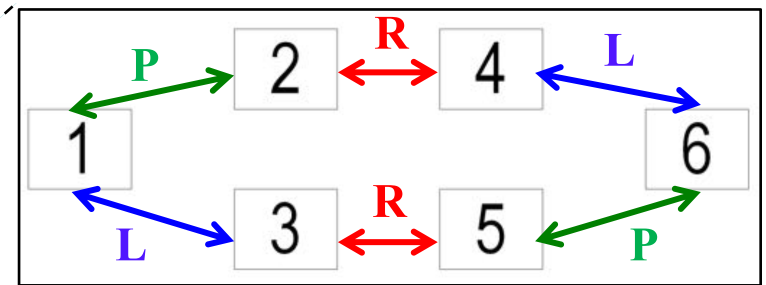
Mme. Evélyne Andreani, Université de Paris VIII
M. Daniel Charles, Université de Nice

Julio Estrada, ***Théorie de la composition : discontinuum – continuum,*** **université de Strasbourg II, 1994**

Permutohedron and *Tonnetz*: a structural inclusion







$\boxed{1} = \boxed{(3\ 4\ 5)}$

$\boxed{2} = \boxed{(4\ 3\ 5)}$

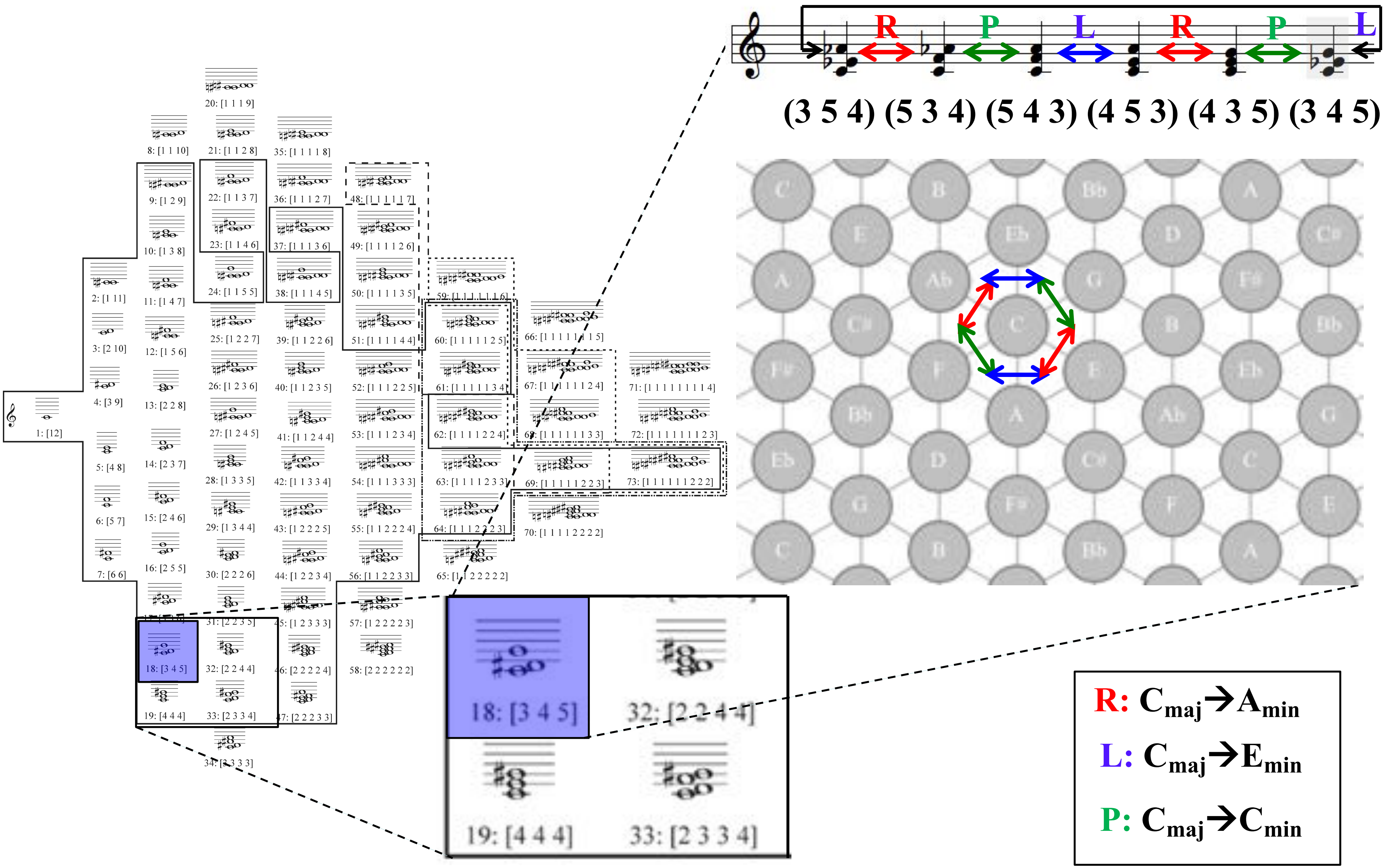
$\boxed{3} = \boxed{(3\ 5\ 4)}$

$\boxed{4} = \boxed{(4\ 5\ 3)}$

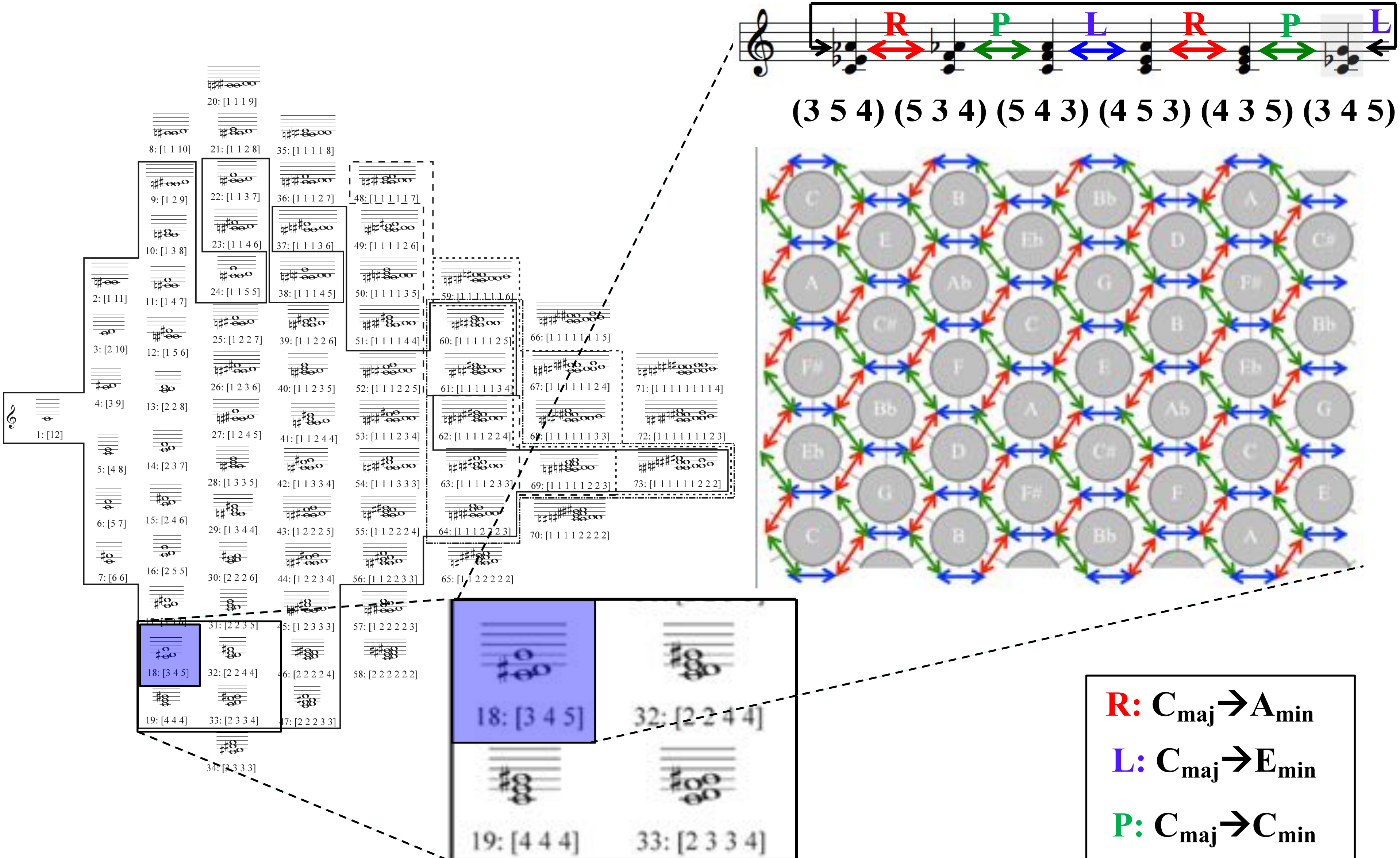
$\boxed{5} = \boxed{(5\ 3\ 4)}$

$\boxed{6} = \boxed{(5\ 4\ 3)}$

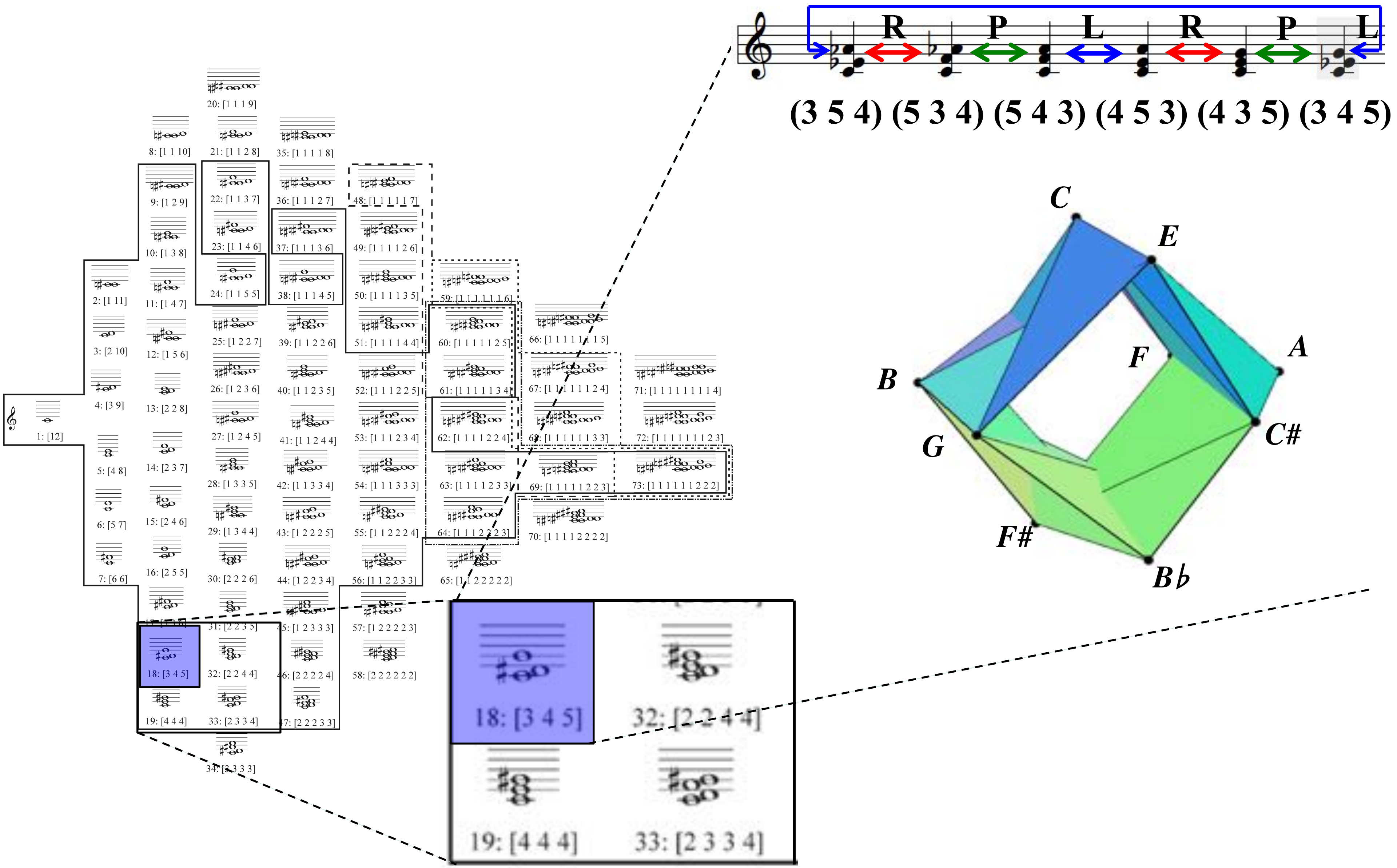
Permutohedron and *Tonnetz*: a structural inclusion



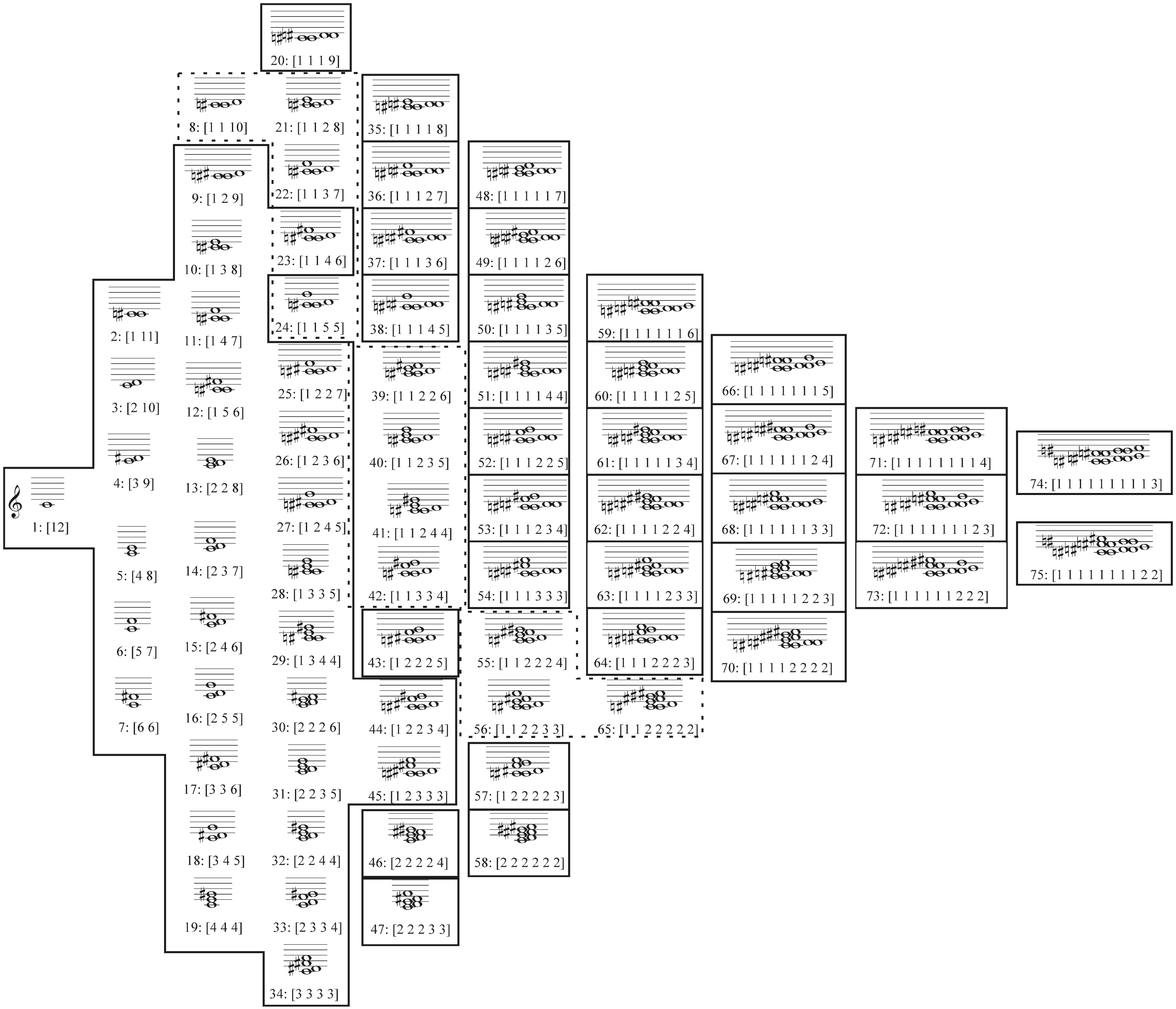
Permutohedron and *Tonnetz*: a structural inclusion

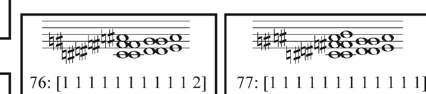


Permutohedron and a topological structural inclusion



The permutohedron as a musical conceptual space

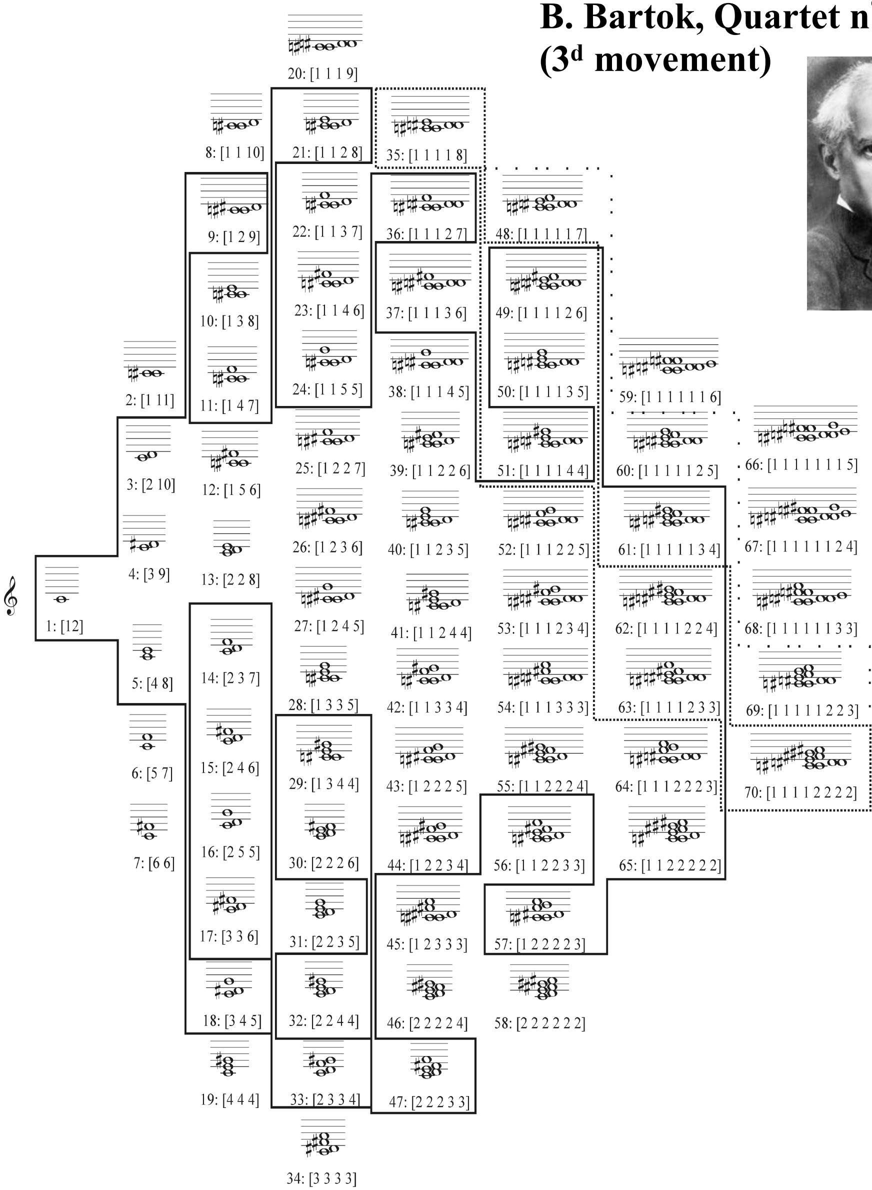




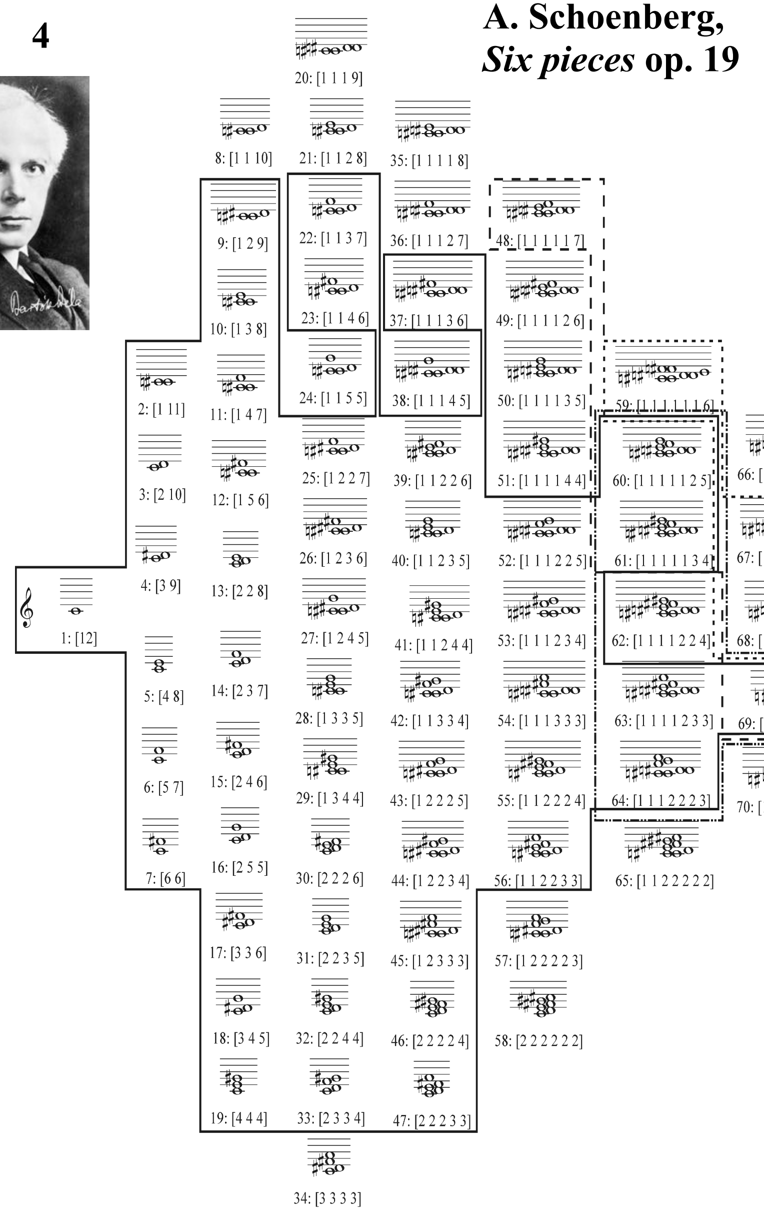
L. Van Beethoven,
Quatuor n° 17

The permutohedron as a musical conceptual space

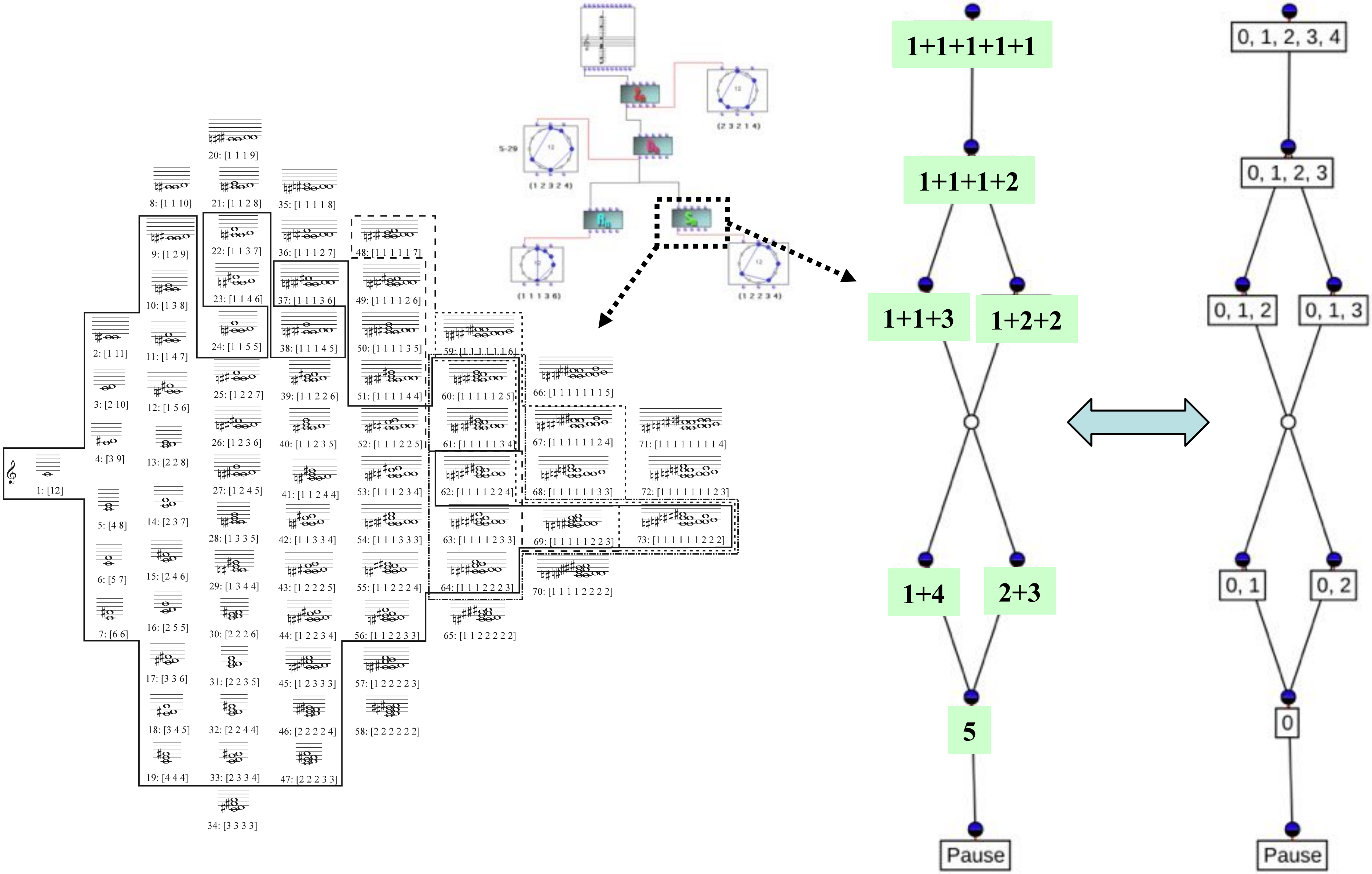
B. Bartok, Quartet n° 4 (3[d] movement)



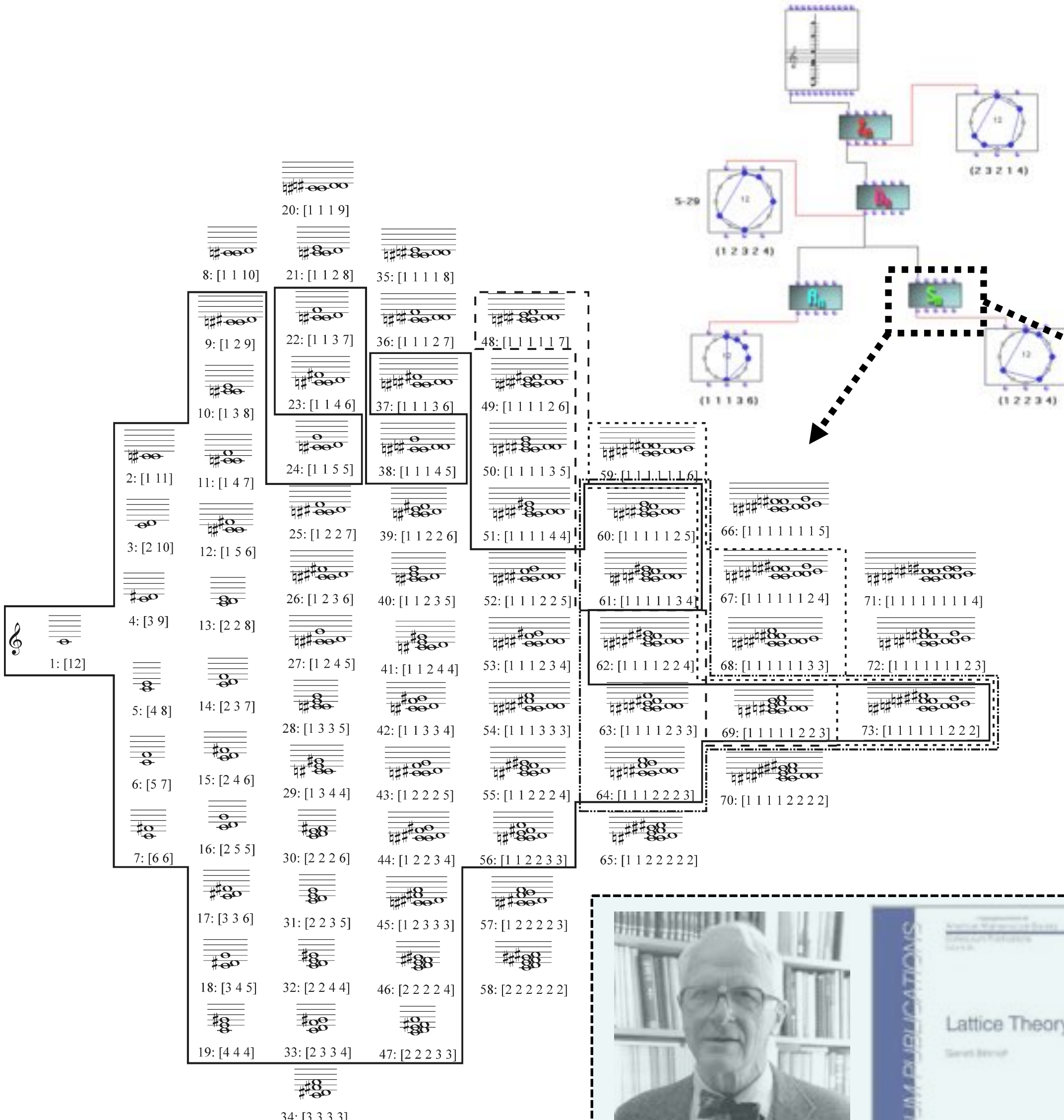
A. Schoenberg, *Six pieces* op. 19

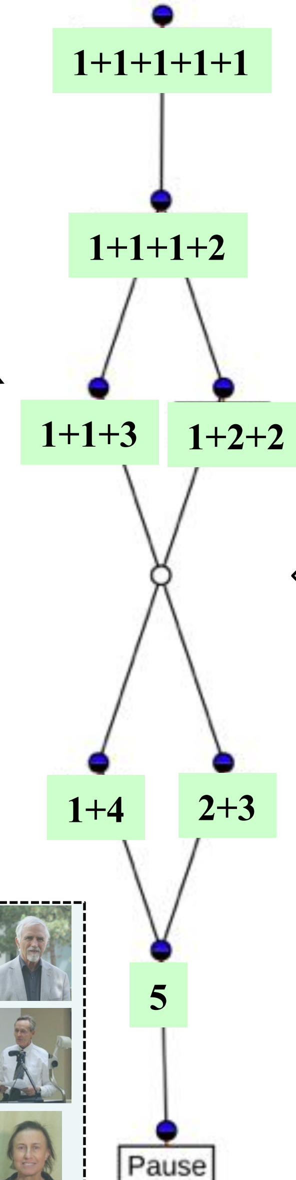


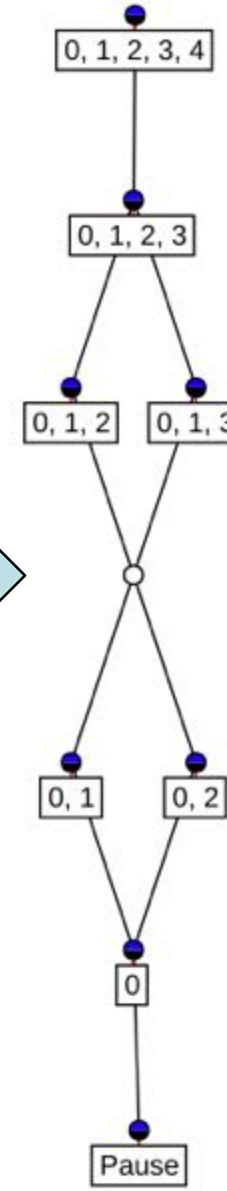

The permutohedron as a lattice of formal concepts

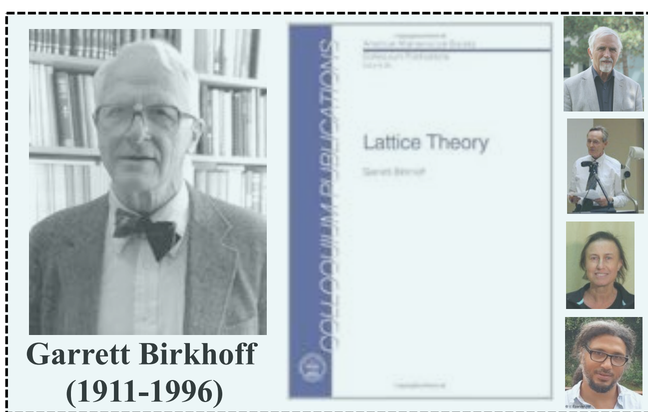


Using MM/FCA to have a ‘signature’ of a musical piece









Garrett Birkhoff
(1911-1996)

THANK YOU FOR YOUR ATTENTION